SISTEMI DI CONTROLLO

[Appunti Di Esercizi, Parte 1]

A CURA DI ALESSANDRO PAGHI

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LINK AL CORSO ANNO 2015/2016:
http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=55078&aa=2015

FREQUENTAZIONE: Consigliata.

17/11/2014 TRACCIA #1

ES. 1

$$G(s) = \frac{Y(s)}{R(s)} = ?$$

$A = \frac{1}{s+1}$

$B = \frac{1}{s}$

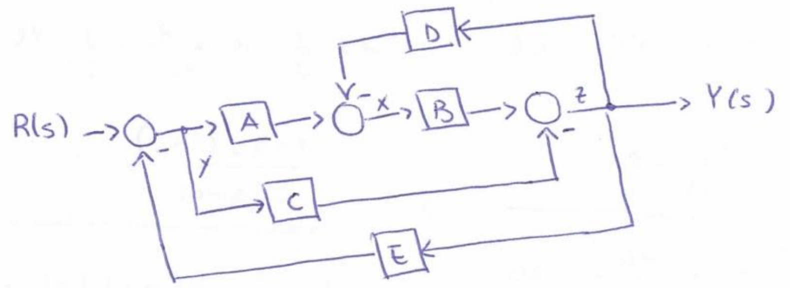
$C = 2$

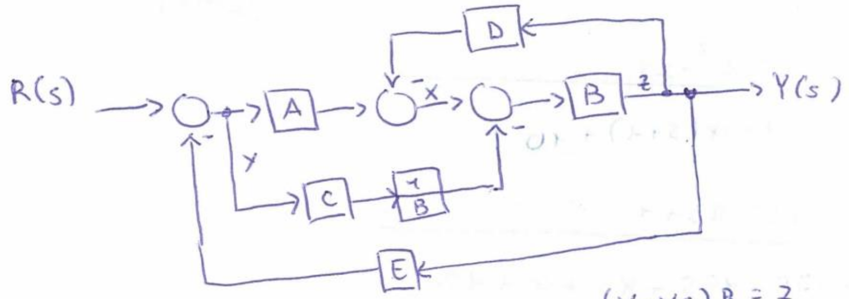
$D = \alpha$

$E = 10$

$\alpha : G(s)$ A.S.

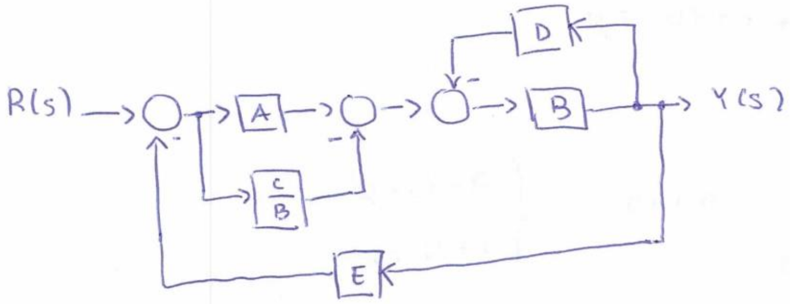
$xB - yC = z$

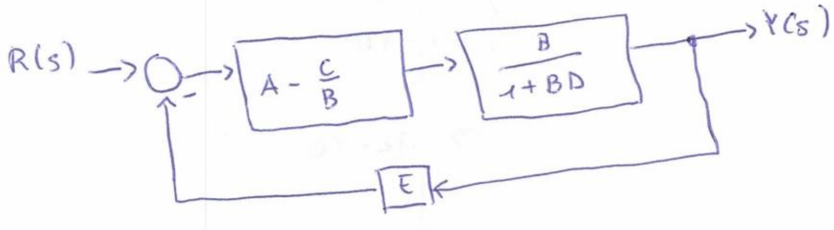


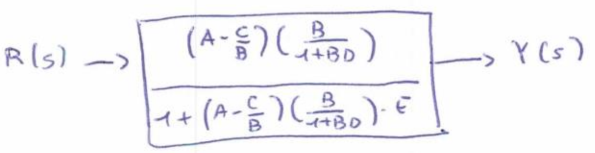


$(x - yC)B = z$

$xB - yCB = z$







$$G(s) = \frac{\left(A - \frac{C}{B}\right)\cdot\left(\frac{B}{1+BD}\right)}{1 + \left(A - \frac{C}{B}\right)\left(\frac{B}{1+BD}\right)E} = \frac{\frac{AB-C}{\cancel{B}}\cdot\frac{\cancel{B}}{1+BD}}{1 + \frac{AB-C}{\cancel{B}}\cdot\frac{\cancel{B}E}{1+BD}} = \frac{\frac{AB-C}{\cancel{1+BD}}}{\frac{1+BD+ABE-CE}{\cancel{1+BD}}} = \frac{AB-C}{1+BD+ABE-CE}$$

$$G(s) = \frac{AB - C}{1 + BD + ABE - CE} = \frac{\frac{1}{s+1} \cdot \frac{1}{s} - 2}{1 + \frac{1}{s} \cdot \alpha + \frac{1}{s+1} \cdot \frac{1}{s} \cdot 10 - 2 \cdot 10}$$

$$= \frac{\frac{1}{s(s+1)} - 2}{1 + \frac{\alpha}{s} + \frac{10}{s(s+1)} - 20} = \frac{\frac{1 - 2s(s+1)}{s(\cancel{s+1})}}{\frac{s(s+1) + \alpha(s+1) + 10 - 20s(s+1)}{s(\cancel{s+1})}}$$

$$= \frac{1 - 2s^2 - 2s}{-19s(s+1) + \alpha(s+1) + 10}$$

$$= \frac{-2s^2 - 2s + 1}{-19s^2 - 19s + \alpha s + \alpha + 10}$$

$$d(s) = -19s^2 + s(\alpha - 19) + \alpha + 10 = \emptyset$$

$$\begin{array}{c|cc} n & & \\ 2 & -19 & \alpha + 10 \\ 1 & \alpha - 19 & \\ \emptyset & \alpha + 10 & \end{array} \qquad \begin{cases} \alpha - 19 < \emptyset \\ \alpha + 10 < \emptyset \end{cases}$$

$$\begin{cases} \alpha < 19 \\ \alpha < -10 \end{cases}$$

$$\Rightarrow \alpha < -10$$

ES. 2

$$G(s) = 4000 \cdot \frac{(s+10)^2}{(s^2+2500)(s-1)(s+200)} = \frac{\cancel{4000}^{4} \cdot \cancel{100}}{\cancel{200} \cdot \underset{5}{\cancel{2500}}} \cdot \frac{\left(1+\frac{s}{10}\right)^2}{(1-s)\left(1+\frac{s}{200}\right)\left(1+\frac{s^2}{2500}\right)}$$

$$= -\frac{4}{5} \cdot \frac{\left(1+\frac{s}{10}\right)^2}{(1-s)\left(1+\frac{s}{200}\right)\left(1+\frac{s^2}{2500}\right)}$$

$K_G = -\frac{4}{5}$

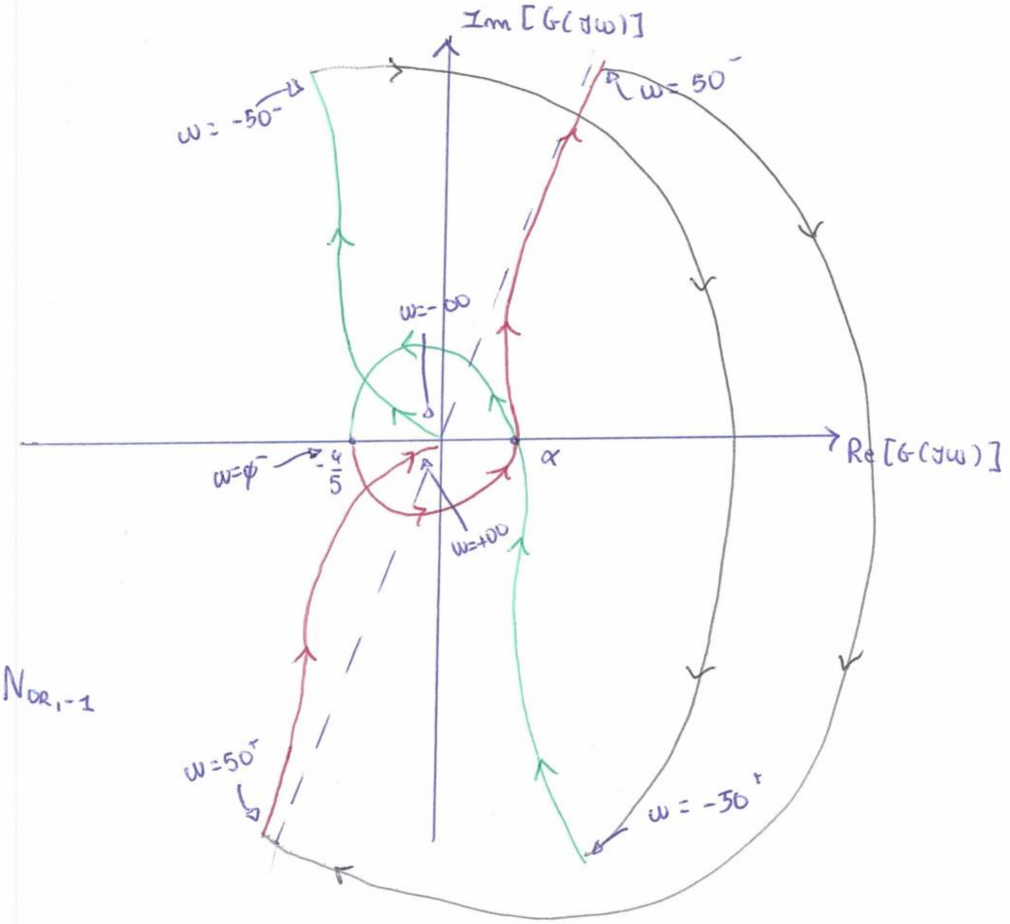
$z_{1/2} = -10$

$p_1 = 1$

$p_2 = -200$

$\omega_m = 50$

$\zeta = \emptyset$



$$N_{Pd}^{AC} = \underset{\underset{1}{\parallel}}{N_{Pd}^{AA}} + N_{OR,-1}$$

$-\alpha = -\frac{4}{5}$

$K > \emptyset$: $K > \frac{5}{4}$: STABILE

$K < \frac{5}{4}$: 1 Pd

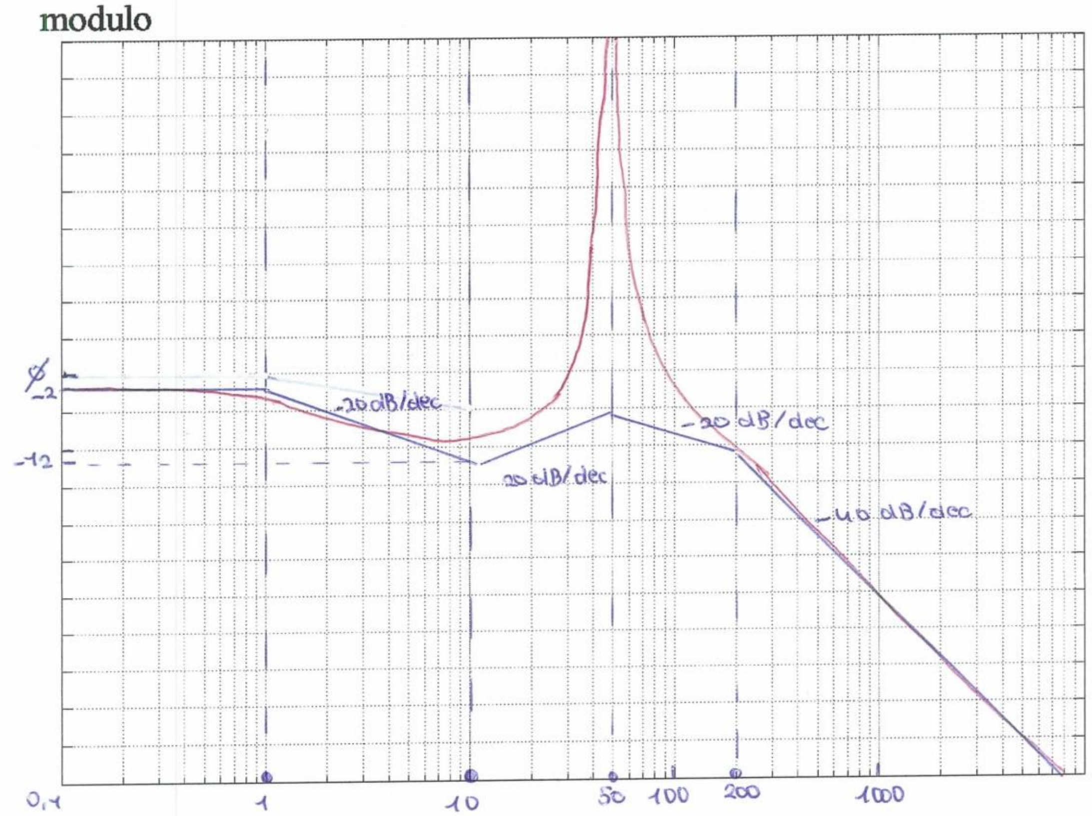
$K = 2$: STABILE

$K < \emptyset$:

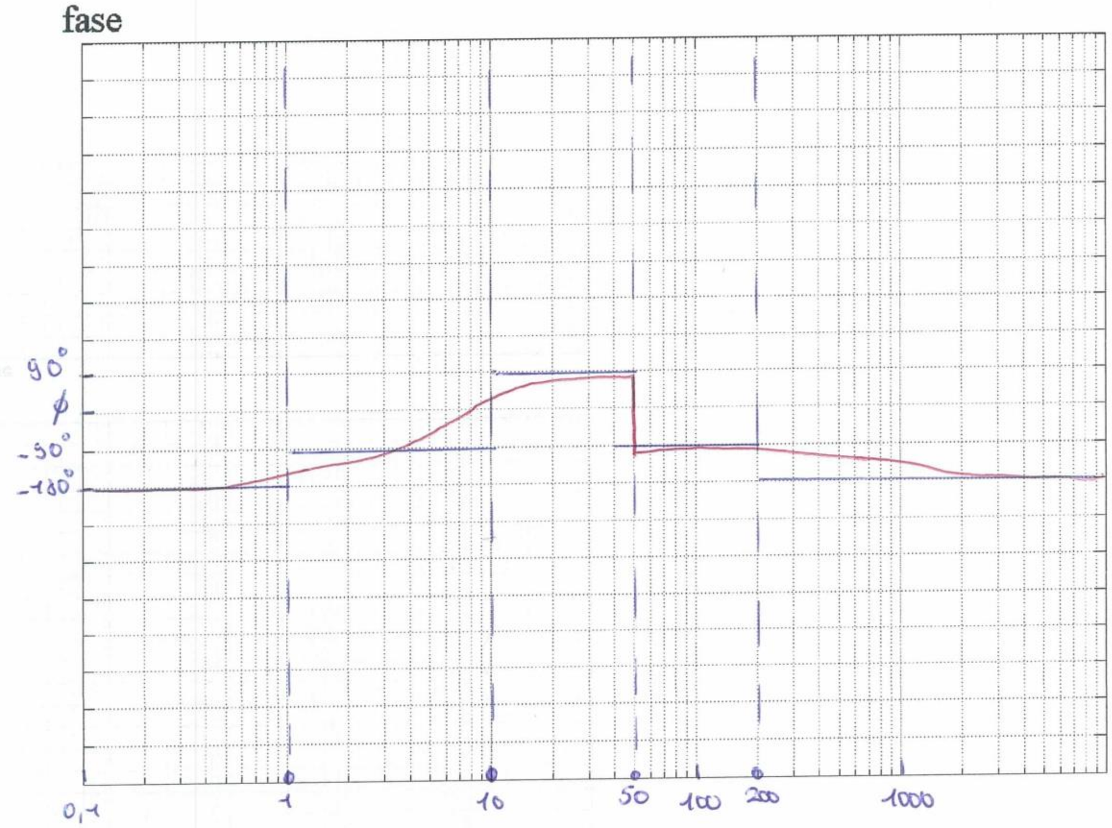
$|K| > \frac{1}{\alpha}$: 1 Pd

$|K| < \frac{1}{\alpha}$: 3 Pd

modulo


∅
-2
-12
-20 dB/dec
20 dB/dec
-20 dB/dec
-40 dB/dec
0,1
1
10
50
100
200
1000

fase


90°
∅
-90°
-180°
0,1
1
10
50
100
200
1000

Es. 3

$$G(s) = \frac{(100+s^2)}{(2+s)(5-s)(5+s)^2} = -\frac{(s^2+100)}{(s+2)(s-5)(s+5)^2}$$

$K_B = -1$

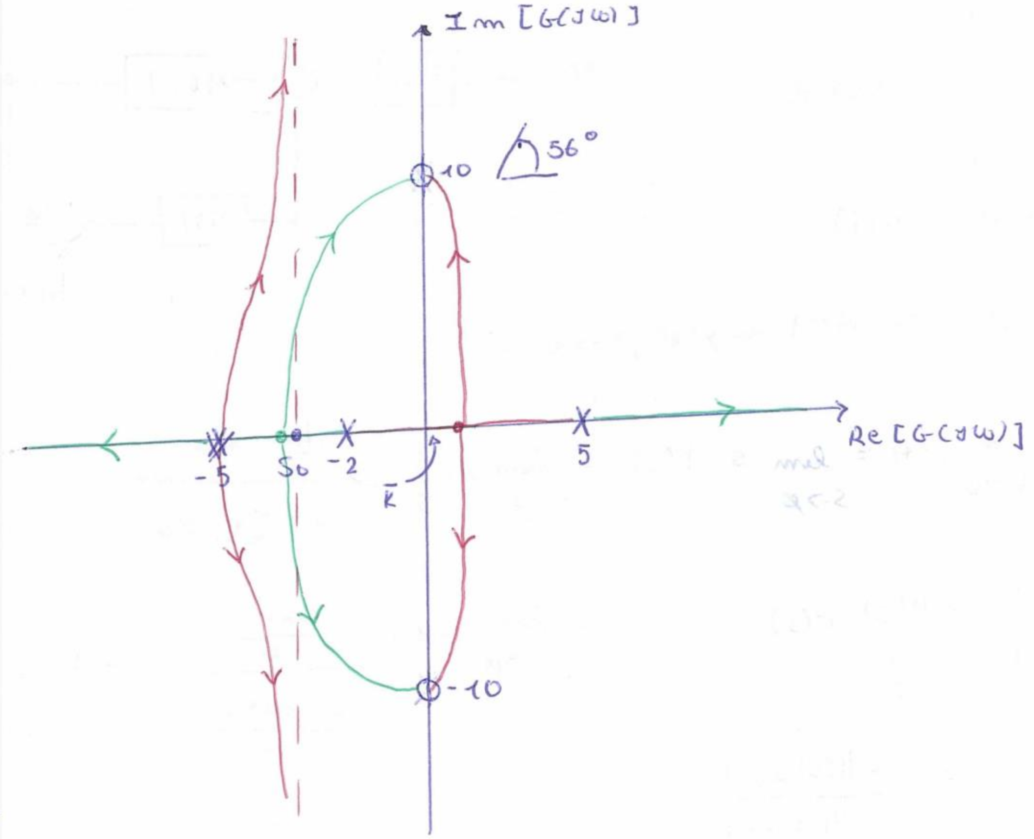
$K^* = -K$

$z_{1/2} = \pm j10$

$p_1 = -2$

$p_2 = 5$

$p_{3/4} = -5$



$n - m = 4 - 2 = 2$

$$S_0 = \frac{-2 + \cancel{5} - 5 - \cancel{5}}{2} = -\frac{7}{2}$$

$$\begin{cases} \dfrac{(2h+1)\pi}{2} = \left\{\dfrac{\pi}{2} ; \dfrac{3}{2}\pi\right\} & LD \\ \dfrac{2h\pi}{2} = \left\{\emptyset ; \pi\right\} & LC \end{cases}$$

$K^* > \emptyset \Rightarrow K < \emptyset$ $\quad |K| > \bar{K}$: 2Pd

$|K| < \bar{K}$: 1Pd

$K^* < \emptyset \Rightarrow K > \emptyset$ $\quad$ 1Pd

ES. 4

F(s), H(s) tipo 1

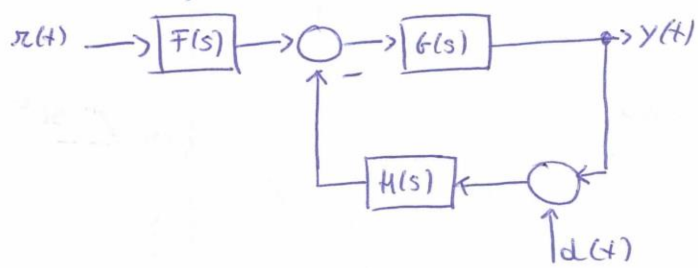
G(s) tipo ∅

K_F, K_H, K_G

AC as. stabile

$r(t) = t$

$d(t) = 3 \cdot u(t)$



Effetto di d(t) su y(t), t->∞

$$\lim_{t\to\infty} y(t) = \lim_{s\to\emptyset} s \cdot Y(s) = \lim_{s\to\emptyset} \cancel{s} \cdot \frac{3}{\cancel{s}} \cdot \frac{-\frac{K_H}{s} \cdot K_G}{1 + \frac{K_H}{s} \cdot K_G}$$

$$Y(s) = R(s) \cdot G(s)$$

$$R(s) = \frac{3}{s}$$

$$G(s) = \frac{-H(s)G(s)}{1 + H(s)G(s)}$$

$$= \lim_{s\to\emptyset} -3 \cdot \frac{\frac{K_H K_G}{\cancel{s}}}{\frac{s + K_H K_G}{\cancel{s}}} = -3 .$$

17/11/2014 TRACCIA #2

Es. 1

$G(s) = \dfrac{Y(s)}{R(s)} = ?$

$A = \alpha$

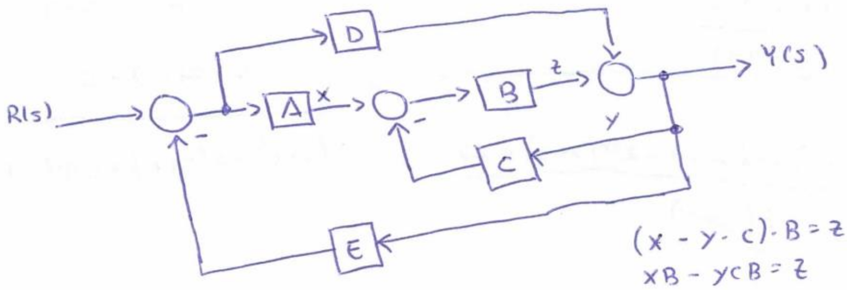
$B = \dfrac{1}{s}$

$C = 3$

$D = \dfrac{1}{s+1}$

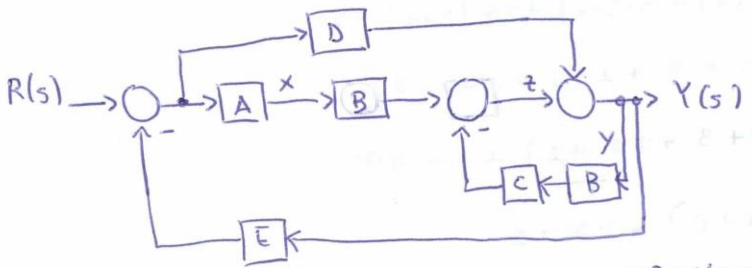
$E = 2$

α : $G(s)$ A.S.

