ELEMENTI DI PROBABILITA’ E STATISTICA

[Formulario Esame]

A CURA DI ALESSANDRO PAGHI

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LINK AL CORSO:
http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=54946&aa=2014

FREQUENTAZIONE: Sconsigliata.

V.A. BERNOULLI

$X = \begin{cases} 1 & \text{successo} \\ \emptyset & \text{fallimento} \end{cases}$

$P(X=1) = p$

$P(X=\emptyset) = 1-p = q$

$E[X] = p$

$E[X^2] = p$

$Var[X] = pq$

$E[X^m] = p$

V.A. BINOMIALE

k successi in N prove

N ripetizione di Bernoulli

X v.a.: "# successi"

$P(X=k) = \binom{N}{k} p^k q^{N-k}$, $\binom{N}{k} = \frac{n!}{k!(n-k)!}$

$\sum_i p(X=x_i) = 1$

$E[X] = np$

$Var[X] = npq$

V.A. POISSON

N # di prove

p probabilità successo

$N \to +\infty$, $p \to \emptyset$

$P(X=k) = \frac{\mu^k}{k!} e^{-\mu}$

$\sum_{k=\emptyset}^{\infty} \frac{\mu^k}{k!} e^{-\mu} = 1$

$E[X] = \mu$

$Var[X] = \mu$

V.A. ESPONENZIALE

$p_X(x) = \begin{cases} \lambda e^{-\lambda x} & \text{se } x \geq 0 \\ \emptyset & \text{se } x < 0 \end{cases}$

$F_X(x) = 1 - e^{-\lambda x}$, $x \geq 0$

$E[X] = 1/\lambda$

$Var[X] = 1/\lambda^2$

$\int_0^{+\infty} p_X(x)\,dx = 1$

V.A. UNIFORME

intervallo $[\alpha, \beta] \in \mathbb{R}$

$p_X(x) = \begin{cases} 1/(\beta-\alpha) & \text{se } X \in D \\ \emptyset & \text{se } X \notin D \end{cases}$

$\int_{-\infty}^{+\infty} p_X(x)\,dx = 1$

$\frac{1}{\beta-\alpha}\int_\alpha^\beta dx = 1$

$E[X] = \alpha+\beta/2$

$E[X^2] = \alpha^2+\alpha\beta+\beta^2/2$

$Var[X] = (\beta-\alpha)^2/12$

V.A. GAUSSIANA

$X \sim N(\mu, \sigma^2)$

$p_X(x) = \frac{1}{\sqrt{2\pi}\cdot\sigma} \cdot e^{-(x-\mu)^2/2\sigma^2}$

$E[X] = \mu$

$E[X^2] = \sigma^2 + \mu^2$

$Var[X] = \sigma^2$

$G(k) = E[e^{ikx}] = e^{ik\mu} \cdot e^{(ik\sigma)^2/2}$

$F_X(x) = \int_{-\infty}^{x} p_X(x)\,dx = \frac{1}{2} + erf(x)$

$\hookrightarrow X \sim N(0,1)$

$erf(x) = \int_\emptyset^x \frac{1}{\sqrt{2\pi}} e^{-\frac{z^2}{2}} dz$, $z = \frac{x-\mu}{2}$

$N(\emptyset, \sigma^2)$

- MOMENTI PARI: $\bar{\mu}_{2m} = (2m-1)!!\,\sigma^{2m} = E[X^{2m}]$

$P(E \cup F) = P(E) + P(F) - P(E \cap F)$

EVENTI INDIPENDENTI

$P(A \cap B) = P(A) \cdot P(B)$

EVENTI DIPENDENTI

$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{P(B|A)\,P(A)}{P(B)}$

T. BAYES

$A \subseteq H_i$ e $H_i \cap H_j = \emptyset \ \forall i,j$

$P(A) = \sum_i P(A|H_i)\,P(H_i)$

V.A. CONTINUA

$P(x < X < x+dx) = p_X(x)\,dx$

$P(x_1 < X < x_2) = \int_{x_1}^{x_2} p_X(x)\,dx$

PROBABILITÀ CUMULATIVA

$P(X < x) = F_X(x)$

$F_X(x) = \int_{estr.\,inf}^{x} p_X(x)\,dx \Rightarrow p_X(x) = F'_X(x)$

$P(X > x) = 1 - P(X < x) = 1 - F_X(x)$

INDICI DI CENTRALITÀ

- VALORE ATTESO

$E[X] = \int x\, p_X(x)\, dx$

- MEDIANA

$\int_{\text{estr. inf.}}^{x_{1/2}} p_X(x)\,dx = \int_{x_{1/2}}^{\text{estr. sup.}} p_X(x)\,dx = F_X(x_{\frac{1}{2}}) = \frac{1}{2}$