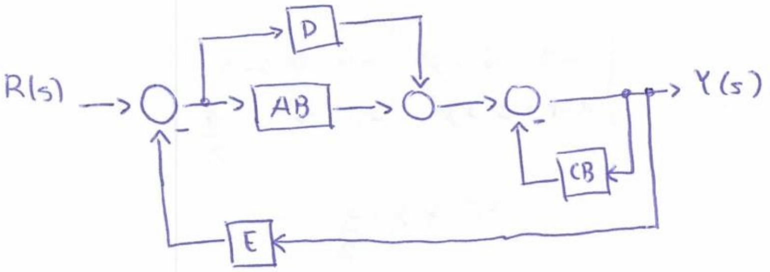


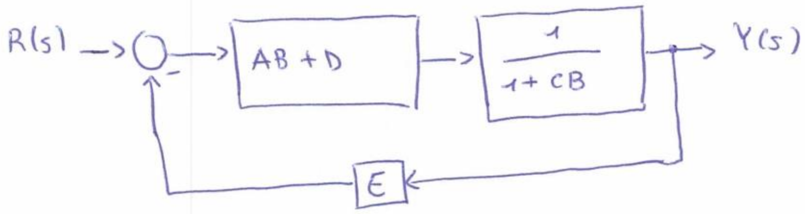
$(x - y \cdot c) \cdot B = z$

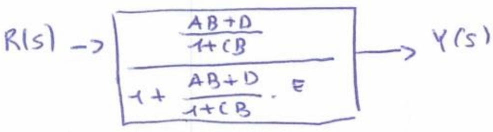
$xB - ycB = z$



$xB - yc = z$







$$G(s) = \frac{\dfrac{AB+D}{1+CB}}{1 + \dfrac{(AB+D)E}{1+CB}} = \frac{AB+D}{1+CB+ABE+DE}$$

$$G(s) = \frac{\alpha \cdot \frac{1}{s} + \frac{1}{s+1}}{1 + 3 \cdot \frac{1}{s} + \alpha \cdot \frac{1}{s} \cdot 2 + \frac{1}{s+1} \cdot 2} = \frac{\frac{\alpha}{s} + \frac{1}{s+1}}{1 + \frac{3\alpha}{s} + \frac{2\alpha}{s} + \frac{2}{s+1}}$$

$$= \frac{\frac{\alpha(s+1) + s}{s\cancel{(s+1)}}}{\frac{s(s+1) + 3(s+1) + 2\alpha(s+1) + 2s}{s\cancel{(s+1)}}} = \frac{\alpha(s+1) + s}{s(s+1) + 3(s+1) + 2\alpha(s+1) + 2s}$$

$$d(s) = s(s+1) + 3(s+1) + 2\alpha(s+1) + 2s$$

$$= s^2 + s + 3s + 3 + 2\alpha s + 2\alpha + 2s$$

$$= s^2 + s(1 + 3 + 2\alpha + 2) + 3 + 2\alpha$$

$$= s^2 + s(2\alpha + 6) + 2\alpha + 3$$

$$\begin{array}{c|cc} n & & \\ 2 & 1 & 2\alpha+3 \\ 1 & 2\alpha+6 & \\ \emptyset & 2\alpha+3 & \end{array}$$

$$\begin{cases} 2\alpha + 6 > \emptyset \rightarrow \alpha > -3 \\ 2\alpha + 3 > \emptyset \rightarrow \alpha > -\frac{3}{2} \end{cases}$$

$$\Rightarrow \alpha > -\frac{3}{2}$$

ES. 2

$$G(s) = 1000 \cdot \frac{(s^2+100)}{s(s+1)(s-10)(s+50)^2}$$

$$= \frac{\overset{4}{\cancel{\overset{20}{\cancel{1000}}}} \cdot 10\cancel{0}}{-1\cancel{0} \cdot \cancel{50} \cdot \cancel{50}} \cdot \frac{\left(1+\frac{s^2}{100}\right)}{s(1+s)\left(1-\frac{s}{10}\right)\left(1+\frac{s}{50}\right)^2}$$

$$= \frac{-4}{s} \cdot \frac{\left(1+\frac{s^2}{100}\right)}{(1+s)\left(1-\frac{s}{10}\right)\left(1+\frac{s}{50}\right)^2}$$

$K_B = -4$

$p_1 = \emptyset$

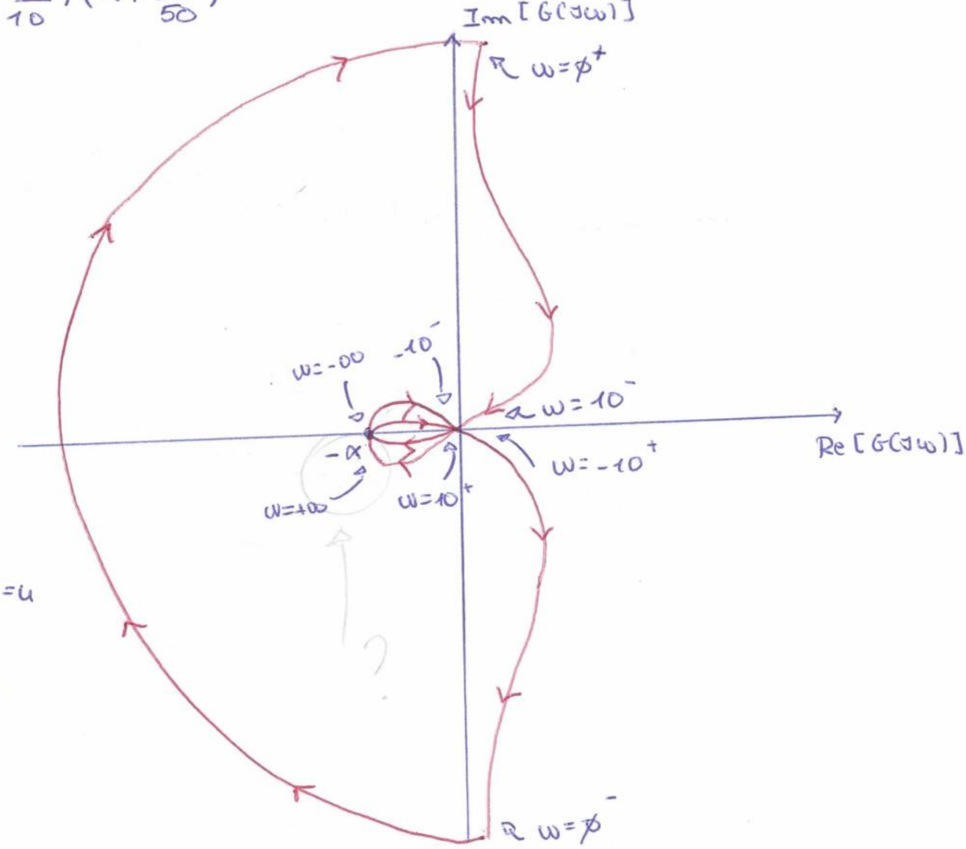
$p_2 = -1$

$p_3 = 10$

$p_{4/5} = -50$

$\alpha_m = 10$

$\xi = \emptyset$



$$\left|-\frac{4}{j\omega}\right| = 1$$

$$\frac{4}{\omega} = 1 \Rightarrow \omega = 4$$

$$\frac{4}{0{,}1} = x_{lim} = 40$$

$$x_{dB} = 32 \text{ dB}$$

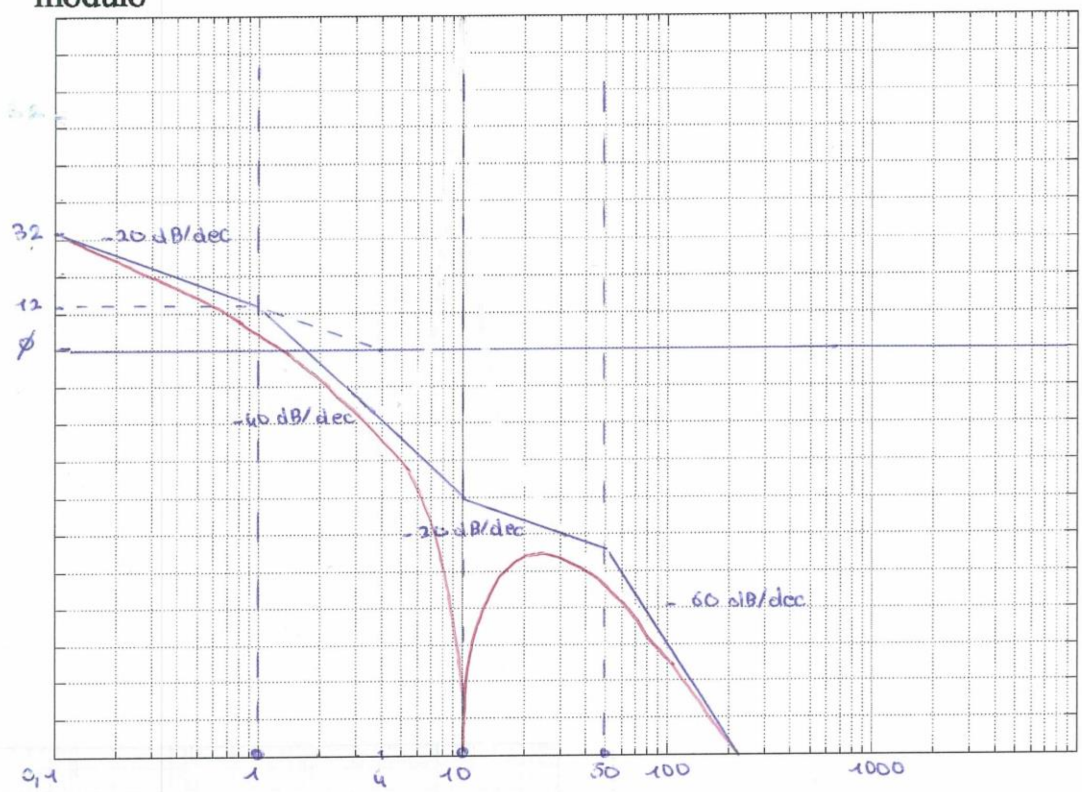
$$N_{Pd}^{AC} = \underset{\substack{\| \\ 1}}{N_{Pd}^{AA}} + N_{oa,-1}$$

$K > \emptyset$: $K > \frac{1}{\alpha}$: 4 Pd

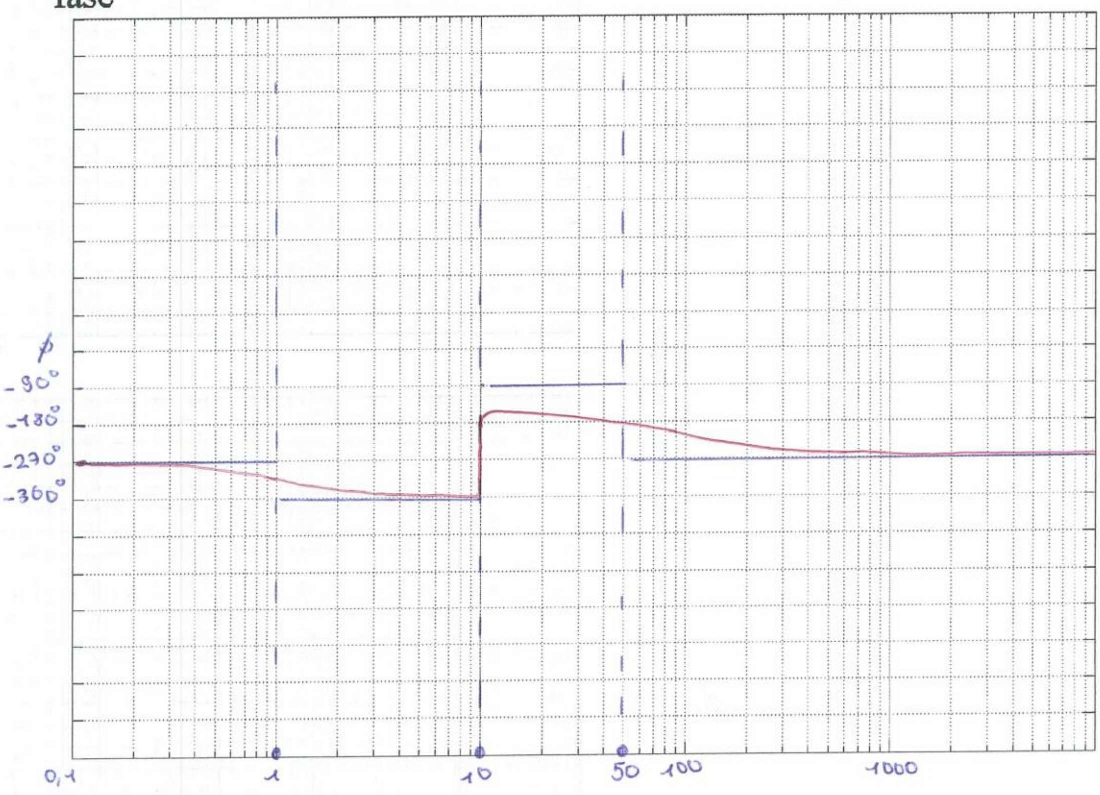
$K < \frac{1}{\alpha}$: 2 Pd

$K < \emptyset$: 1 Pd

modulo


32
12
∅
-20 dB/dec
-40 dB/dec
-20 dB/dec
-60 dB/dec
0,1
1
4
10
50
100
1000

fase


∅
-90°
-180°
-270°
-360°
0,1
1
10
50
100
1000

Es. 3

$$G(s) = \frac{(s+4)^2}{(s^2+400)(s-4)(s+50)}$$

$z_{1/2} = -4$

$p_1 = 4$

$p_2 = -50$

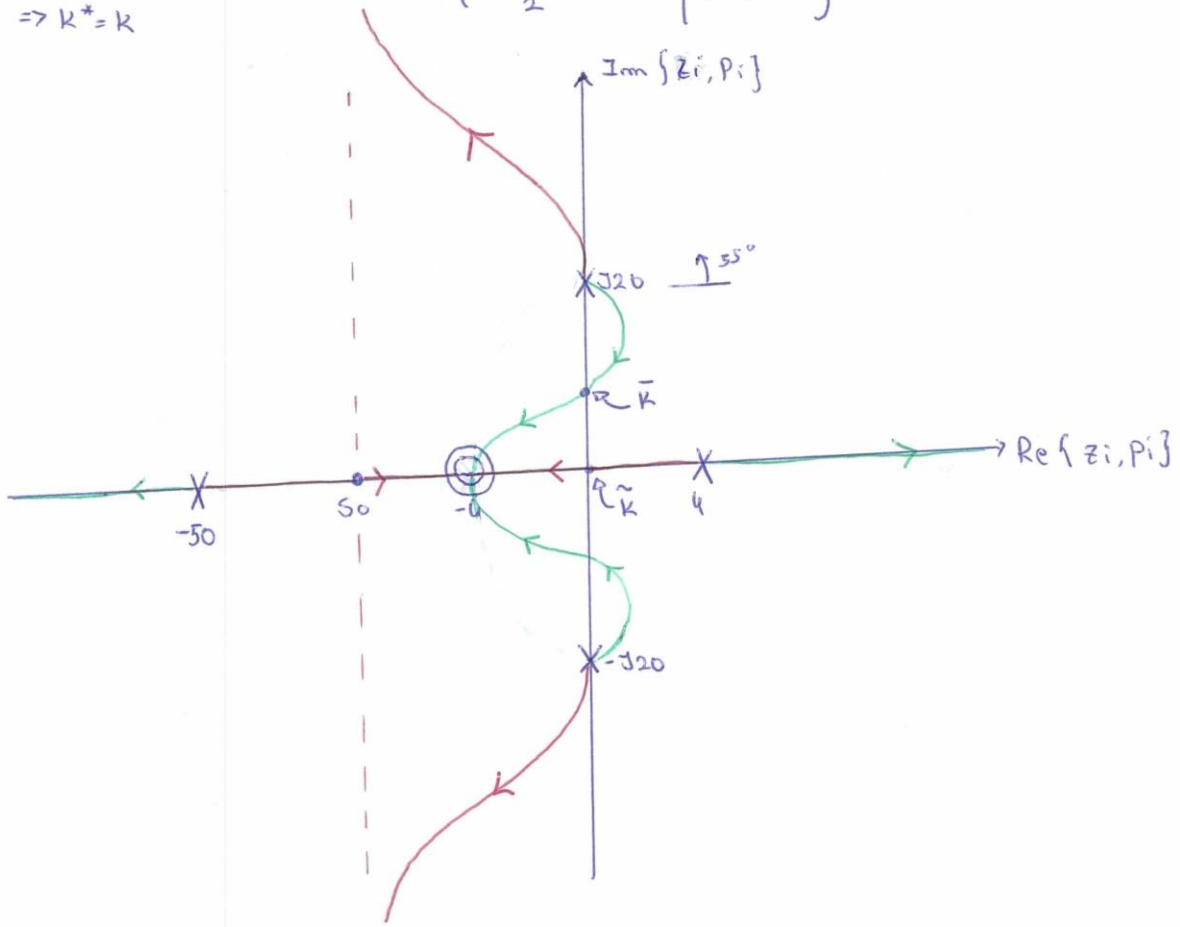
$p_{3/4} = \pm j20$

$K_B = 1$

$\Rightarrow K^* = K$

$n - m = 4 - 2 = 2$

$$\begin{cases} \dfrac{(2h+1)\pi}{2} = \left\{ \dfrac{\pi}{2} ; \dfrac{3}{2}\pi \right\} & LD \\[2ex] \dfrac{2h\pi}{2} = \left\{ \emptyset ; \pi \right\} & LC \end{cases}$$



$$S_0 = \frac{4 - 50 + 4 + 4}{2} = -19$$

$\measuredangle s-p_1 + \measuredangle s-p_2 + \measuredangle s-p_3 + \measuredangle s-p_4 - \measuredangle s-z_1 - \measuredangle s-z_2 = \emptyset$

$$\underbrace{tg^{-1}\left(\frac{20}{-4}\right)}_{+\pi} + tg^{-1}\left(\frac{20}{50}\right) + tg^{-1}\left(\frac{40}{\emptyset}\right) - tg^{-1}\left(\frac{20}{4}\right) \cdot 2 = -\measuredangle s-p_3$$

$$-78 + 180 + 21 + 90 - 2 \cdot 78 = -\measuredangle s-p_3 = 55^\circ$$

$K = K^*$

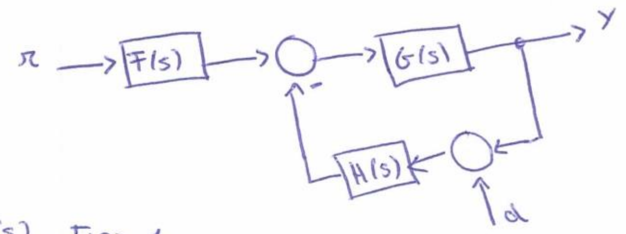
$K > \emptyset$: $K < \tilde{K}$: 1 Pd

$K > \tilde{K}$: STABILE

$K < \emptyset$: $|K| < \bar{K}$: 3 Pd

$|K| > \bar{K}$: 1 Pd

ES. 4



$G(s), H(s)$ tipo 1

$F(s)$ tipo $\emptyset$

K_G, K_H, K_F

AC asintoticamente stabile

$r(t) = u(t) = 1$

$d(t) = 7u(t) = 7$

? Effetto di $d(t)$ su $y(t)$, $t \to \infty$

$$Y(s) = G(s) \cdot D(s)$$

$$D(s) = \frac{7}{s}$$

$$G(s) = \frac{Y(s)}{D(s)} = \frac{-H(s)G(s)}{1+G(s)H(s)}$$

$$\lim_{t\to\infty} y(t) = \lim_{s\to\emptyset} s \cdot Y(s) = \lim_{s\to\emptyset} \cancel{s} \cdot \frac{7}{\cancel{s}} \cdot \frac{-\frac{K_H}{s}\cdot\frac{K_G}{s}}{1+\frac{K_H}{s}\cdot\frac{K_G}{s}}$$

$$= \lim_{s\to\emptyset} 7 \frac{\frac{-K_H K_G}{\cancel{s^2}}}{\frac{s^2+K_H K_G}{\cancel{s^2}}} = -7$$

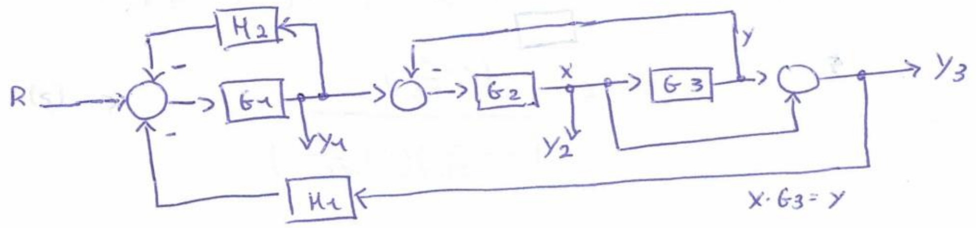
19/11/2013 TRACCIA #1

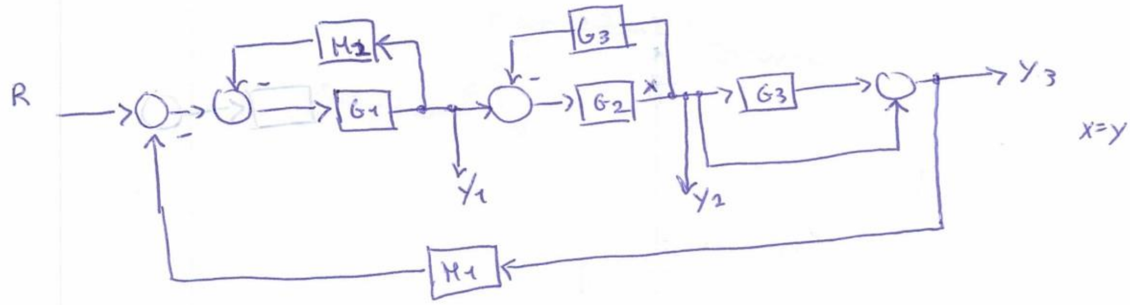
Es. 1

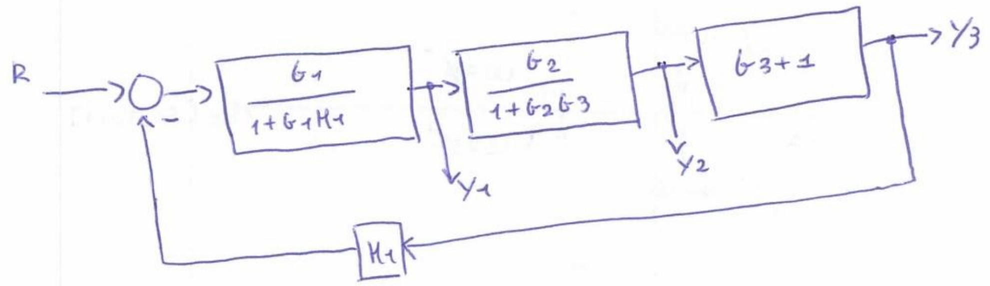
$F_1(s) = \frac{Y_1(s)}{R(s)}$

$F_2(s) = \frac{Y_2(s)}{R(s)}$

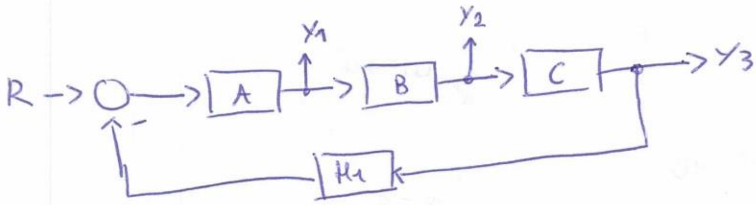
$F_3(s) = \frac{Y_3(s)}{R(s)}$







$A = \frac{G_1}{1+G_1H_1}$; $B = \frac{G_2}{1+G_2G_3}$; $C = G_3+1$



$F_1(s) = \frac{A}{1+ABCH_1}$

$F_2(s) = \frac{AB}{1+ABCH_1}$

$F_3(s) = \frac{ABC}{1+ABCH_1}$

ES. 2

$$G(s) = \frac{-1000(10-s)}{(s-50)(s^2+100)} = + \frac{10\cancel{00} \cdot \cancel{10}}{5\cancel{0} \cdot \cancel{100}} \frac{\left(1-\frac{s}{10}\right)}{\left(1-\frac{s}{50}\right)\left(1+\frac{s^2}{100}\right)}$$

$K_B = 2$

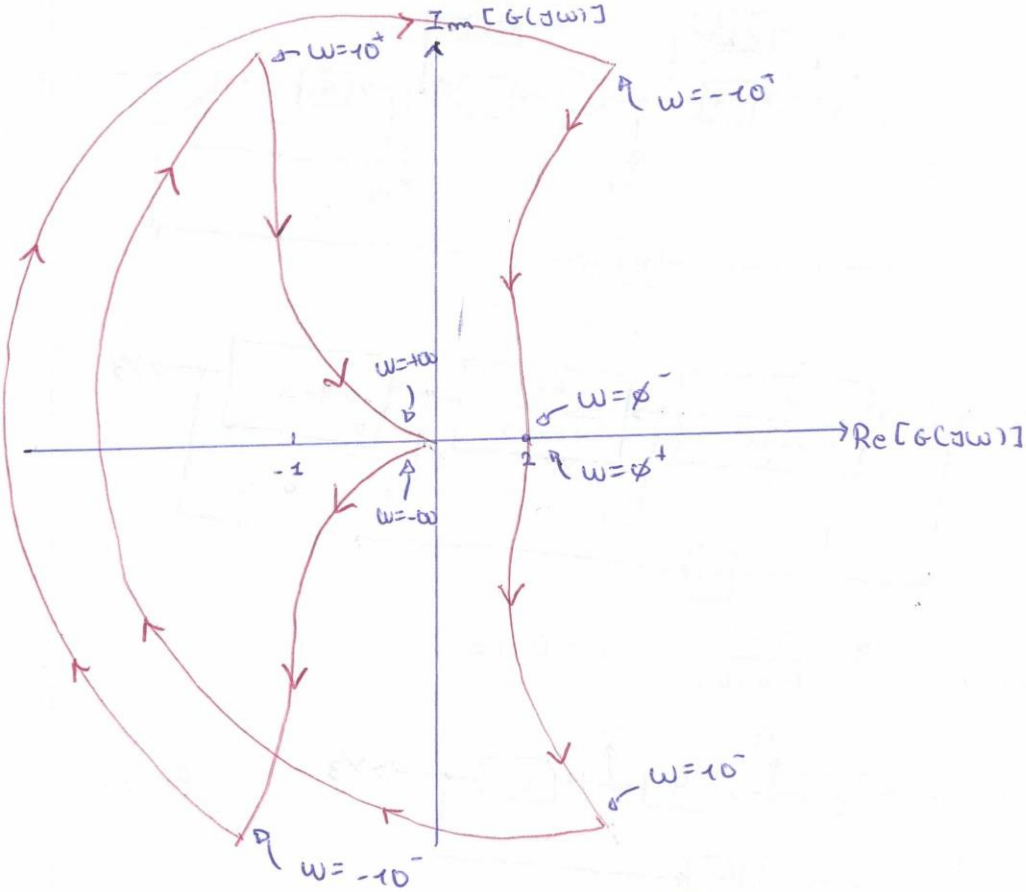
$$= 2 \cdot \frac{\left(1-\frac{s}{10}\right)}{\left(1-\frac{s}{50}\right)\left(1+\frac{s^2}{100}\right)}$$

$z_1 = 10$

$p_1 = 50$

$\omega_m = 10$

$g = \emptyset$



$\alpha = 2$

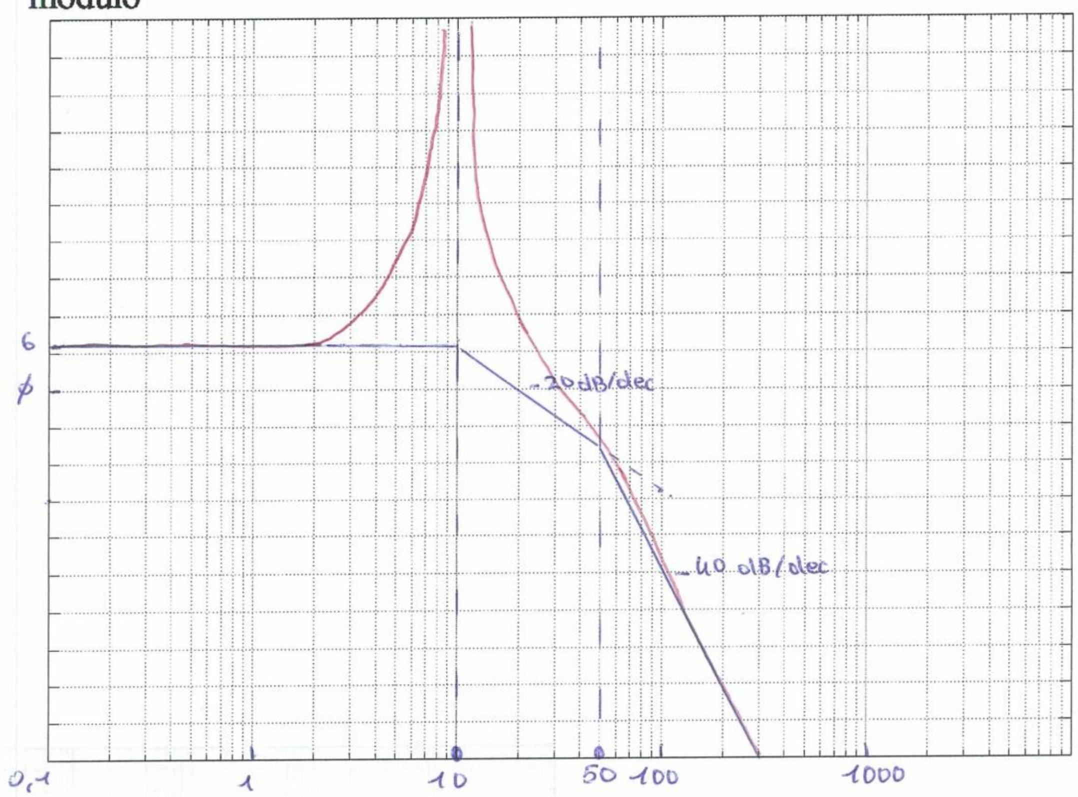
$$N_{Pd}^{AC} = N_{Pd}^{AA} + N_{oR,-1}$$
$$\overset{\shortparallel}{1}$$

$K > \emptyset$: 3 Pd

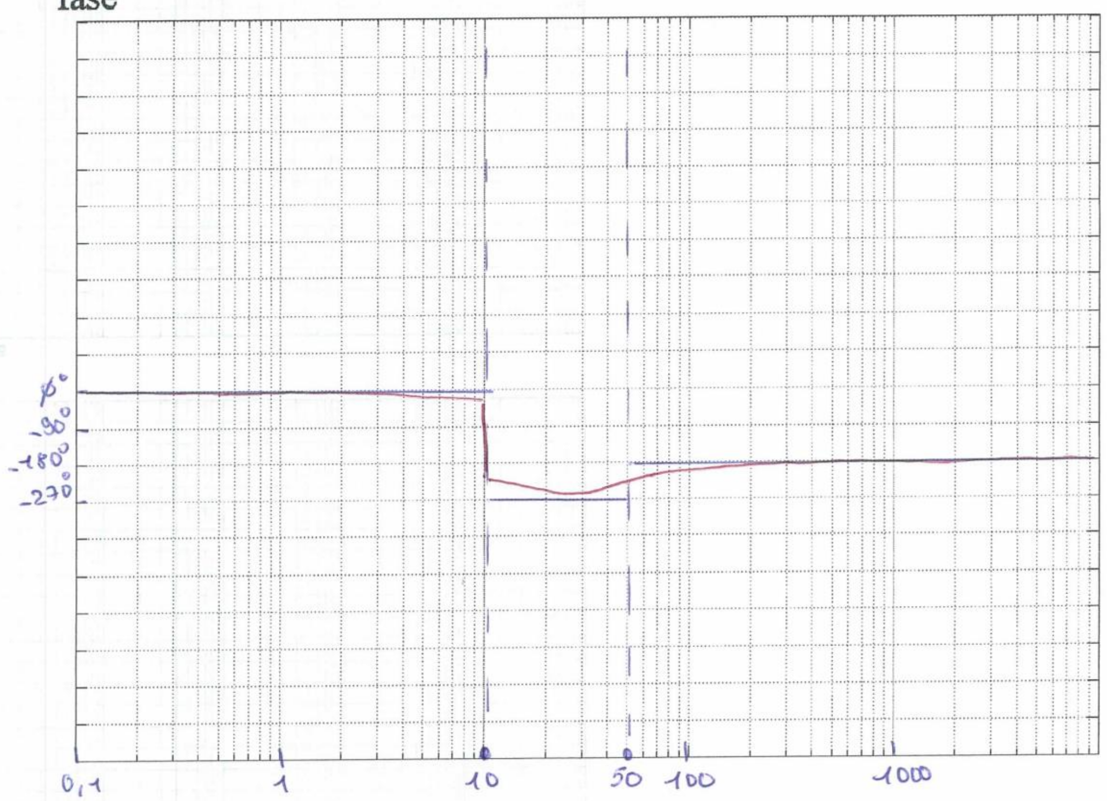
$K < \emptyset$: $|K| > \frac{1}{2}$: 2 Pd

$|K| < \frac{1}{2}$: 1 Pd

modulo


6
ϕ
-20 dB/dec
-40 dB/dec
0,1
1
10
50
100
1000

fase


ϕ°
-90°
-180°
-270°
0,1
1
10
50
100
1000

RautR :

$$d(s) = -1000\,(10-s)\cdot K + (s-50)(s^2+100)$$

$$= -10000K + 1000Ks + s^3 + 100s - 50s^2 - 5000$$

$$= s^3 - 50s^2 + s(1000K+100) + (-10000K - 5000) = 0$$

n		
3	1	$1000K + 100$
2	$-5\cancel{0}$	$-1000\cancel{0}K - 500\cancel{0}$
1	K	
0	$-2K-1$	

$$\begin{cases} K < 0 \\ -2K - 1 < 0 \end{cases}$$

$$-\cancel{2}K < \frac{1}{2}$$

$$K > -\frac{1}{2} \quad \rightarrow 1\ Pd$$

ES. 3

$$G(s) = \frac{10s}{(s^2+4s+68)(s+10)(10-s)} \qquad n-m = 4-1 = 3$$

$$= -\frac{10s}{(s+10)(s-10)(s^2+4s+68)}$$

$K_B = -10$

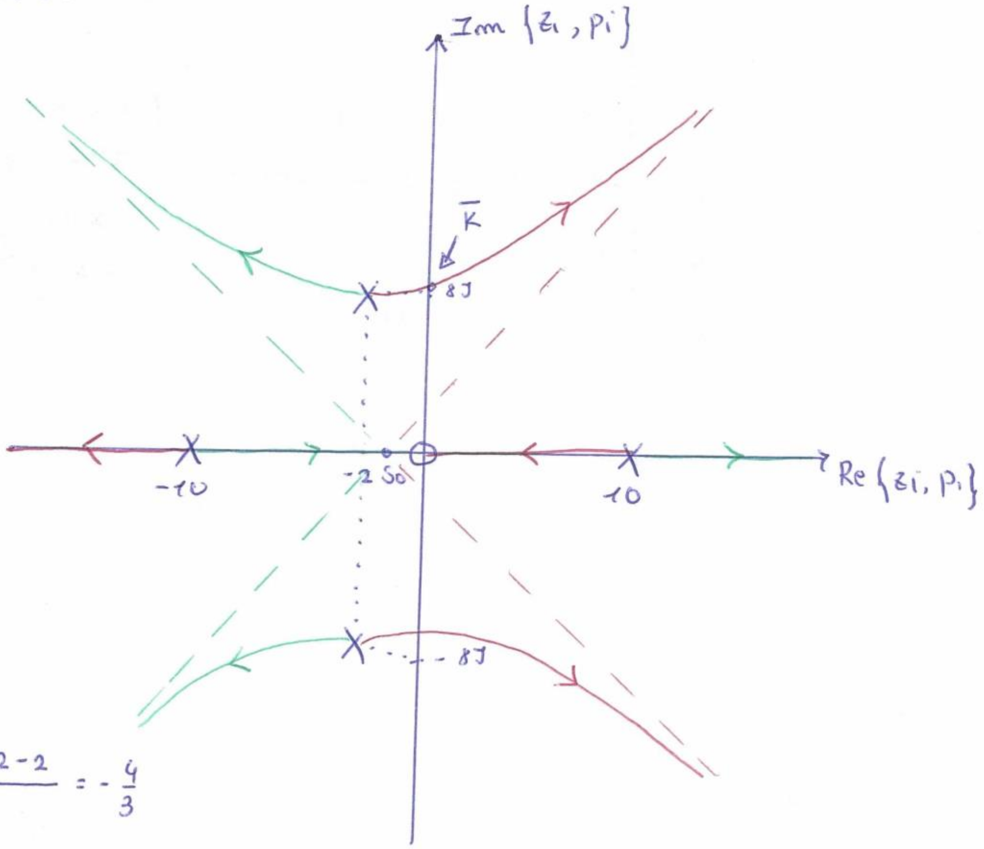
$K^* = -10 \cdot K$

$z_1 = \emptyset$

$p_1 = -10$

$p_2 = +10$

$p_{3/4} = -2 \pm 8J$



$$S_0 = \frac{\cancel{-10}\cancel{+10}-2-2}{3} = -\frac{4}{3}$$

$$\begin{cases} \dfrac{(2h+1)\pi}{3} = \left\{ \dfrac{1}{3}\pi ; \dfrac{3}{3}\pi ; \dfrac{5}{3}\pi \right\} & LD \\ \dfrac{2h\pi}{3} = \left\{ \emptyset ; \dfrac{2}{3}\pi ; \dfrac{4}{3}\pi \right\} & LC \end{cases}$$

$K^* > \emptyset \Rightarrow K < \emptyset$: $|K| > \bar{K}$: 3 Pd

$|K| < \bar{K}$: 1 Pd

$K^* < \emptyset \Rightarrow K > \emptyset$: 1 Pd

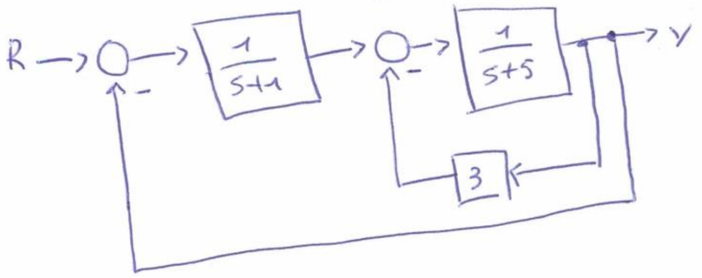
Es. 4

$r(t) = 3$

$\frac{Y(s)}{R(s)}$ stabile ?

$\lim_{t \to \infty} y(t) = ?$

$$\frac{\frac{1}{s+5}}{1+\frac{3}{s+5}} = \frac{1}{s+8}$$



$$G(s) = \frac{\frac{1}{(s+1)(s+8)}}{1+\frac{1}{(s+1)(s+8)}} = \frac{1}{1+(s+1)(s+8)}$$

$$d(s) = 1+s^2+8s+s+8 = s^2+9s+9$$

n		
2	1	9
1	9	
0	9	

STABILE!

$$\lim_{t \to \infty} y(t) = \lim_{s \to 0} s \cdot Y(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{3}{\cancel{s}} \cdot \frac{1}{1+(\cancel{s}+1)(\cancel{s}+8)}$$

$$= \frac{1}{3}.$$

$Y(s) = R(s)\,G(s)$

$R(s) = \frac{3}{s}$

$G(s) = \frac{1}{1+(s+1)(s+8)}$

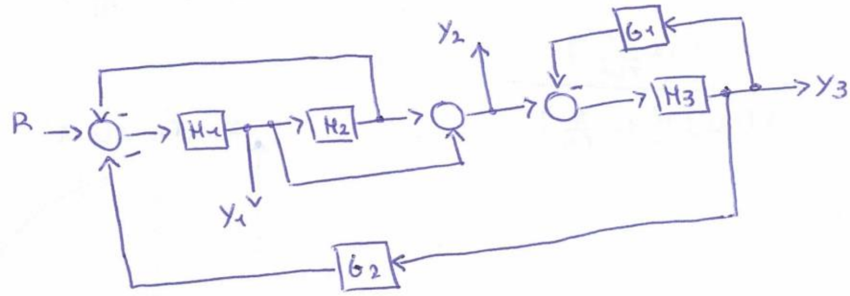
3/11/2012 TRACCIA #2

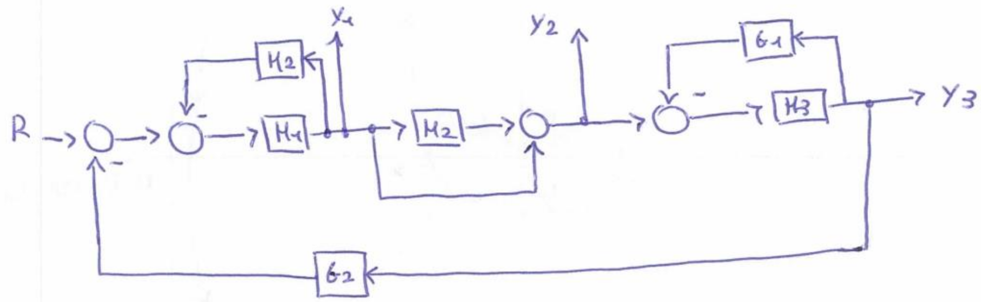
ES. 1

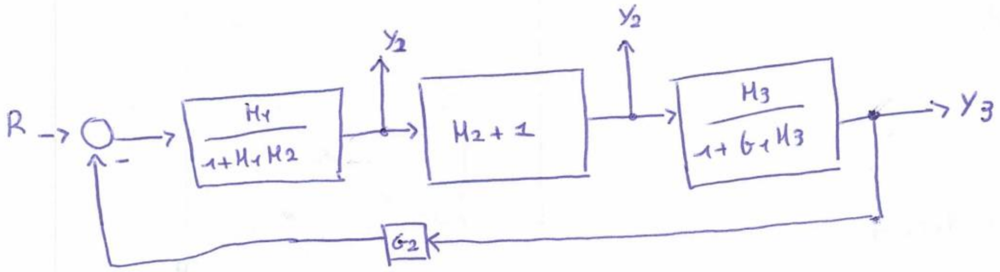
$$F_1(s) = \frac{Y_1(s)}{R(s)}$$

$$F_2(s) = \frac{Y_2(s)}{R(s)}$$

$$F_3(s) = \frac{Y_3(s)}{R(s)}$$







$$A = \frac{H_1}{1+H_1H_2} \quad ; \quad B = H_2 + 1 \quad ; \quad C = \frac{H_3}{1+G_1H_3}$$

$$F_1(s) = \frac{A}{1+ABCG_2}$$

$$F_2(s) = \frac{AB}{1+ABCG_2}$$

$$F_3(s) = \frac{ABC}{1+ABCG_2}$$

ES. 2

$$G(s) = \frac{(s^2+100)}{s(s-1)(s-10)} = + \frac{10\cancel{0}}{1\cancel{0}} \cdot \frac{\left(1+\frac{s^2}{100}\right)}{s(1-s)\left(1-\frac{s}{10}\right)}$$

$$= +10 \cdot \frac{\left(1+\frac{s^2}{100}\right)}{s(1-s)\left(1-\frac{s}{10}\right)}$$

$K_B = 10$

$\alpha_n = 10$

$\xi = \emptyset$

$p_1 = \emptyset$

$p_2 = 1$

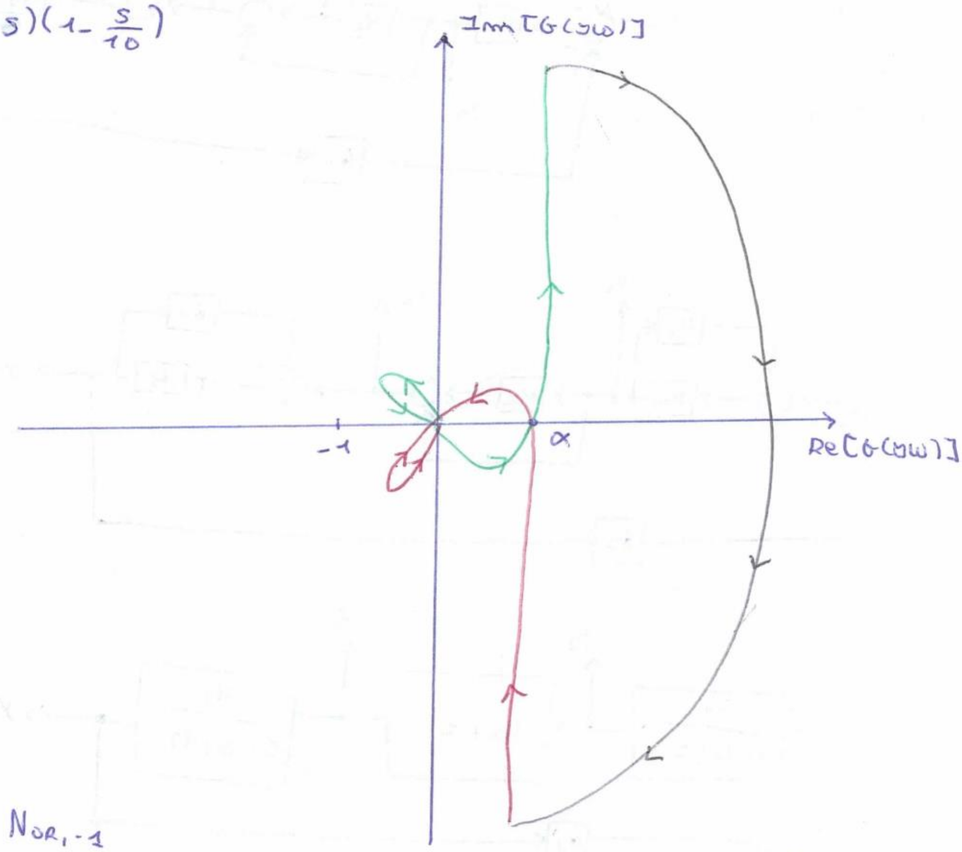
$p_3 = 10$

$\left|\frac{10}{j\omega}\right| = \frac{10}{\omega} = 1$

$\Rightarrow \omega = 10$

$\frac{10}{0,1} = x_{lin} = 100$

$x_{dB} = 40$



$N_{Pd}^{AC} = \underset{\substack{\parallel \\ 2}}{N_{Pd}^{AA}} + N_{OR,-1}$

$\alpha = \frac{9}{11}$.