- MODA

$p_X'(x) = \varnothing$

FUNZIONE CARATTERISTICA

$G(k) = E[e^{ikx}] = \int e^{ikx} p_X(x)\,dx$

$= \sum_{m=0}^{+\infty} \mu_m \frac{(ik)^m}{m!}$

ALTRO:

- X, Y indipendenti

$\Rightarrow E[XY] = E[X] \cdot E[Y]$

$\Rightarrow \mathrm{Cov}[XY] = \varnothing$

- $\mathrm{Cov}[XY] = \mathrm{cov}[YX]$

- $\mathrm{Var}[X+b] = \mathrm{Var}[X]$

$\mathrm{Var}[aX] = a^2 \mathrm{Var}[X]$

- $E[b] = b$

$E[aX] = a E[X]$

- X, Y indipendenti

$\Rightarrow p(x,y) = p_X(x)\, p_Y(y)$

- $Y = |X|$

Trovo $F_X(x)$

$P(Y < y) = P(|X| < y) = P(-y < X < y) = F_X(y) - F_X(-y)$.

$\int_0^{+\infty} x^m e^{-\lambda x}\,dx = m!/\lambda^{m+1}$

$\int_0^{+\infty} x^m e^{-x/\lambda}\,dx = m!\,\lambda^{m+1}$

$\sum_{m=0}^{\infty} z^m = \frac{1}{1-z}$; $\sum_{m=0}^{\infty} m z^m = \frac{z}{(1-z)^2}$; $\sum_{m=0}^{\infty} m^2 z^m = z \cdot \frac{z+1}{(1-z)^3}$

- $\int_{-\infty}^{+\infty} \frac{|x| e^{-x^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\,dx = 2\int_0^{+\infty} \frac{x\, e^{-x^2/2\sigma^2}}{\sqrt{2\pi}\,\sigma}\,dx$

INDICATORI DI DISPERSIONE

- VARIANZA

$\mathrm{Var}[X] = E[X^2] - E[X]^2$

- COVARIANZA

$\mathrm{Cov}[XY] = E[XY] - E[X]\,E[Y]$

- MOMENTI DI X

$E[X^m] = \int x^m p_X(x)\,dx$

- MOMENTI CENTRATI

$\mu_m^c = E[(X-\mu_1)^m] = \int (x-\mu_1)^m p_X(x)\,dx$

F. GENERATRICE CUMULANTI

$\log G(k) = \sum_{m=1}^{+\infty} c_m \frac{(ik)^m}{m!}$

$c_1 = \mu_1$

$c_2 = \mu_2 - \mu_1^2 = \mathrm{Var}[X]$

$c_3 = \mu_3^c = \mu_3 - 3\mu_2\mu_1 + 2\mu_1^3$

$c_4 = \mu_4^c - 3(\mu_2^c)^2$

- $p_X(x) = \int dy\, p(x,y)$

$p_Y(y) = \int dx\, p(x,y)$

- QUARTILI

$F(x_{1/4}) = 1/4$

- $p_{X|Y}(x|y) = \frac{p(x,y)}{p_Y(y)}$

$p_{Y|X}(y|x) = \frac{p(x,y)}{p_X(x)}$

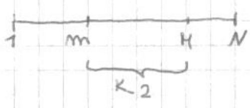
- min $\lambda_1, \lambda_2, \lambda_3$

ancora va

$\lambda = \lambda_1 + \lambda_2 + \lambda_3$

- MATRICE COVARIANZE

$\begin{pmatrix} \mathrm{Var}[X] & \mathrm{Cov}[XY] \\ \mathrm{Cov}[YX] & \mathrm{Var}[Y] \end{pmatrix}$

- 

$= \frac{\binom{M-m}{K-2}}{\binom{N}{K}}$

- $\sum_{m=0}^{\infty} q^m = \frac{1}{1-q}$ $p < 1$.

- $P(A) = 2P(B)$ $\Rightarrow P(A)$

$\frac{N}{2} P(A) + \frac{N}{2} P(B) = 1$.

pari / dispari

- $p_{T_1+T_2}(z) = \int_0^t p_{T_1}(z-w)\, p_{T_2}(w)\,dw$