$K > \emptyset$: 2 Pd

$K < \emptyset$: $|K| > \frac{1}{\alpha}$: 1 Pd

$|K| < \frac{1}{\alpha}$: 3 Pd

m		
3	1	10
2	K-11	100 K
1	$\frac{-11-9K}{K-11}$	
∅	100 K	

$d(s) = s(s-1)(s-10) + k(s^2+100)$

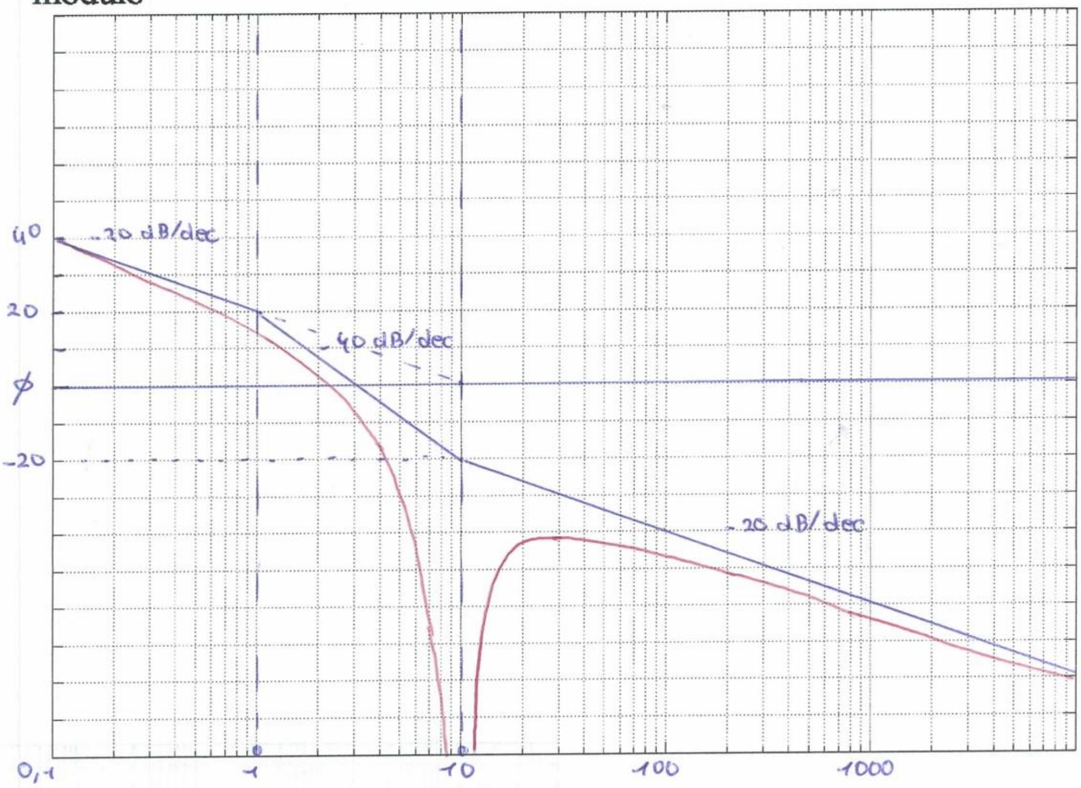
$= (s^2-s)(s-10) + ks^2 + 100k$

$= s^3 - 10s^2 - s^2 + 10s + ks^2 + 100k$

$= s^3 + s^2(-11+k) + s(10) + 100k = \emptyset$

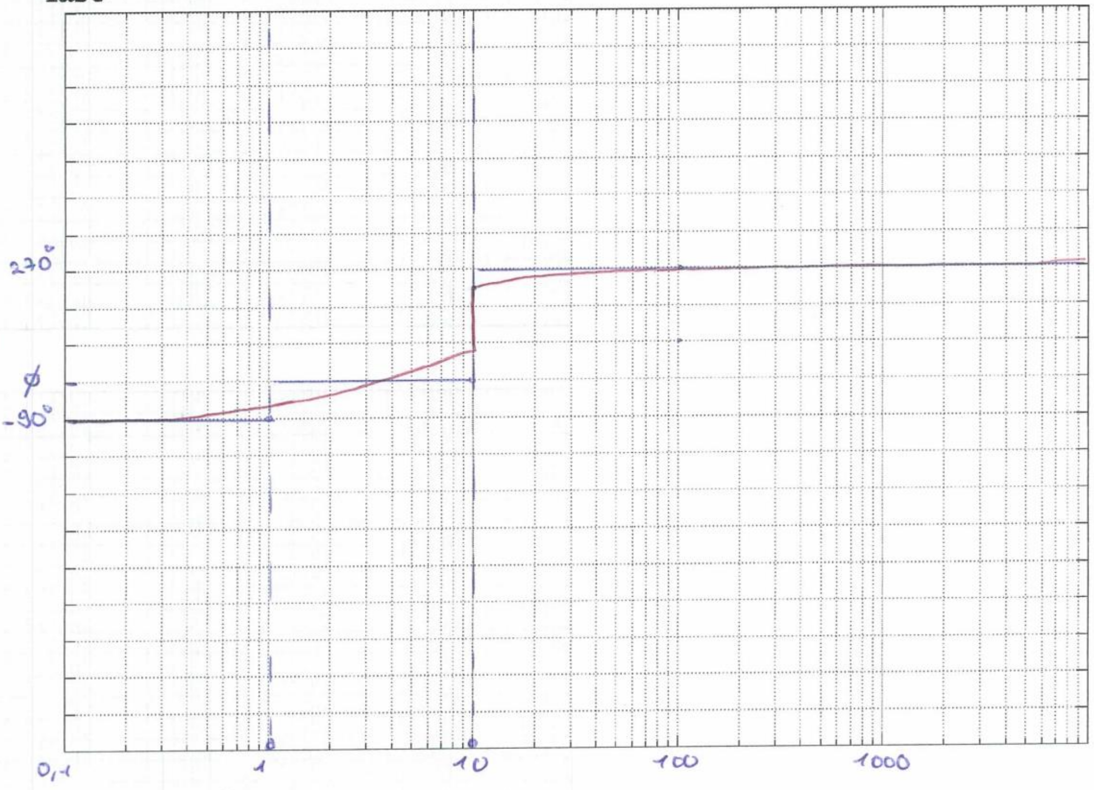
$$\begin{cases} K-11 > \emptyset \\ K > \emptyset \\ \frac{-11-9K}{K-11} > \emptyset \end{cases}$$

modulo



40
20
ϕ
-20
-20 dB/dec
-40 dB/dec
-20 dB/dec
0,1
1
10
100
1000

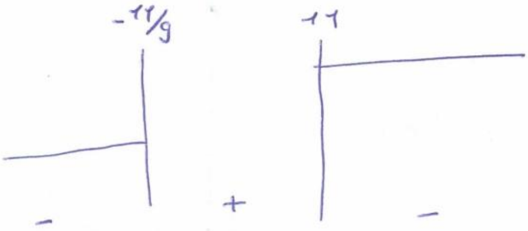
fase



270°
ϕ
-90°
0,1
1
10
100
1000

$$\frac{-11-9K}{K-11} > \emptyset$$

$$\begin{cases} K < -\frac{11}{9} \\ K > 11 \end{cases}$$

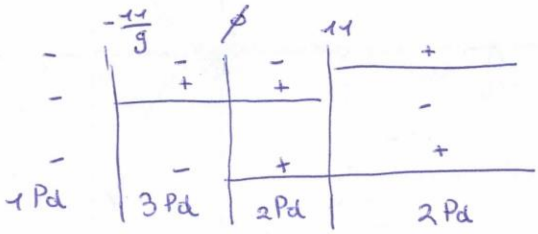


$$-\frac{11}{9} < K < 11$$

$K > 11$

$-\frac{11}{9} < K < 11$

$K > \emptyset$



$$\begin{cases} K < -\frac{11}{9} & : \ 1\,Pd \quad \Rightarrow \quad \alpha = \frac{11}{11} \\ -\frac{11}{9} < K < \emptyset & : \ 3\,Pd \\ K > \emptyset & : \ 2\,Pd \end{cases}$$

Es. 3

$$G(s) = \frac{-(s^2 - 8s + 80)}{s^3(s+10)}$$

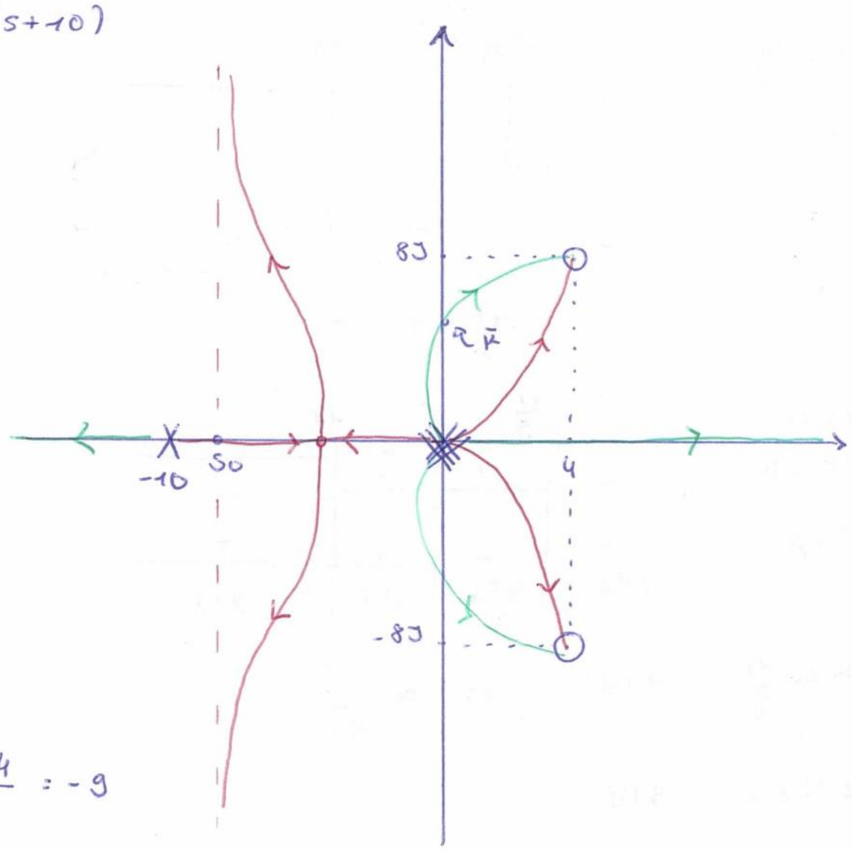
$K_B = -1$

$K^* = -K$

$p_{1/2/3} = \emptyset$

$p_4 = -10$

$z_{1/2} = 4 \pm 8j$



$n - m = 4 - 2 = 2$

$$S_0 = \frac{-10 - 4 - 4}{2} = -9$$

$$\begin{cases} \dfrac{(2h+1)\pi}{2} = \left\{ \dfrac{\pi}{2} ; \dfrac{3}{2}\pi \right\} & LD \\[2ex] \dfrac{2h\pi}{2} = \left\{ \emptyset ; \pi \right\} & LC \end{cases}$$

$$\measuredangle s+10 + \measuredangle s+\emptyset - \measuredangle s-(4+8j) - \measuredangle s(-4-8j) = \emptyset$$

$$\cancel{tg^{-1}(\emptyset)} - tg^{-1}\left(\frac{8}{-4}\right) + \pi - tg^{-1}\left(\frac{\pm 8}{+4}\right) = -\measuredangle s+\emptyset$$

$$\cancel{+63} + 180 \cancel{-63} = -\measuredangle s+\emptyset$$

$$\measuredangle s+\emptyset = -180°$$

$K^* > \emptyset \Rightarrow K < \emptyset$ 2Pd

$K^* < \emptyset \Rightarrow K > \emptyset$: $K > \bar{K}$: 3Pd

$K < \bar{K}$: 1 Pd

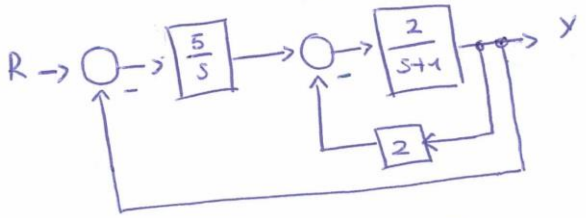
ES. 4

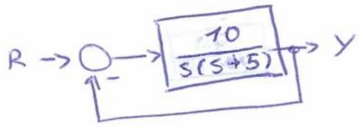
$r(t) = 5$

$F(s) = \dfrac{Y(s)}{R(s)}$

$\lim\limits_{t\to\infty} y(t) = ?$

$$\underbrace{\frac{\frac{2}{s+1}}{1+\frac{4}{s+1}}} = \frac{2}{s+5}$$





$$F(s) = \frac{\frac{10}{s(s+5)}}{1+\frac{10}{s(s+5)}} = \frac{10}{s(s+5)+10}$$

$d(s) = s^2 + 5s + 10$

n		
2	1	10
1	5	
0	10	

STABILE!

$$\lim_{t\to\infty} y(t) = \lim_{s\to 0} s\cdot Y(s) = \lim_{s\to 0} \cancel{s}\cdot\frac{5}{\cancel{s}}\cdot\frac{\cancel{10}}{\cancel{s(s+5)}+\cancel{10}} = 5.$$

$Y(s) = G(s)\cdot R(s)$

$R(s) = \dfrac{5}{s}$

13/04/2015 ES. 1

$$G(s) = \frac{500(2s+100)}{(s^2+100s)(s^2+10s+100)}$$

$$= \frac{\cancel{500}\cdot\cancel{100}}{\cancel{100}\,100} \cdot \frac{\left(1+\frac{s}{50}\right)}{s\left(1+\frac{s}{100}\right)\left(\frac{s^2}{100}+\frac{10s}{10}+100\right)} = \frac{5}{s}\cdot\frac{\left(1+\frac{s}{50}\right)}{\left(1+\frac{s}{100}\right)\left(1+\frac{1}{10}s+\frac{s^2}{100}\right)}$$

$K_B = 5$

$p_1 = \emptyset$

$p_2 = -100$

$\omega_n = 10$

$\xi = 1/2$

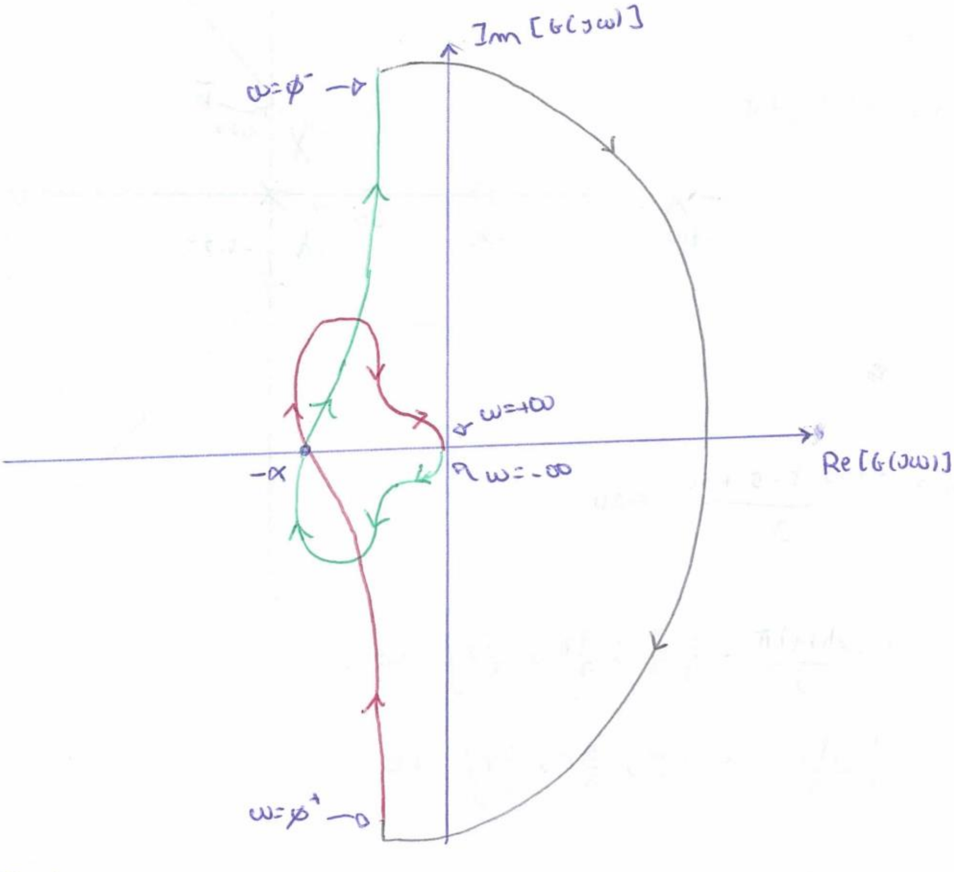
$z_1 = -50$

$$\left|\frac{5}{j\omega}\right| = 1$$

$$\frac{5}{\omega} = 1 \Rightarrow \omega = 5$$

$$\frac{5}{0,1} = X_{lim} = 50$$

$X_{dB} = 33\ dB$



$$N_{Pd}^{AC} = \cancel{N_{Pd}^{AA}} + N_{OR,-1}$$

$K > \emptyset$: $K > \frac{1}{\alpha}$: 2 Pd

$K < \frac{1}{\alpha}$: STABILE

$K < \emptyset$: 1 Pd

$$G(s) = 500 \cdot 2 \frac{(s+50)}{s(s+100)(s^2+10s+100)}$$

$m - m = 4 - 1 = 3$

$K_B = 1000$

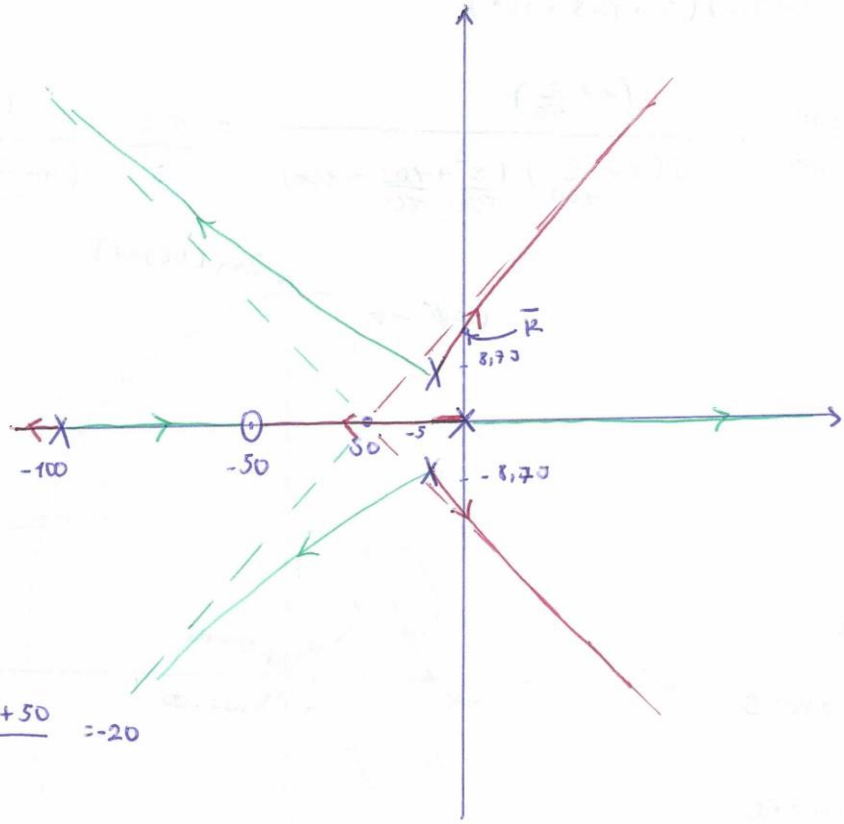
$K^* = 1000\,K$

$z_1 = -50$

$p_1 = \emptyset$

$p_2 = -100$

$p_{3/4} = -5 \pm 8{,}7\,j$



$$S_0 = \frac{-100 - 5 - 5 + 50}{3} = -20$$

$$\begin{cases} \dfrac{(2h+1)\pi}{3} = \left\{ \dfrac{\pi}{3} ; \dfrac{3}{3}\pi ; \dfrac{5}{3}\pi \right\} & \text{LD} \\ \dfrac{2h\pi}{3} = \left\{ \emptyset ; \dfrac{2}{3}\pi ; \dfrac{4}{3}\pi \right\} & \text{LC} \end{cases}$$

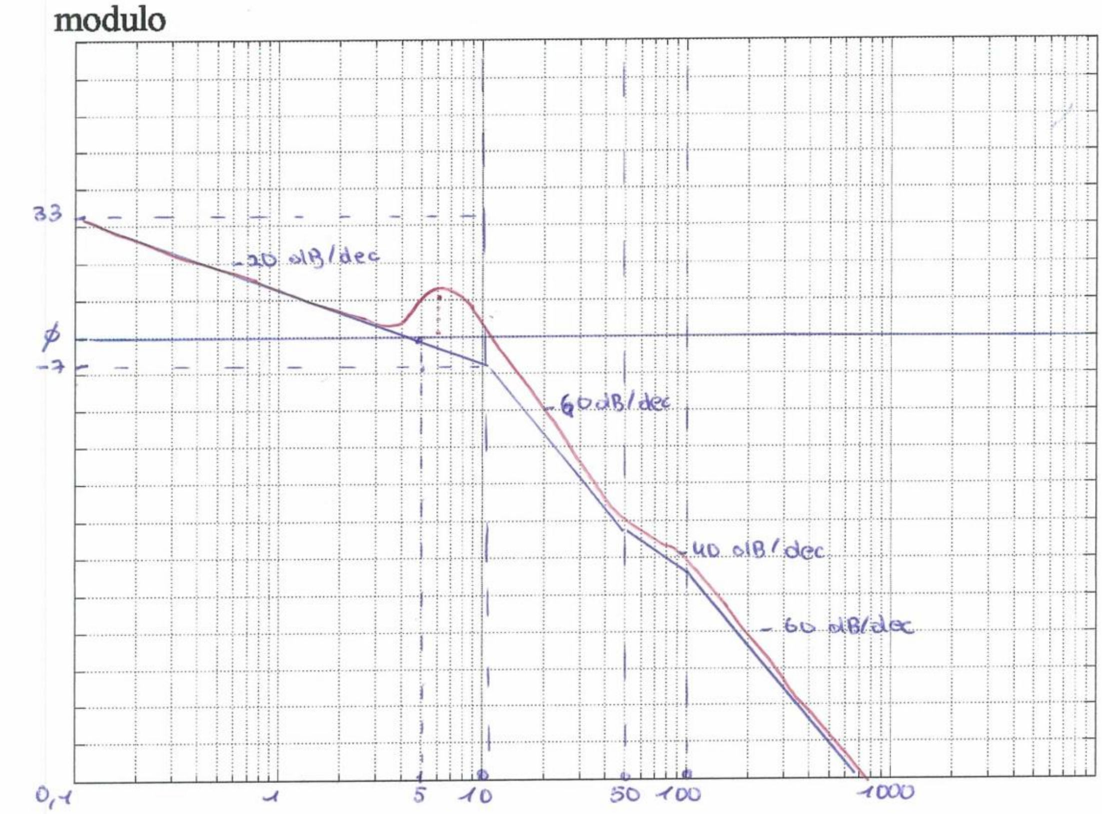
$K > \emptyset$:

$K > \bar{K}$: 2 Pd

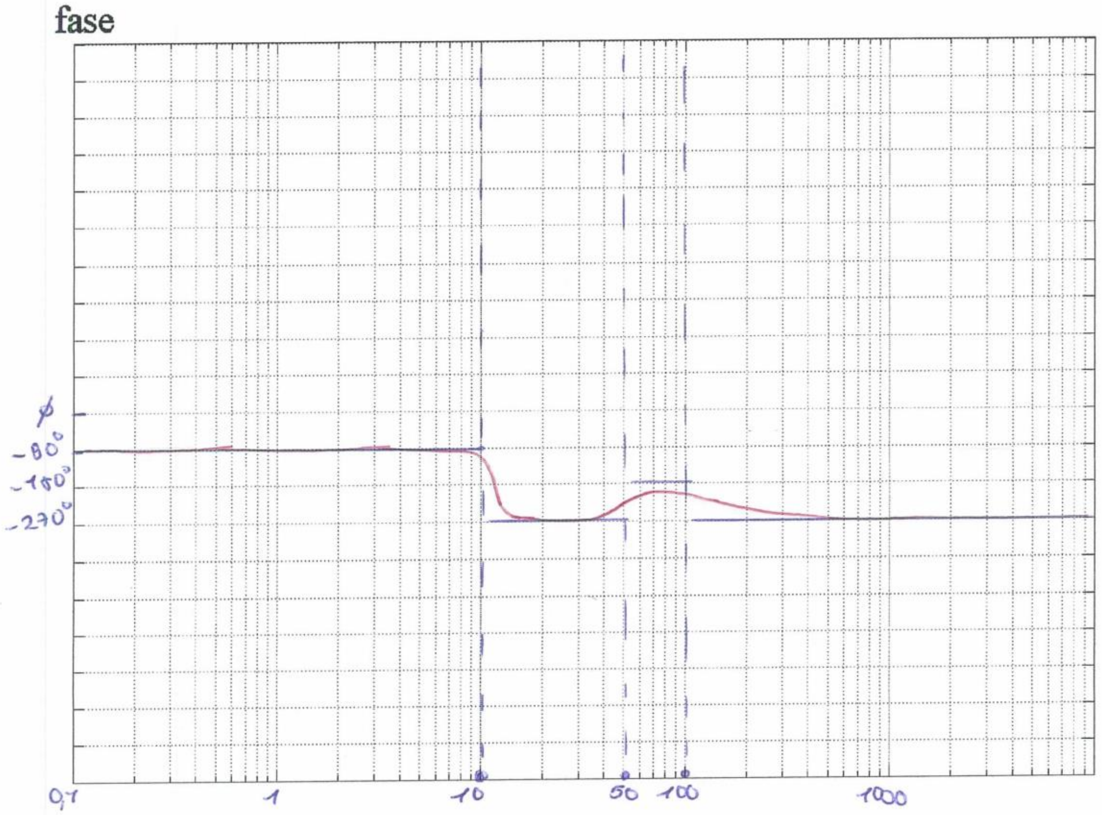
$K < \bar{K}$: STABILE

$K < \emptyset$: 1 Pd

modulo


33
-20 dB/dec
ϕ
-7
-60 dB/dec
-40 dB/dec
-60 dB/dec
0,1
1
5
10
50
100
1000

fase


ϕ
-90°
-180°
-270°
0,1
1
10
50
100
1000

11/02/2015 ES. 1

$$G(s) = \frac{(s+1)(s^2+900)}{s^2(s+10)^2} = \frac{90\cancel{0}}{10\cancel{0}} \cdot \frac{(1+s)\left(1+\frac{s^2}{900}\right)}{s^2 \cdot \left(1+\frac{s}{10}\right)^2} = \frac{9}{s^2} \cdot \frac{(1+s)(1+s^2/900)}{(1+s/10)^2}.$$

$K_B = 9$

$p_{1/2} = \emptyset$

$p_{3/4} = -10$

$z_1 = -1$

$\alpha_m = 30$

$\xi = \emptyset$

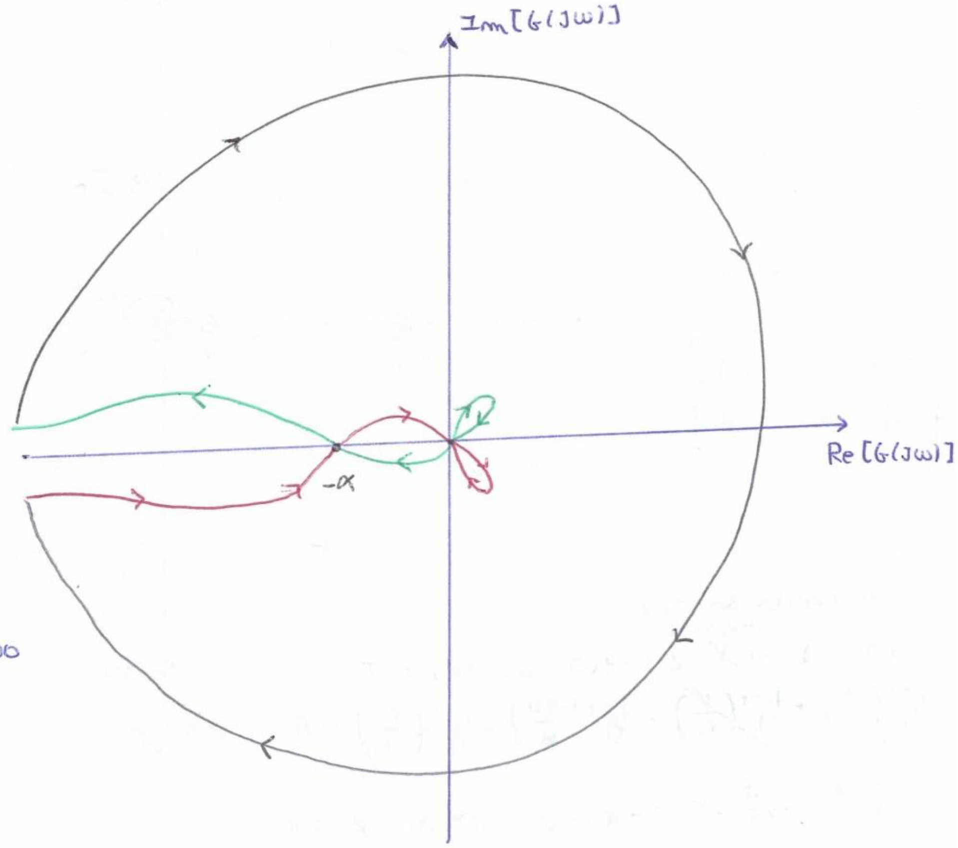
$$\left|\frac{9}{(-\omega^2)}\right| \, 1$$

$$\frac{9}{\omega^2} = 1$$

$$\Rightarrow \omega = 3$$

$$\frac{9}{(0,01)^2} = x_{lim} = 90000$$

$x_{dB} = 99$



$$N_{Pd}^{AC} = N_{Pd}^{AA} + N_{OR, -1}$$

$K > \emptyset$: $K > \frac{1}{4}$: 2 Pd $\quad$ $K < \emptyset$: 1 Pd

$K < \frac{1}{9}$: STABILE

$$G(s) = \frac{(s+1)(s^2+900)}{s^2(s+10)^2} \qquad n - m = 4 - 3 = 1$$

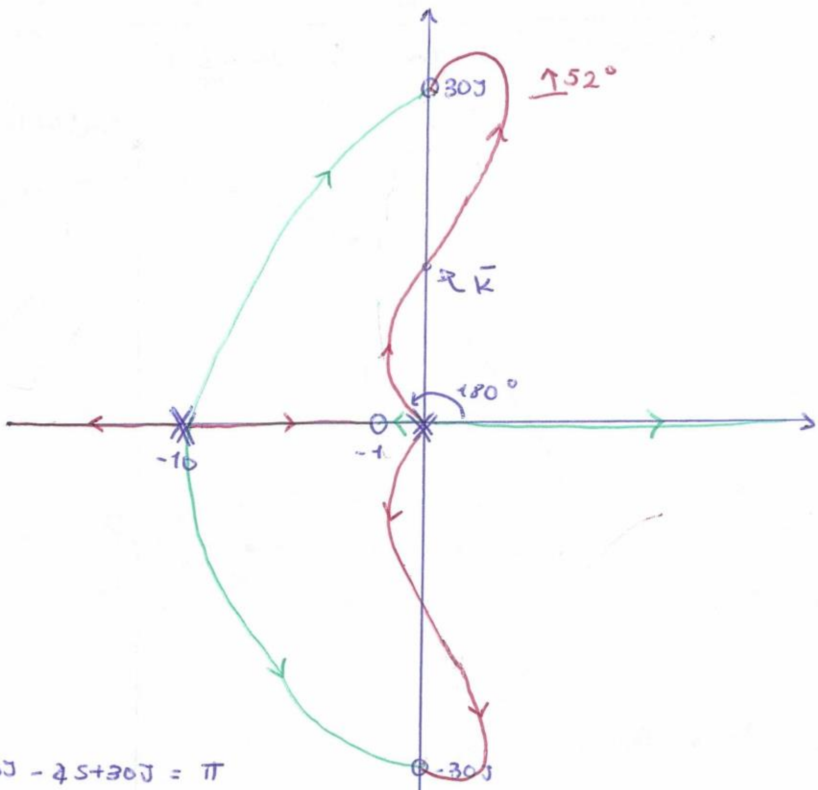
$K^* = K$

$P_{1/2} = \emptyset$

$P_{3/4} = -10$

$z_1 = -1$

$z_{2/3} = \pm 30j$



$$S_0 = \frac{-10-10+1}{1} = -19$$

$$\begin{cases} (2h+1)\pi = \{\pi\} \\ 2h\pi = \{\emptyset\} \end{cases}$$

LD:

Fase di uscita da $p = \emptyset$

$$\measuredangle s+10 + \measuredangle s+10 + \measuredangle s-\emptyset - \measuredangle s-30j - \measuredangle s+30j = \pi$$

$$\cancel{tg^{-1}\left(\frac{\emptyset}{10}\right)} + \cancel{tg^{-1}\left(\frac{\emptyset}{10}\right)} - tg^{-1}\left(\frac{-30}{\emptyset}\right) - tg^{-1}\left(\frac{30}{\emptyset}\right) - \pi = -\measuredangle s-\emptyset$$

$$-\left(-\cancel{\frac{\pi}{2}}\right) - \cancel{\frac{\pi}{2}} - \pi = -\measuredangle s-\emptyset \quad \Rightarrow \measuredangle s-\emptyset = \pi$$

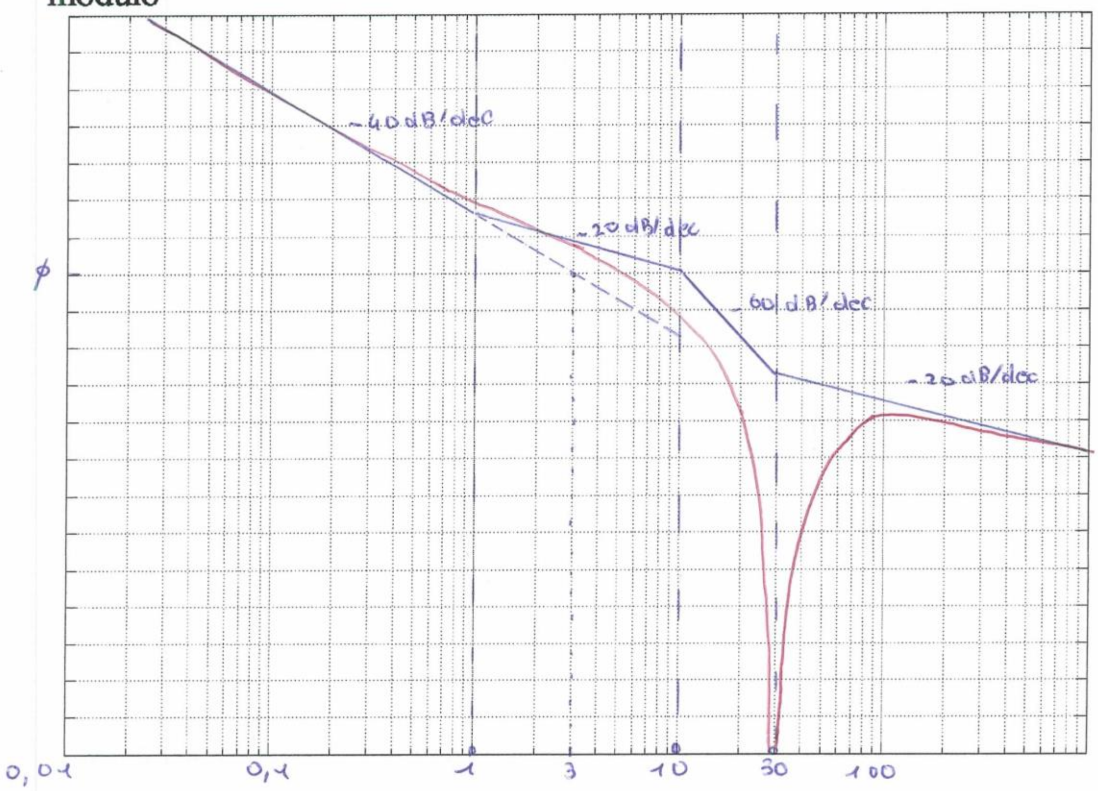
Fase di ingresso in $z = 30j$

$$2\measuredangle s+10 + 2\cdot\measuredangle s-\emptyset - \measuredangle s+30j = \measuredangle s-30j$$

$$2\cdot tg^{-1}\left(\frac{30}{10}\right) + 2\cdot tg^{-1}\left(\frac{30}{\emptyset}\right) + 2\cdot tg^{-1}\left(\frac{60}{\emptyset}\right) = \measuredangle s-30j$$

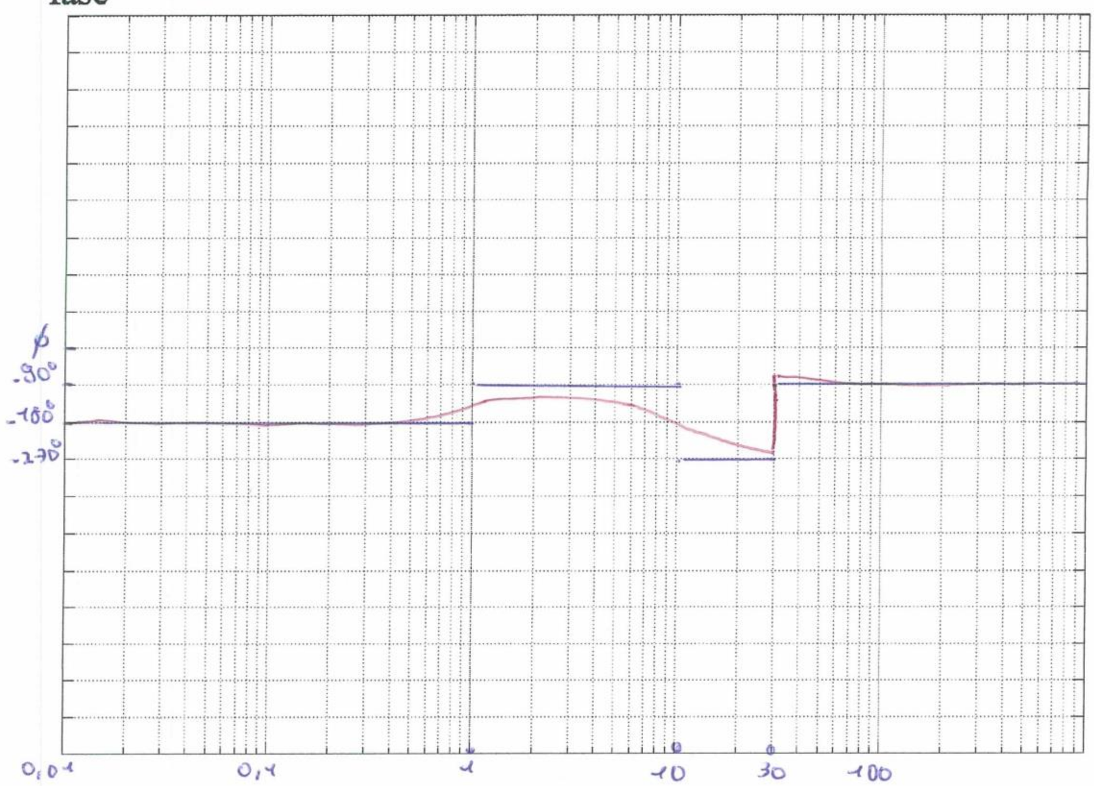
$$2\cdot 71 + 2\cdot 90 + 90 = 412 = \measuredangle s-30j$$

modulo



-40 dB/dec
-20 dB/dec
-60 dB/dec
-20 dB/dec
ϕ
0,01
0,1
1
3
10
30
100

fase



ϕ
-90°
-180°
-270°
0,01
0,1
1
10
30
100

21/01/2015 ES. 1

$$G(s) = -8 \cdot \frac{(10s+1)}{(s-1)^2(s^2+400)} = +\frac{\overset{1}{\cancel{8}}}{\underset{50}{\cancel{400}}} \cdot \frac{(1+10s)}{(1-s)^2\left(1+\frac{s^2}{400}\right)}$$

non dà -

$$= \frac{1}{50} \cdot \frac{(1+10s)}{(1-s)^2\left(1+\frac{s^2}{400}\right)}$$

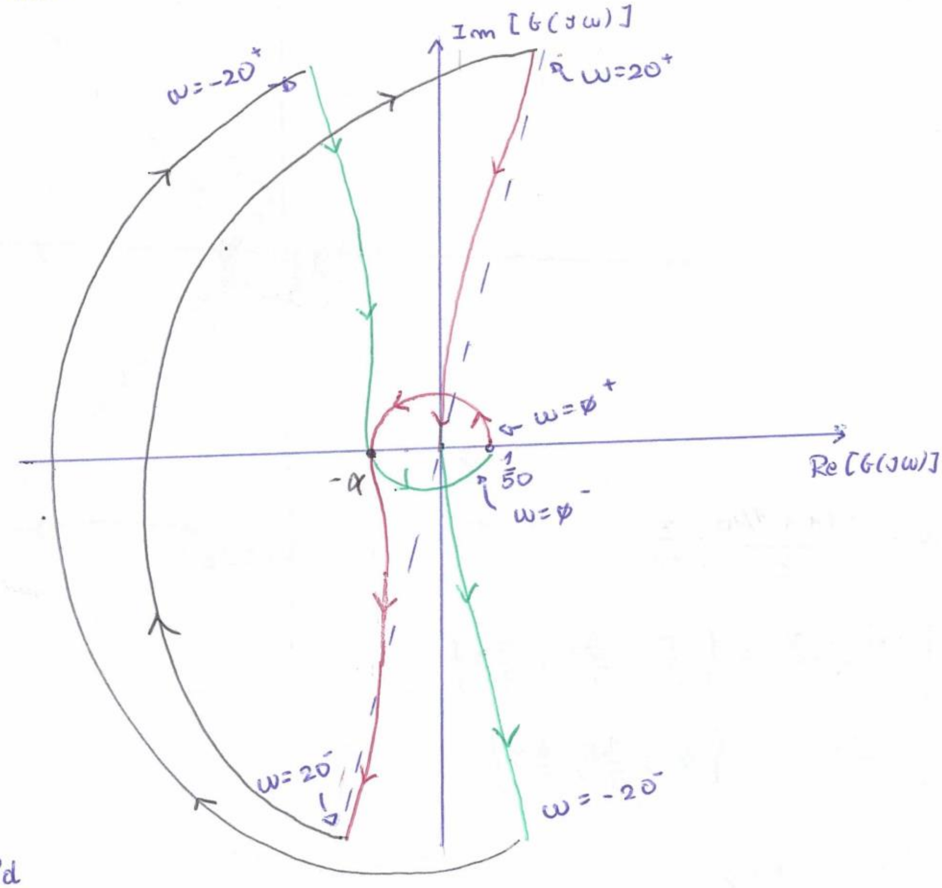
$K_B = \frac{1}{50}$

$z_1 = -\frac{1}{10}$

$p_{1/2} = 1$

$\omega_m = 20$

$\xi = \emptyset$



$N_{Pd}^{AC} = \underset{2}{N_{Pd}^{AA}} + N_{OR,-1}$

$K > \emptyset$:

$K > \frac{1}{\alpha}$: 2 Pd

$K < \frac{1}{\alpha}$: 4 Pd

$K < \emptyset$: $|K| > 50$: 1 Pd

$|K| < 50$: 2 Pd

$$G(s) = -8 \cdot 10 \frac{(s + 1/10)}{(s-1)^2(s^2+400)} = -80 \cdot \frac{(s+\frac{1}{10})}{(s-1)^2(s^2+400)}$$

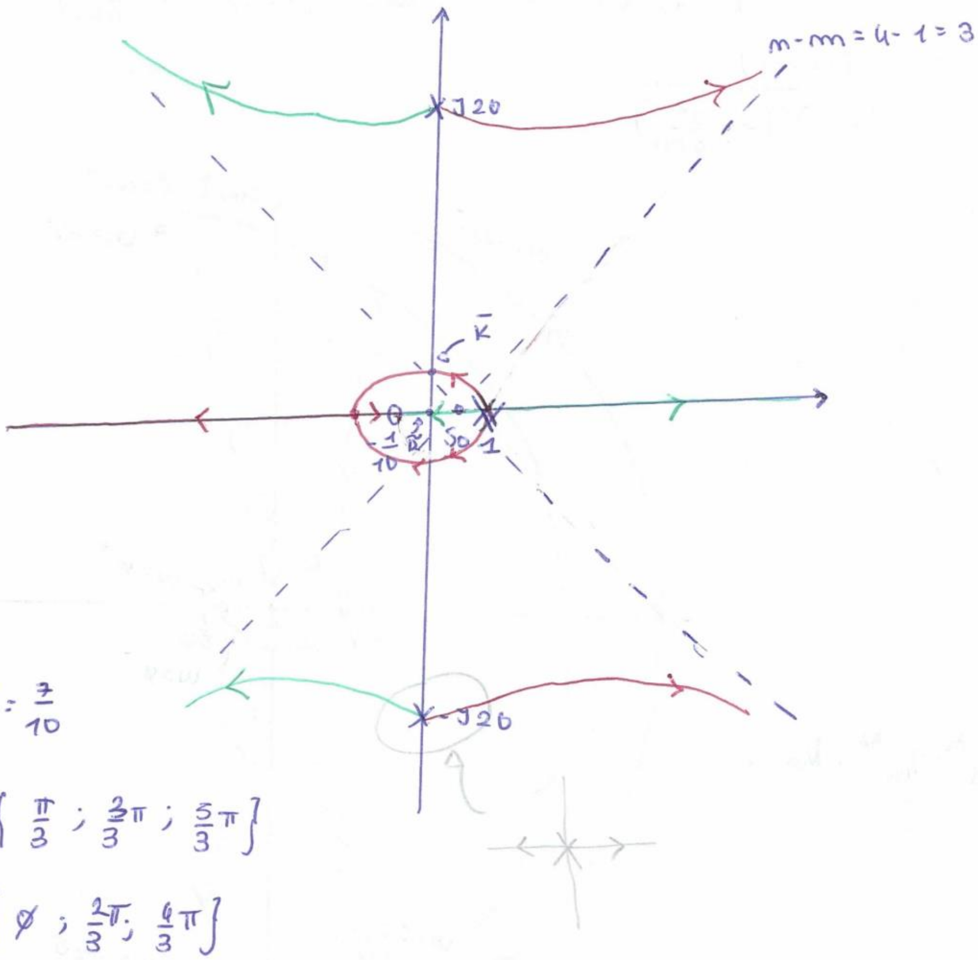
$K_G = -80$

$K^* = -80K$

$z_1 = -\frac{1}{10}$

$p_{1/2} = 1$

$p_{3/4} = \pm j20$



$$s_0 = \frac{1+1+1/10}{3} = \frac{7}{10}$$

$$\begin{cases} \frac{(2h+1)\pi}{3} = \left\{ \frac{\pi}{3} ; \frac{3}{3}\pi ; \frac{5}{3}\pi \right\} \\ \frac{2h\pi}{3} = \left\{ \emptyset ; \frac{2}{3}\pi ; \frac{4}{3}\pi \right\} \end{cases}$$

$K^* > \emptyset$: $K < \emptyset$

$|K| < \bar{K}$: 4Pd

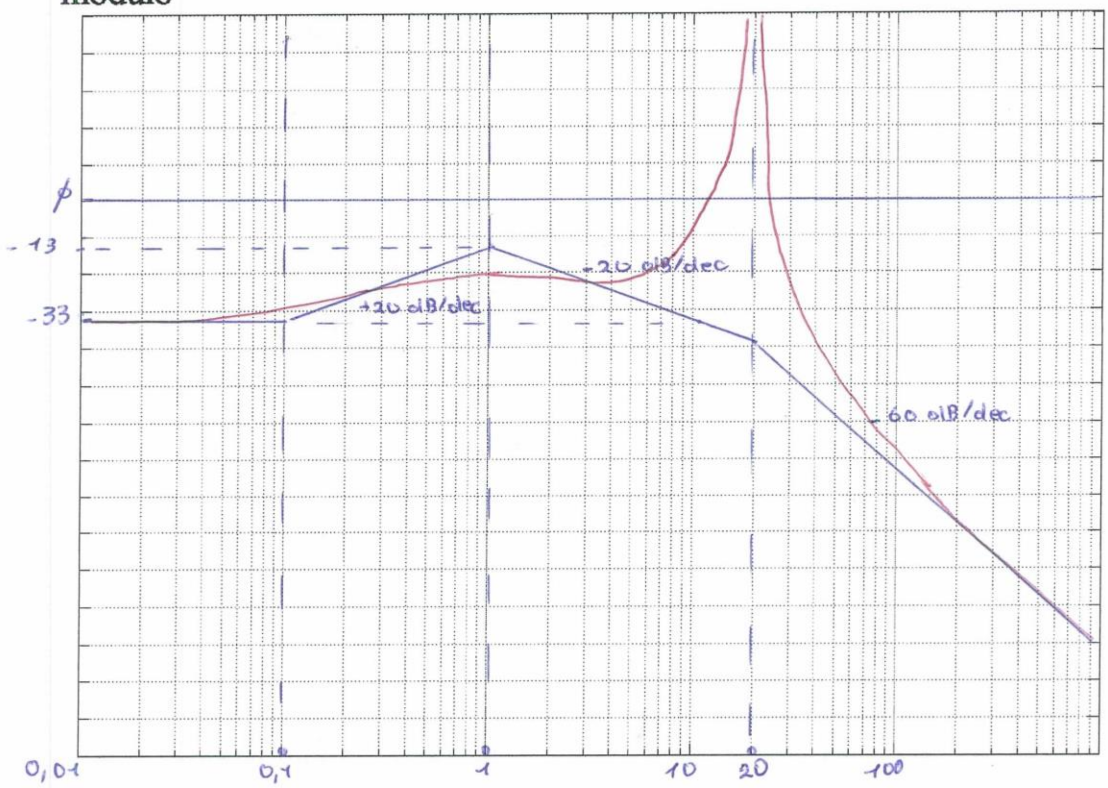
$|K| > \bar{K}$: 2Pd

$K^* < \emptyset$: $K > \emptyset$

$K > \tilde{K}$: 1Pd

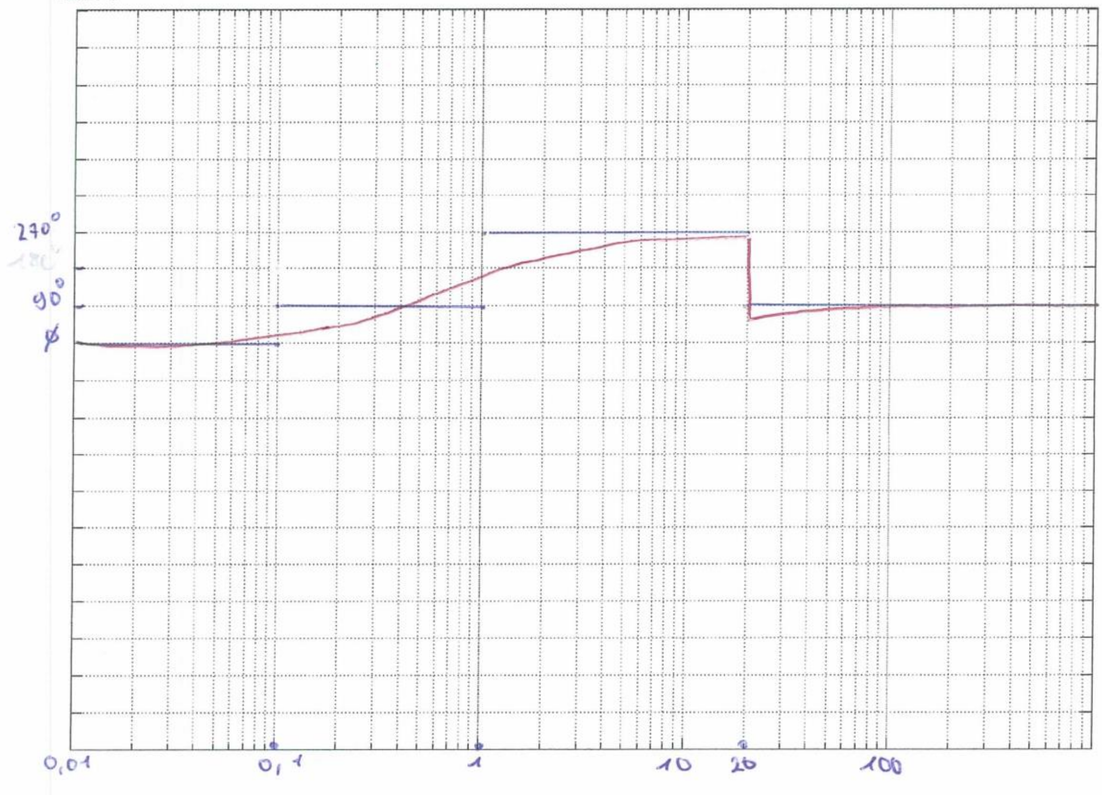
$K < \tilde{K}$: 2Pd

modulo



ϕ
-13
-33
+20 dB/dec
-20 dB/dec
-60 dB/dec
0,01
0,1
1
10
20
100

fase



270°
90°
ϕ
0,01
0,1
1
10
20
100

17/06/2015 ES. 1

$$G(s) = \frac{1000\,(s-10)^2}{(s+10)(s^2+2500)} = + \frac{\overset{2}{\cancel{1000}}\cdot \cancel{100}}{\cancel{10}\cdot \underset{5}{\cancel{2500}}} \cdot \frac{\left(1-\frac{s}{10}\right)^2}{\left(1+\frac{s}{10}\right)\left(1+\frac{s^2}{2500}\right)}$$

$$= 4 \cdot \frac{(1-s/10)^2}{(1+s/10)(1+s^2/2500)}$$

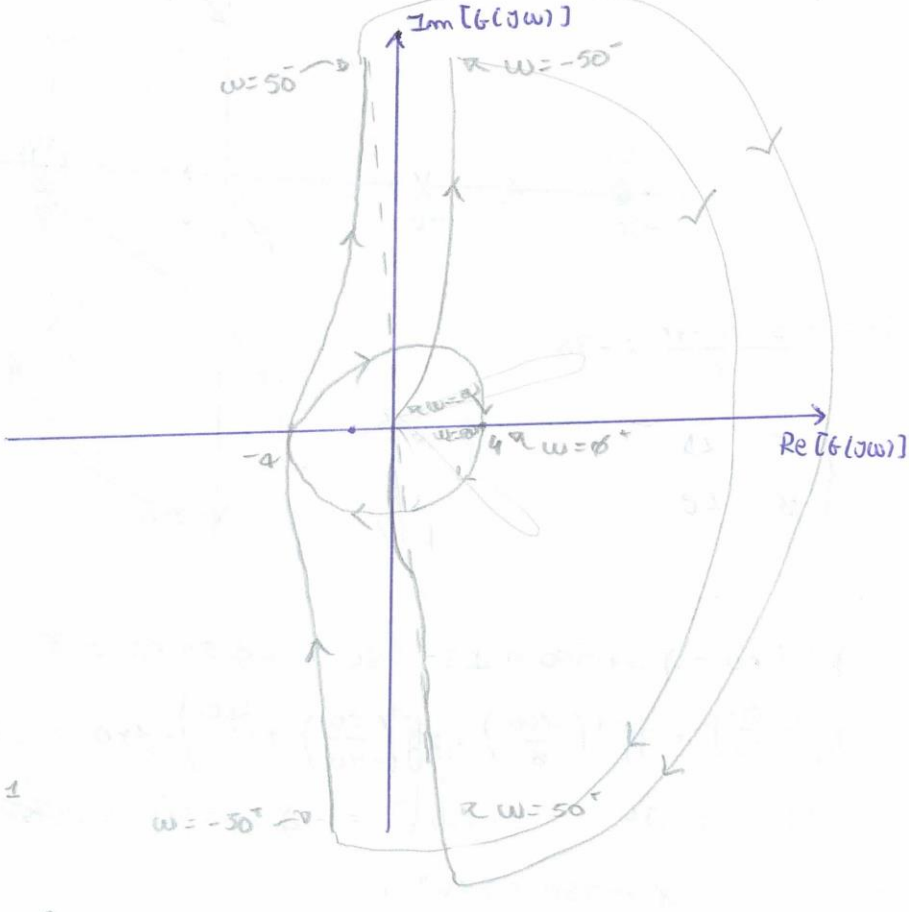
$K_B = 4$

$z_{1/2} = 10$

$p_1 = -10$

$\omega_n = 50$

$g = \emptyset$



$N_{pd}^{LC} = \cancel{N_{pd}^{AA}} \xrightarrow{} \emptyset + N_{oe,-1}$

$k>\emptyset$:

$k > \frac{1}{\alpha} \rightarrow 2$ Pd

$k < \frac{1}{\alpha} \rightarrow$ STAB

$k<\emptyset$: ~~2Pd~~ $|k| > \frac{1}{?} \rightarrow 3$ Pd

$|k| < \frac{1}{4} \rightarrow 2$ Pd

$$G(s) = 1000 \cdot \frac{(s-10)^2}{(s+10)(s^2+2500)}$$

$n - m = 1$

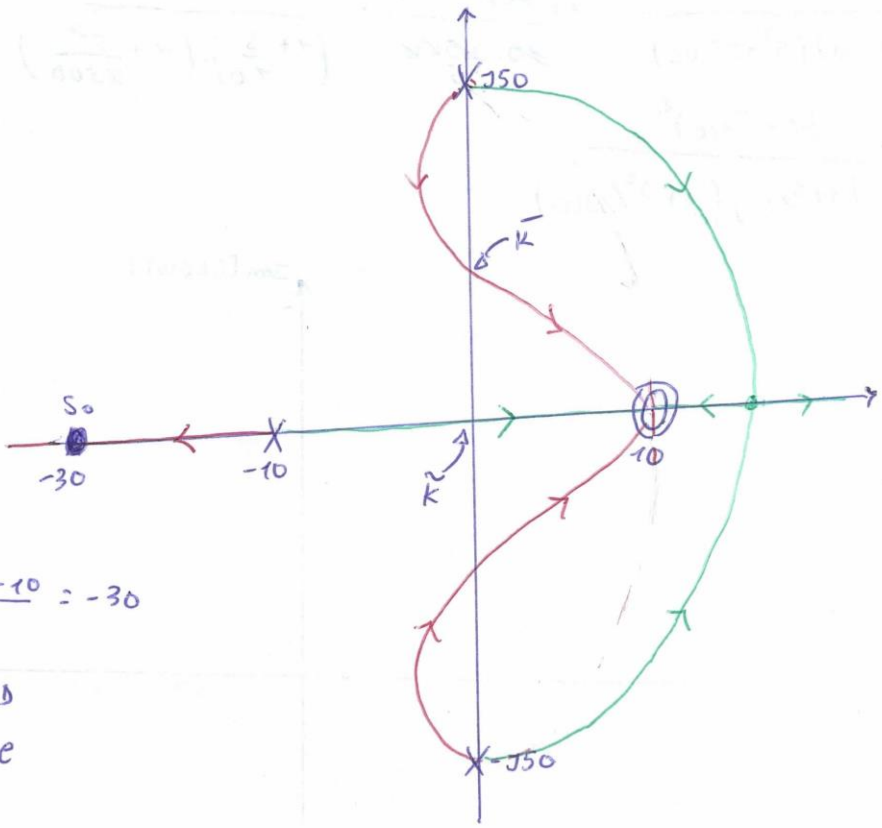
$K_B = 1000$

$K^* = 1000 \cdot K$

$z_{1/2} = 10$

$p_1 = -10$

$p_{2,3} = \pm j50$



$$s_0 = \frac{-10-10-10}{1} = -30$$

$$\begin{cases} \pi & LD \\ 0 & LC \end{cases}$$

$$\measuredangle s+10 + \measuredangle s+j50 + \measuredangle s-j50 - 2\measuredangle s+10 = \pi$$

$$tg^{-1}\left(\frac{50}{10}\right) + tg^{-1}\left(\frac{100}{0}\right) - \left(2tg^{-1}\left(\frac{50}{-10}\right) + 360\right) - 180 = -\measuredangle s - j50$$

$$78 + 90 \quad = -\measuredangle s - j50 = \cancel{324°} -214°$$

$$\measuredangle s - j50 = 214°$$

$K > 0$

$K > \bar{K}$: 2Pd

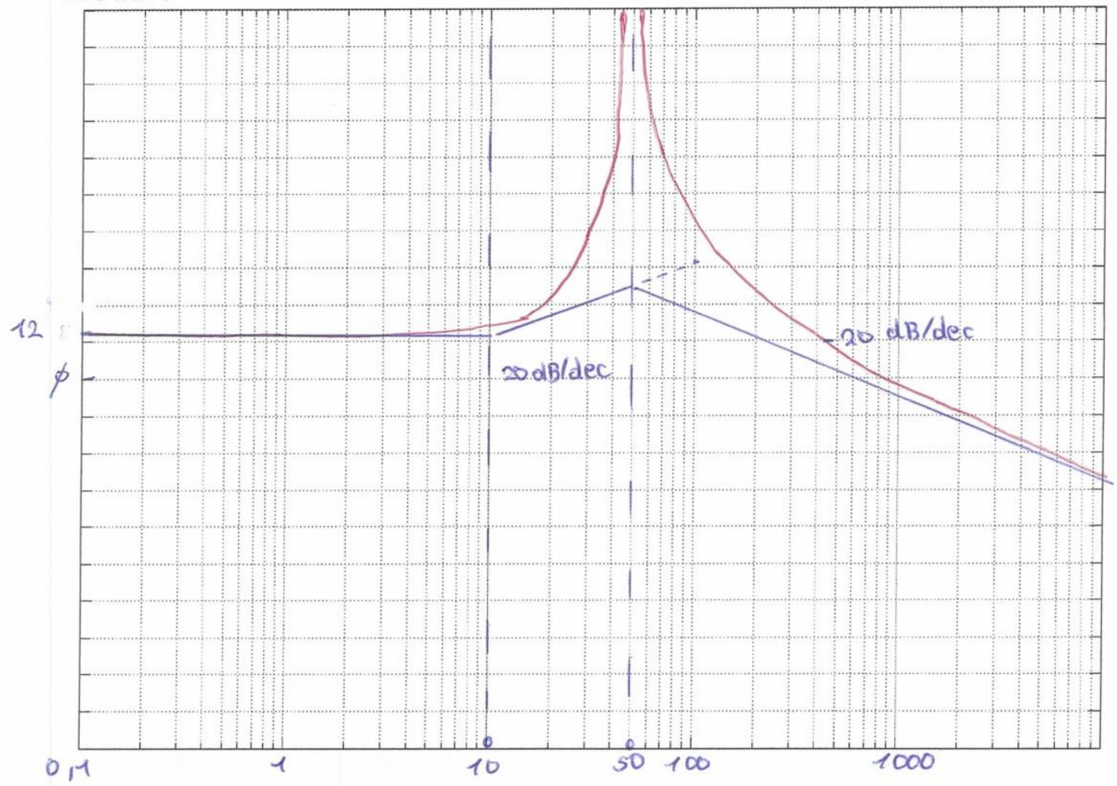
$K < \bar{K}$: STABILE

$K < 0$:

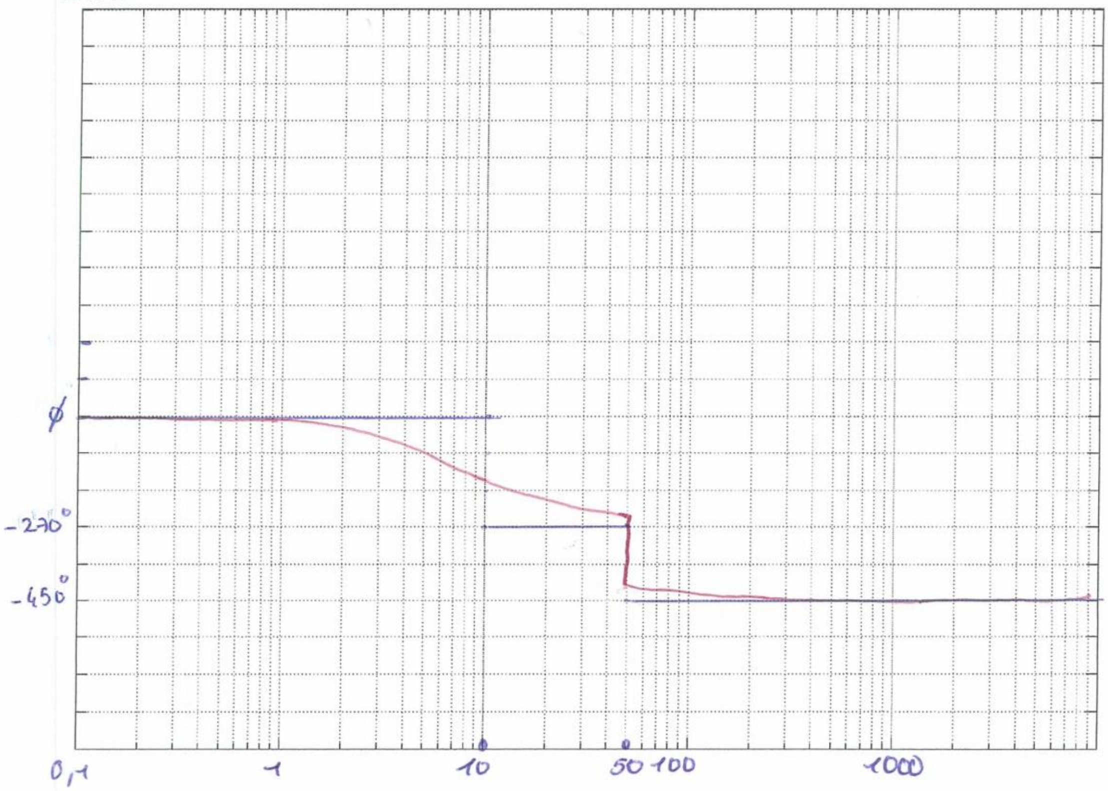
$|K| > \tilde{K}$: 3Pd

$|K| < \tilde{K}$: 2Pd

modulo


12
ϕ
20 dB/dec
-20 dB/dec
0,1
1
10
50
100
1000

fase


ϕ
-270°
-450°
0,1
1
10
50
100
1000

10/08/2015 Es. 1

$$G(s) = \frac{100(s+10)(s-100)}{s^2(s+100)^2} = \frac{\cancel{100}\cdot 10\cdot \cancel{100}}{\cancel{100}\cdot\cancel{100}}\cdot\frac{\left(1+\frac{s}{10}\right)\left(1-\frac{s}{100}\right)}{s^2\left(1+\frac{s}{100}\right)^2}$$

$$= -10\cdot\frac{\left(1+\frac{s}{10}\right)\left(1-\frac{s}{100}\right)}{s^2\left(1+\frac{s}{100}\right)^2}$$

$K_B = -10$

$z_1 = -10$

$z_2 = 100$

$p_{1/2} = \emptyset$

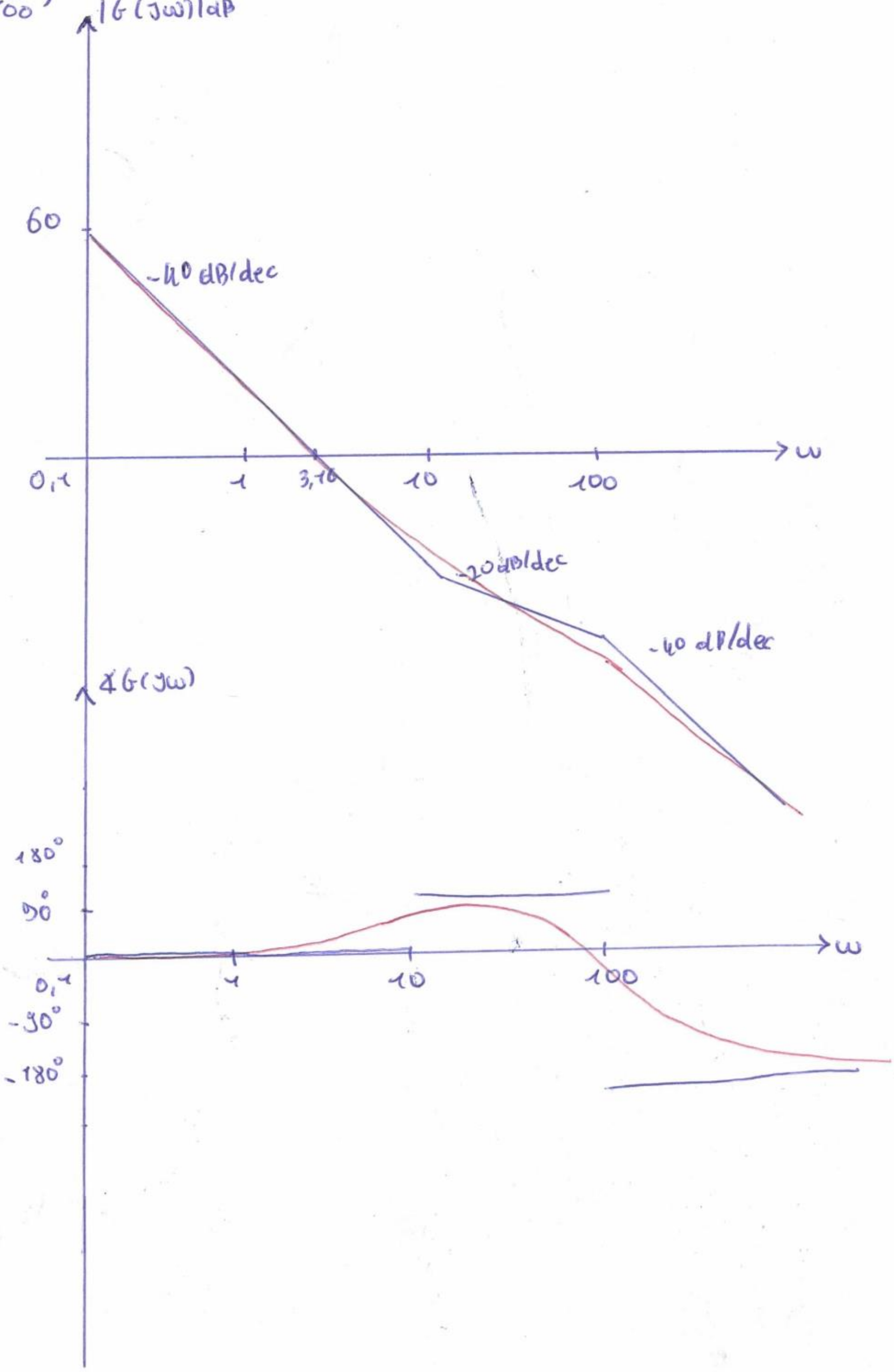
$p_{3/4} = -100$

$\frac{10}{\omega^2} = 1$

$\rightarrow \omega = 3,16.$

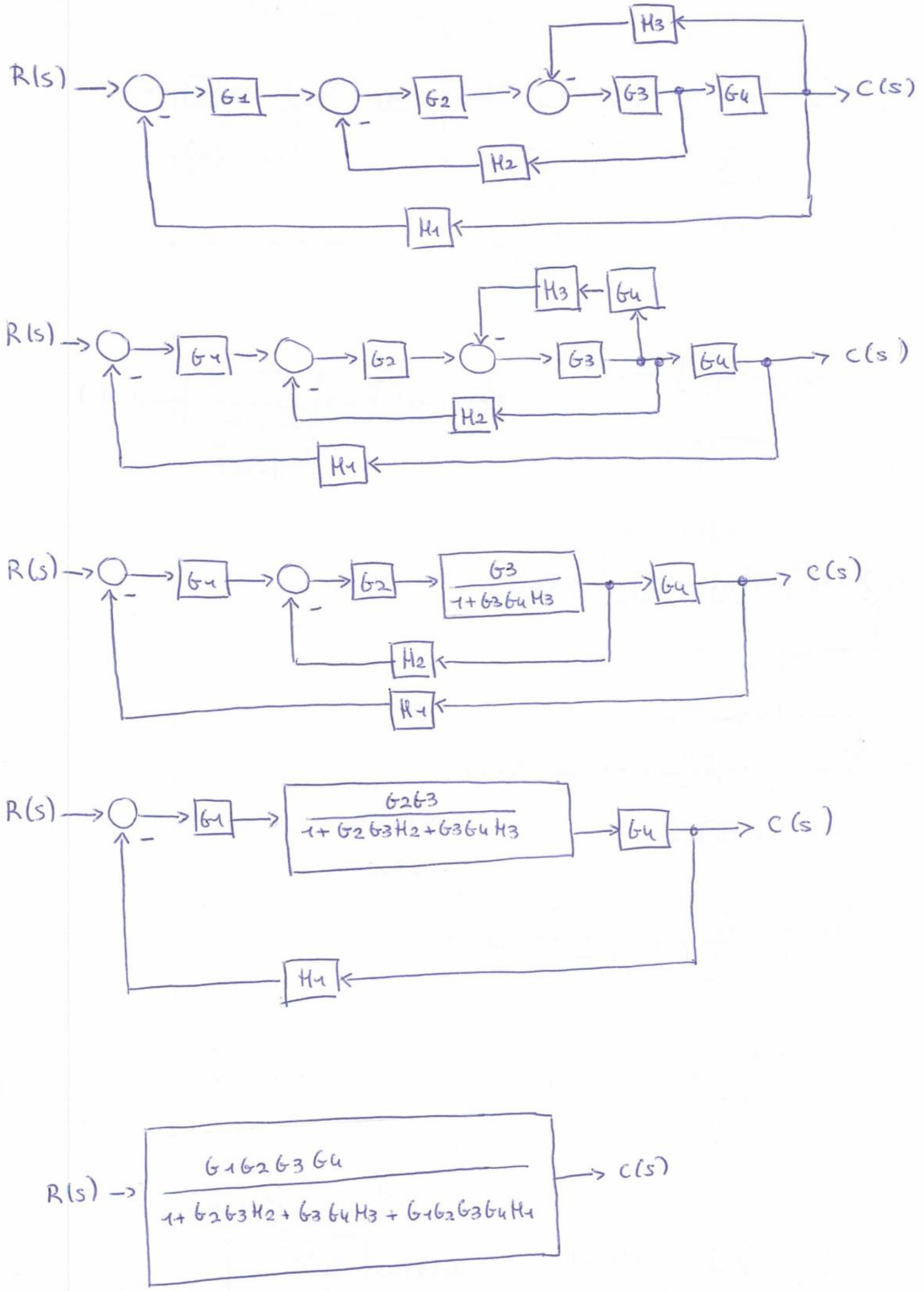
$\frac{10}{(0,1)^2} = x_{lin} = 1000$

$x_{dB} = 60$

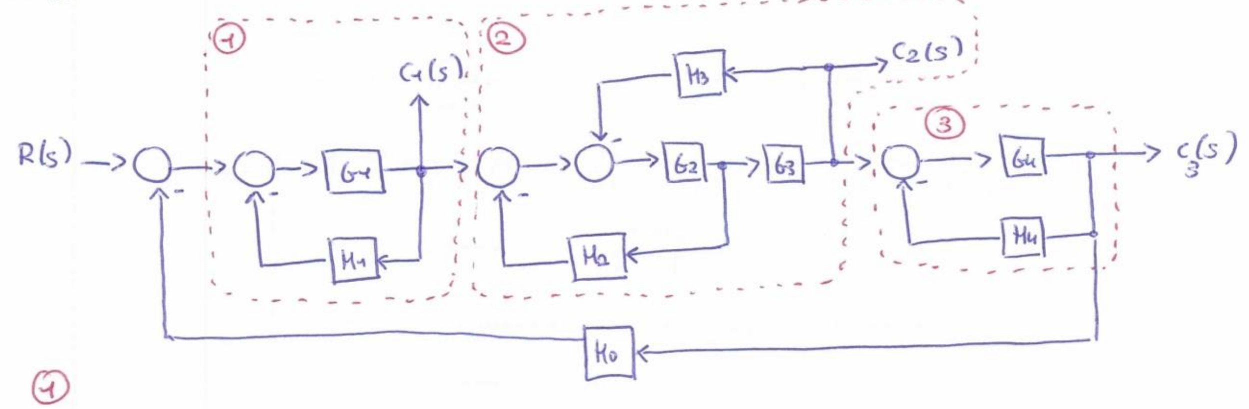


SCHEMI A BLOCCHI

ES. 1

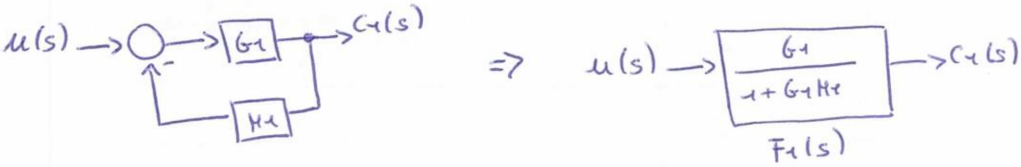


ES. 2



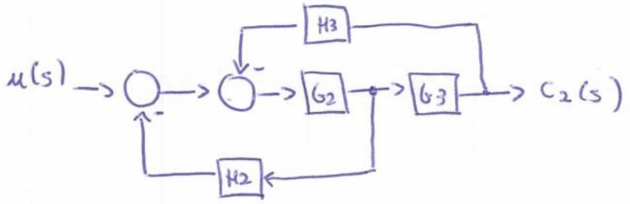
1
2
3
R(s)
G1
H1
C1(s)
H3
G2
G3
H2
C2(s)
G4
H4
C3(s)
H0

1

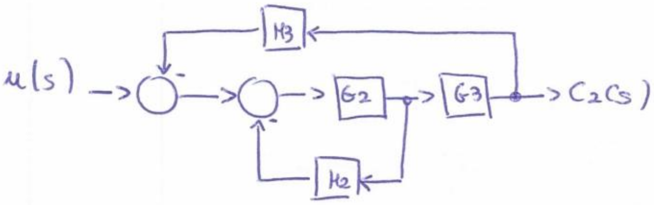


u(s)
G1
H1
C1(s)
=>
u(s)
$\frac{G_1}{1+G_1H_1}$
C1(s)
F1(s)

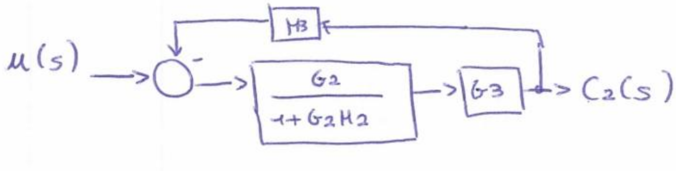
2



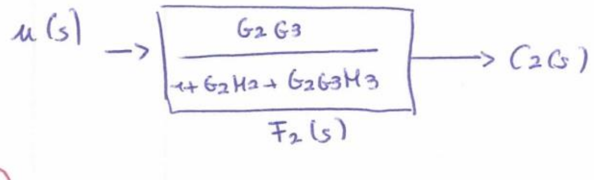
u(s)
H3
G2
G3
H2
C2(s)



u(s)
H3
G2
G3
H2
C2(s)

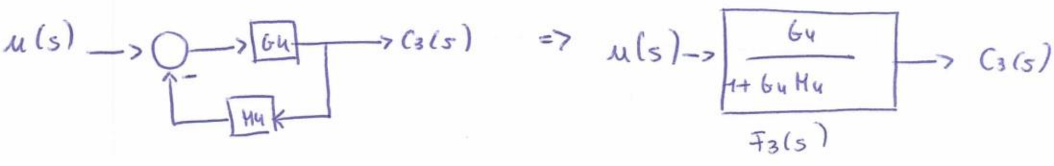


u(s)
H3
$\frac{G_2}{1+G_2H_2}$
G3
C2(s)

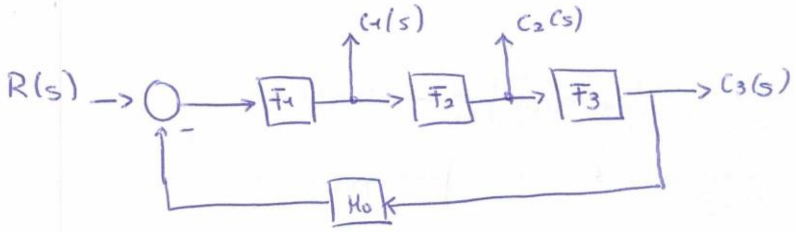


u(s)
$\frac{G_2G_3}{1+G_2H_2+G_2G_3H_3}$
C2(s)
F2(s)

3



u(s)
G4
H4
C3(s)
=>
u(s)
$\frac{G_4}{1+G_4H_4}$
C3(s)
F3(s)

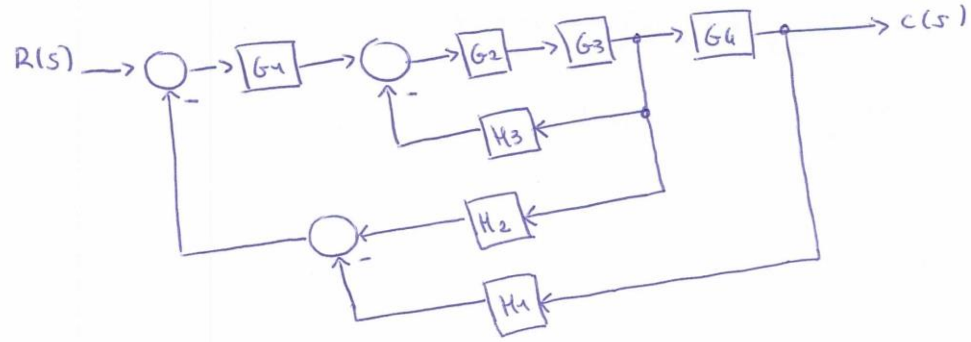


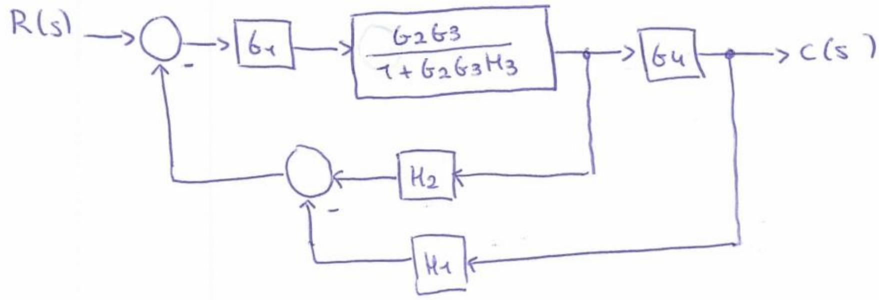
$$\frac{C_1(s)}{R(s)} = \frac{F_1}{1+F_1F_2F_3H_0}$$

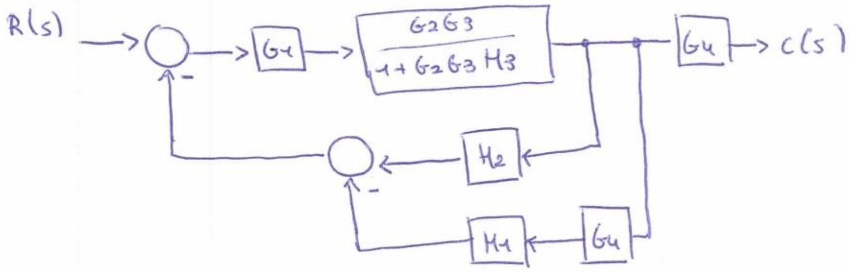
$$\frac{C_2(s)}{R(s)} = \frac{F_1F_2}{1+F_1F_2F_3H_0}$$

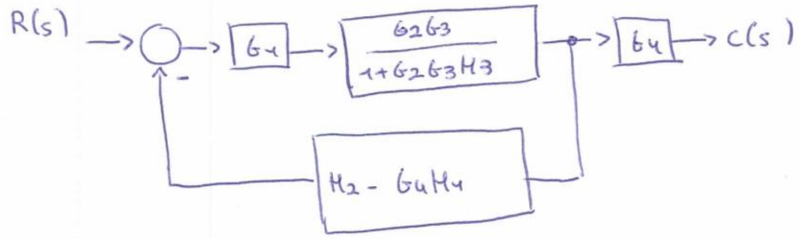
$$\frac{C_3(s)}{R(s)} = \frac{F_1F_2F_3}{1+F_1F_2F_3H_0}$$

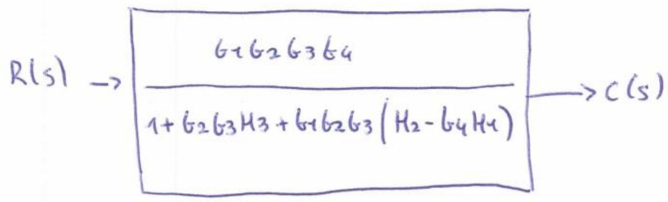
Es. 3


R(s)
G1
G2
G3
G4
C(s)
H3
H2
H1
-
-
-

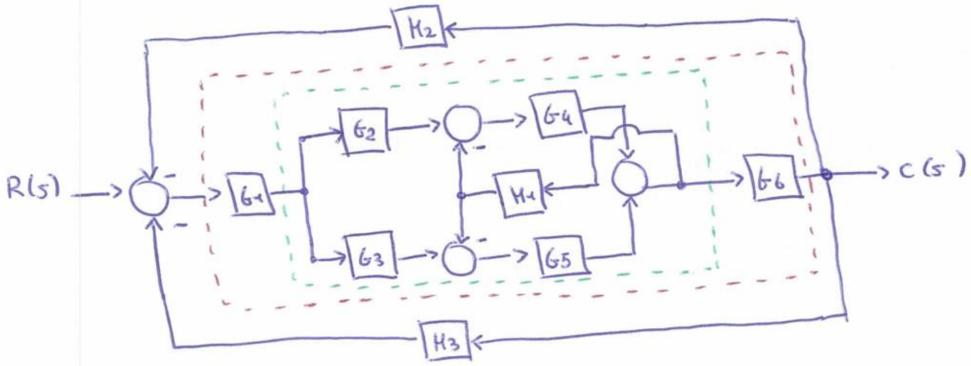

R(s)
G1
$\frac{G_2G_3}{1+G_2G_3H_3}$
G4
C(s)
H2
H1
-
-

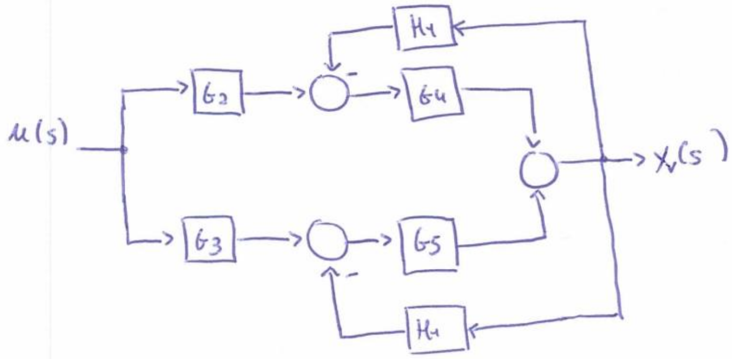

R(s)
G1
$\frac{G_2G_3}{1+G_2G_3H_3}$
G4
C(s)
H2
H1
G4
-
-

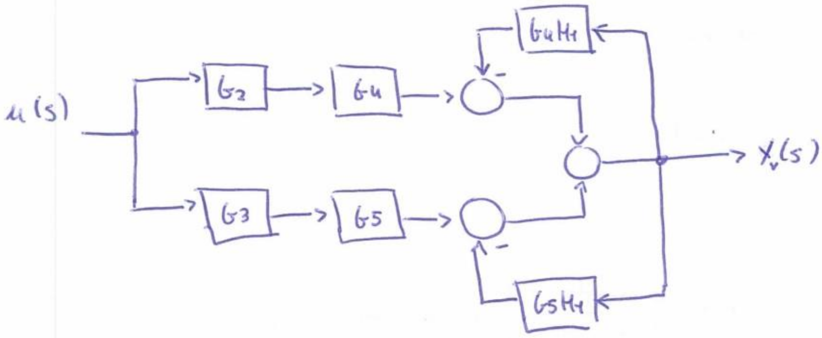

R(s)
G1
$\frac{G_2G_3}{1+G_2G_3H_3}$
G4
C(s)
$H_2 - G_4H_1$
-

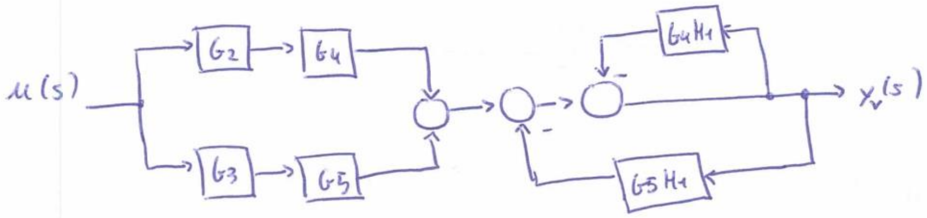

R(s)
$\frac{G_1G_2G_3G_4}{1+G_2G_3H_3+G_1G_2G_3(H_2-G_4H_1)}$
C(s)

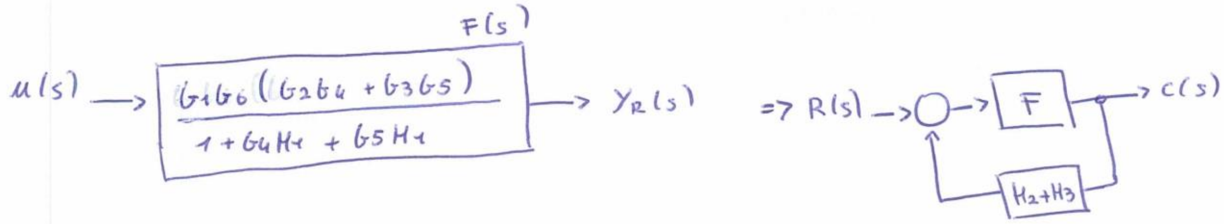
Es. 4


H2
R(s)
G1
G2
G4
H1
G3
G5
G6
C(s)
H3

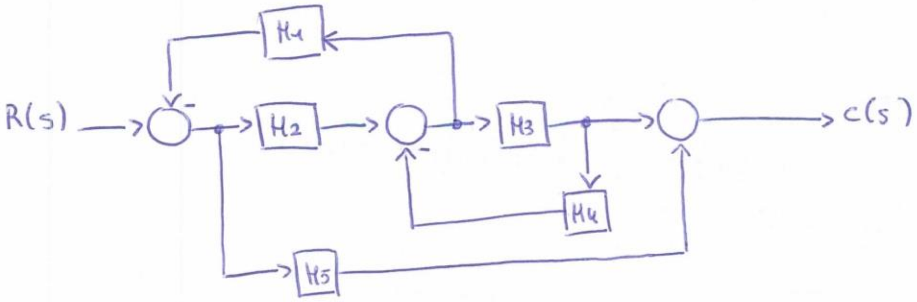

H1
u(s)
G2
G4
G3
G5
H1
$X_v(s)$

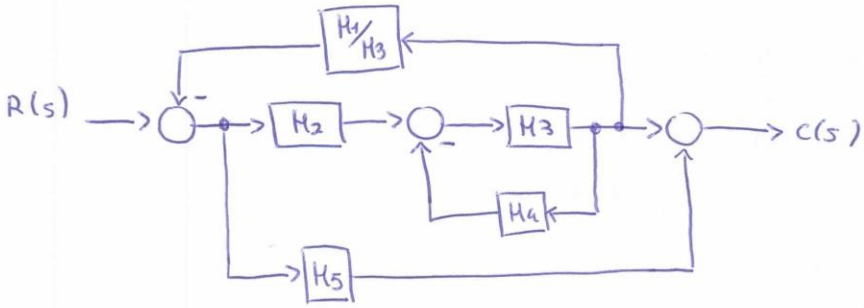

G4H1
u(s)
G2
G4
G3
G5
$X_v(s)$
G5H1

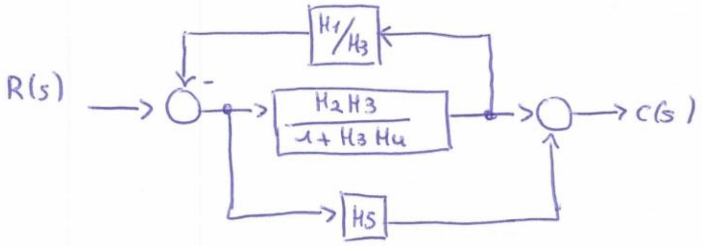

u(s)
G2
G4
G3
G5
G4H1
$X_v(s)$
G5H1

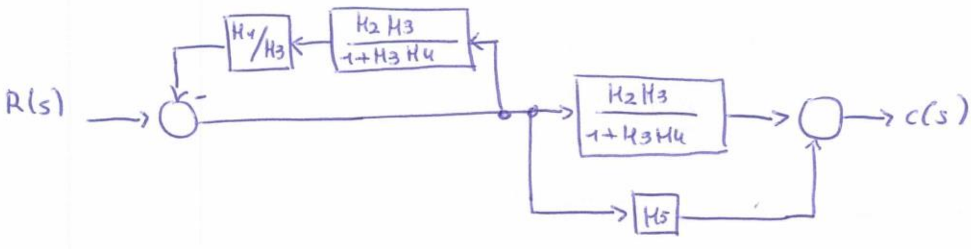

F(s)
u(s)
$\frac{G_1G_6(G_2G_4+G_3G_5)}{1+G_4H_1+G_5H_1}$
$Y_R(s)$
=> R(s)
F
C(s)
H2+H3

Es. 5



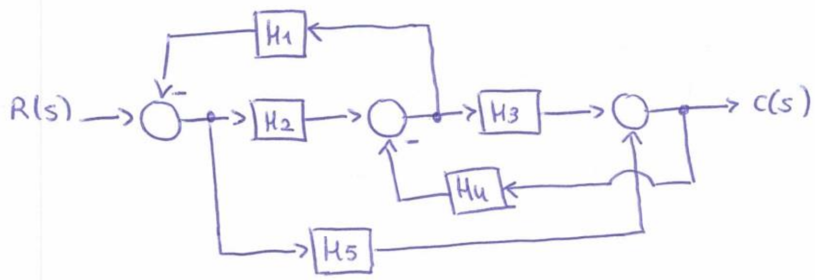


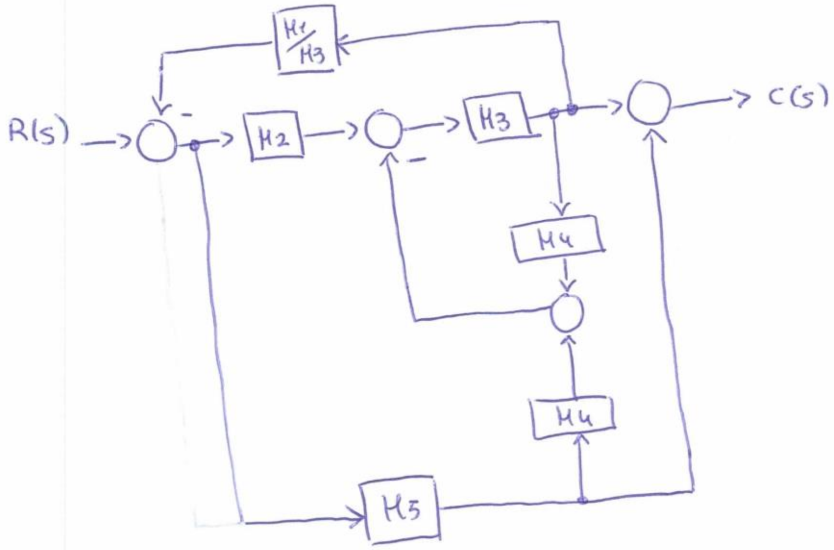


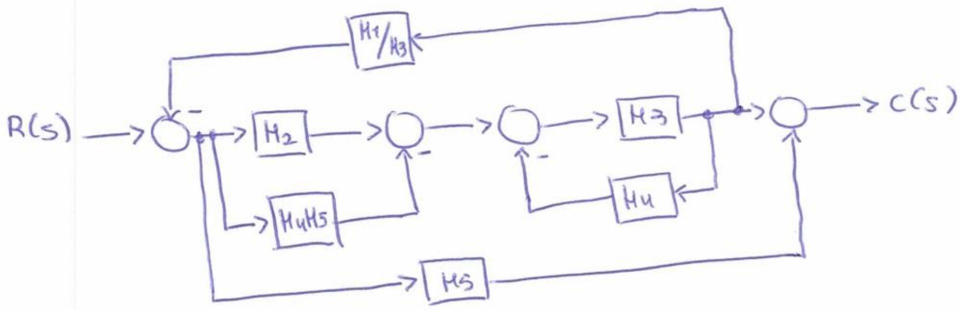


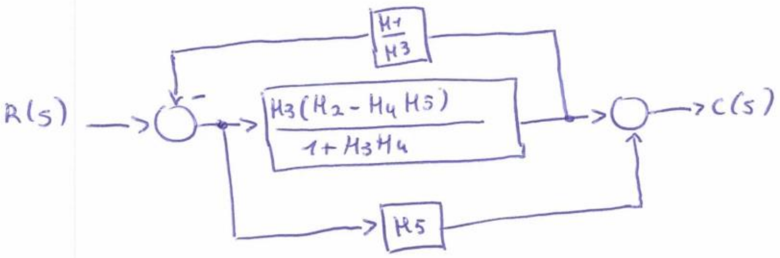
$$R(s) \longrightarrow \boxed{\frac{H_5 + H_2 H_3 + H_3 H_4 H_5}{1 + H_1 H_2 + H_3 H_4}} \longrightarrow C(s)$$

Es. 6


R(s)
C(s)
H1
H2
H3
H4
H5


R(s)
C(s)
H1/H3
H2
H3
H4
H4
H5


R(s)
C(s)
H1/H3
H2
H4H5
H3
H4
H5


R(s)
C(s)
H1/H3
H3(H2 - H4 H5) / 1 + H3H4
H5

$$R(s) \longrightarrow \boxed{\frac{H_2H_3 + H_5}{1+H_1H_2+H_3H_4-H_1H_4H_5}} \longrightarrow C(s)$$

CRITERIO DI ROUTH

ES. 1

$$P(s) = 4s^4 + 3s^3 + 5s^2 + 2s + 1$$

n			
4	4	5	1
3	3	2	
2	7	3	
1	5		
∅	3		

4 Radice $Re < \emptyset$.

ES. 2

$$P(s) = s^3 + 3s - 2$$

$\varepsilon > \emptyset$

n		
3	1	3
2	ε	-2
1	$\frac{2+\varepsilon}{\varepsilon}$	
∅	-2	

2 Radici $Re < \emptyset$

1 Radice $Re > \emptyset$

ES. 3

$$P(s) = s^6 + s^5 - 2s^4 - 3s^3 - 7s^2 - 4s - 4$$

n				
6	1	-2	-7	-4
5	1	-3	-4	
4	1	-3	-4	
3	∅	∅		
3'	4	-6		
2	-3	-8		
1	-50			
∅	-8			

Re<∅

Re<∅

Re<∅

Re>∅

$\Rightarrow s^4 - 3s^2 - 4$

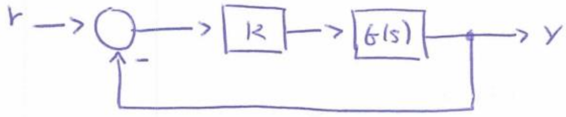
3 Radici $Re < \emptyset$

1 Radice $Re > \emptyset$

6-4 = 2 Radici Im.

Es. 4

$$G(s) = \frac{1}{s^4 + 6s^3 + 11s^2 + 6s + 2}$$



$$F(s) = \frac{K\,G(s)}{1 + K\,G(s)} = \frac{\dfrac{K}{s^4 + 6s^3 + 11s^2 + 6s + 2}}{1 + \dfrac{K}{s^4 + 6s^3 + 11s^2 + 6s + 2}}$$

$$= \frac{K}{s^4 + 6s^3 + 11s^2 + 6s + 2 + K}$$

n			
4	1	11	$2+K$
3	6	6	
2	10	$2+K$	
1	$48-6K$		
$\emptyset$	$2+K$		

$$\begin{cases} 48 - 6K > \emptyset \\ 2 + K > \emptyset \end{cases} \qquad \begin{cases} K < 8 \\ K > -2 \end{cases} \qquad \Rightarrow \qquad -2 < K < 8$$

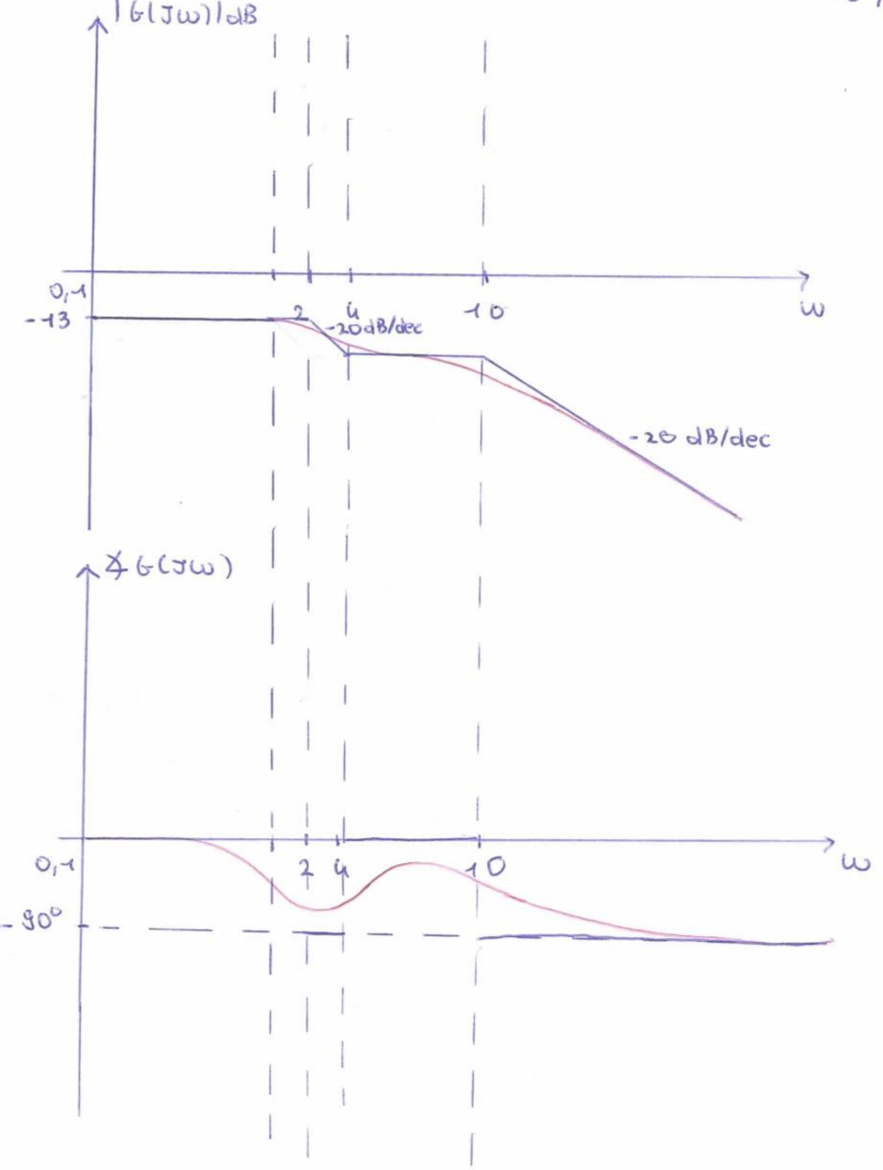
Sistema stabile!

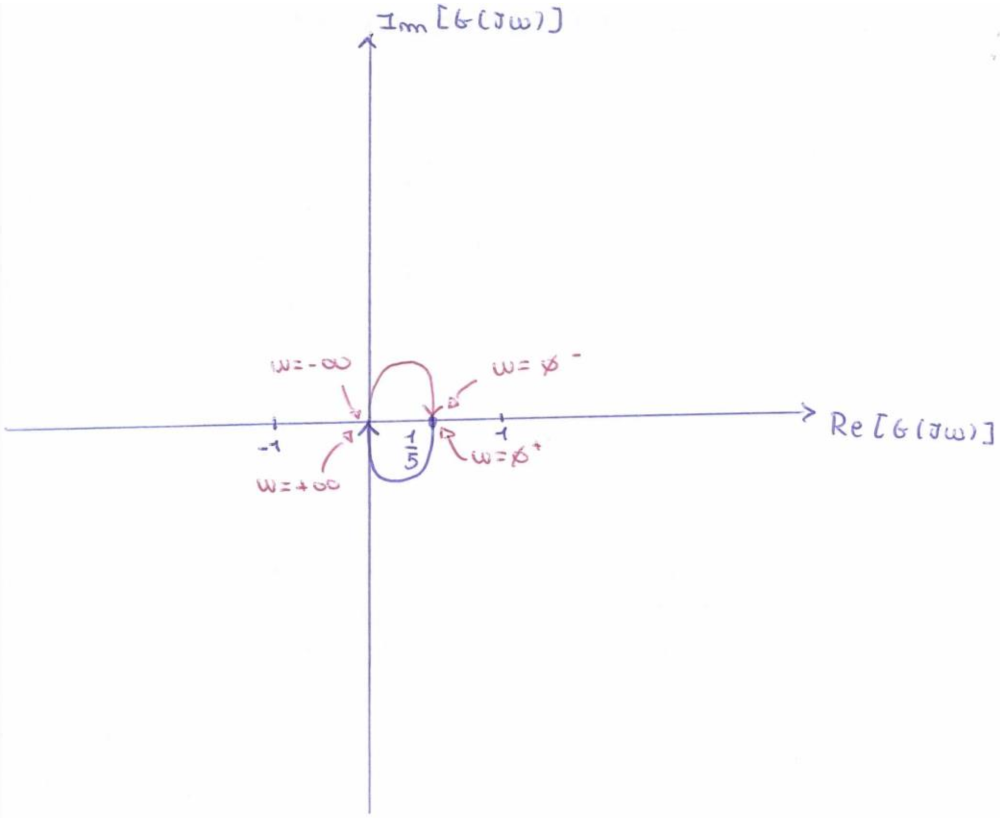
STABILITÀ SISTEMI IN RETROAZIONE

ES. 1

$$G(s) = \frac{s+4}{(s+2)(s+10)} = \frac{4}{2\cdot 10}\,\frac{\left(1+\frac{s}{4}\right)}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)} = \frac{1}{5}\cdot\frac{\left(1+\frac{s}{4}\right)}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}.$$

$$\begin{cases} K_B = 1/5 \\ z_1 = -4 \\ p_1 = -2 \\ p_2 = -10 \end{cases}$$




Im [G(jω)]
Re [G(jω)]
ω = -∞
ω = ∅⁻
-1
1
1/5
ω = +∞
ω = ∅⁺

$N_{Pd}^{AC} = \cancel{N_{Pd}^{AA}} + N_{OR,-1}$

$N_{Pd}^{AA} = \varnothing$

$N_{OR,-1} = \varnothing \Rightarrow N_{Pd}^{AC} = \varnothing$, questo sistema è già stabile in retroazione.

$K > \varnothing$: STABILE

$K < \varnothing$:

$\alpha = \frac{1}{5} \Rightarrow \frac{1}{\alpha} = 5$.

- $|K| > 5$: INSTABILE 1 Pd $\quad K < -5$
- $|K| < 5$: STABILE $\quad K > -5$

$$\begin{cases} K > \varnothing & \text{STABILE} \\ K > -5 & \text{STABILE} \\ K < -5 & \text{INSTABIE } 1Pd \end{cases}$$

$L(s) = (s+2)(s+10) + K(s+4) = s^2 + 10s + 2s + 20 + K \cdot s + 4K$

$= s^2 + s(12+K) + 20 + 4K$

n		
2	1	20+4K
1	12+K	
$\varnothing$	20+4K	

$$\begin{cases} 12 + K > \varnothing \\ 20 + 4K > \varnothing \end{cases} \quad \begin{cases} K > -12 \\ K > -5 \end{cases}$$

$K > -5$: STABILE

ES. 2

$$G(s) = \frac{10}{s^2(s+10)} = \frac{1\cancel{0}}{s^2} \cdot \frac{1}{\left(1+\frac{s}{10}\right)\cdot 1\cancel{0}} = \frac{1}{s^2} \cdot \frac{1}{\left(1+\frac{s}{10}\right)}$$

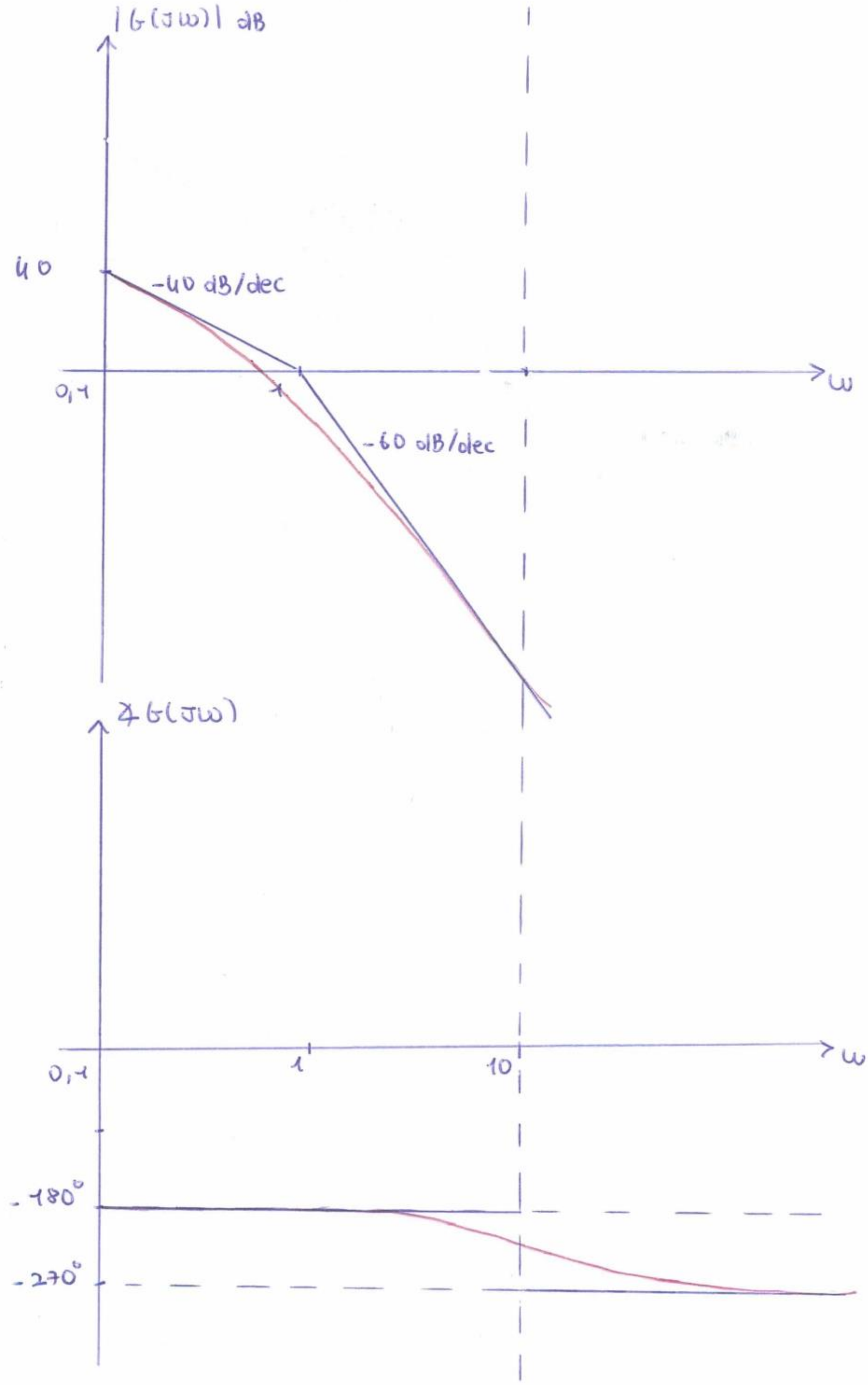
$$\begin{cases} K_B = 1 \\ P_1 = \emptyset \\ P_2 = \emptyset \\ P_3 = -10 \end{cases}$$

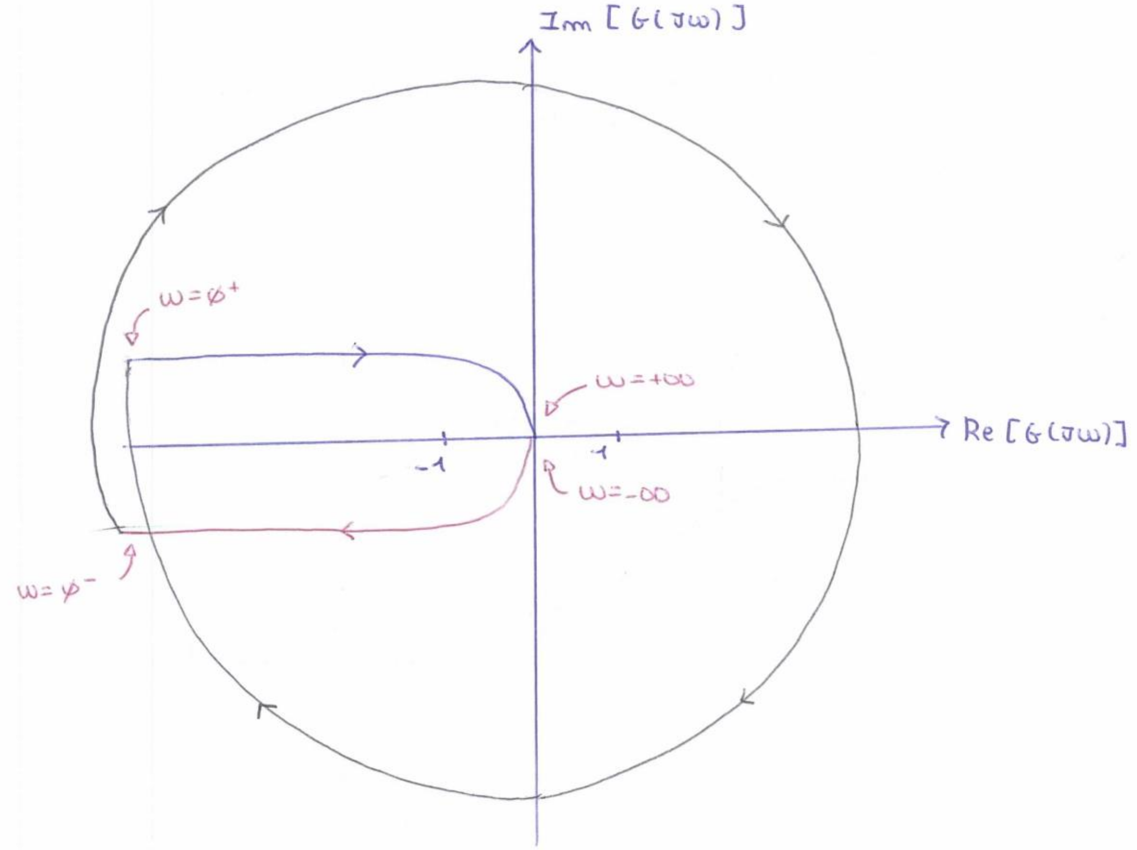
$$\left|\left|\frac{1}{s^2}\right|\right|_{j\omega} = \emptyset \text{ dB}$$

$$\left|\frac{1}{(j\omega)^2}\right| = \emptyset \text{ dB}$$

$$\frac{1}{\omega^2} = \emptyset \text{ dB}$$

$$\omega = 1$$





Im [G(jω)]
Re [G(jω)]
ω = 0+
ω = +∞
-1
1
ω=-∞
ω = 0-

$N_{Pd}^{AC} = \cancel{N_{Pd}^{AA}}^{\to \emptyset} + N_{OR,-1}$

$N_{Pd}^{AA} = \emptyset$

$N_{OR,-1} = 2$

$N_{Pd}^{AC} = 2$, il sistema in retroazione non è stabile.

$K > \emptyset$: INSTABILE 2 Pd

$K < \emptyset$: INSTABILE 1 Pd

$L(s) = s^2(s+10) + 10K = s^3 + 10s^2 + 10K$

n		
3	1	$\emptyset$
2	10	$10K$
1	$-K$	
$\emptyset$	$10K$	

$$\begin{cases} -K > \emptyset \\ 10K > \emptyset \end{cases} \qquad \begin{cases} K < \emptyset \\ K > \emptyset \end{cases}$$

ES. 3

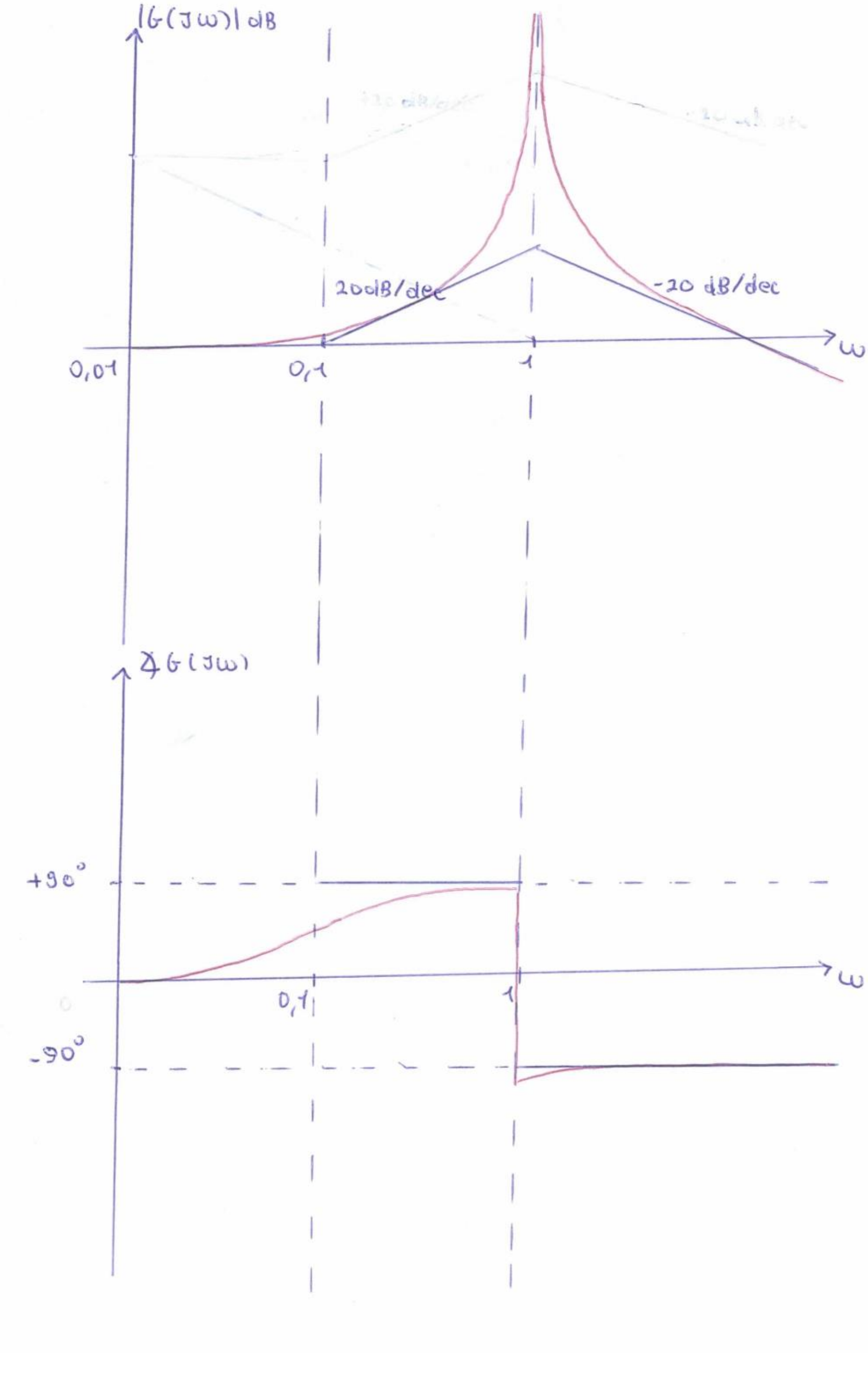
$$G(s) = \frac{10s+1}{s^2+1} = \frac{\left(1+\frac{s}{1/10}\right)}{s^2+1}$$

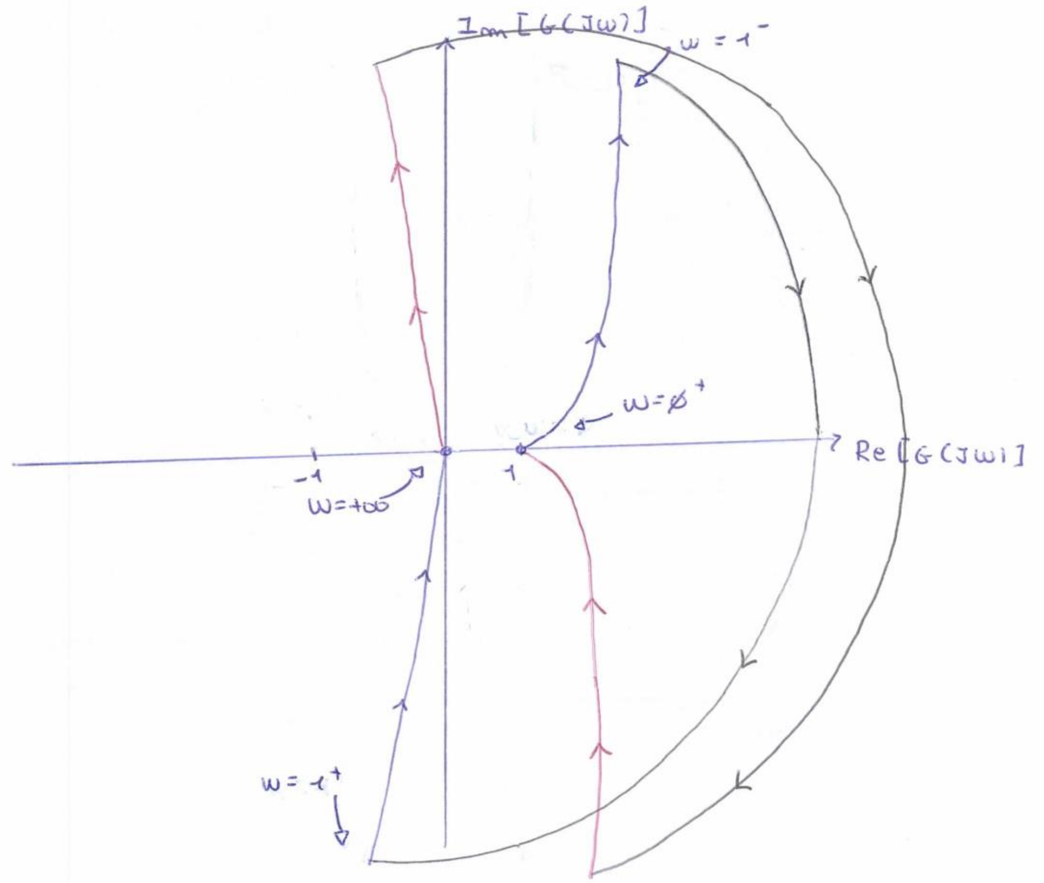
Poli Complessi:

$$1 + \frac{2}{\omega_m}\zeta s + \frac{s^2}{\omega_m^2}$$

$$\begin{cases} K_B = 1 \\ z_1 = -\frac{1}{10} \\ \omega_m = 1 \\ \zeta = \varnothing \end{cases}$$

$$\begin{cases} \frac{2}{\omega_m}\cdot\zeta = \varnothing \\ \frac{1}{\omega_m^2} = 1 \end{cases} \Rightarrow \begin{cases} \zeta = \varnothing \\ \omega_m = 1 \end{cases}$$





Im [G(jω)]
ω = 1⁻
ω = 0⁺
Re [G(jω)]
-1
1
ω = +∞
ω = 1⁺

$N_{Pd}^{AC} = \cancel{N_{Pd}^{AA}}^{\varnothing} - N_{OR,-1}$

$N_{OR,-1} = \varnothing$ $\Rightarrow$ il sistema in retroazione è già stabile.

$K > \varnothing$: STABILE

$K < \varnothing$: $\alpha = 1 \Rightarrow \frac{1}{\alpha} = 1$

- $|K| > 1$: INSTABILE 1 Pd $\rightarrow K < -1$
- $|K| < 1$: INSTABILE 2 Pd $\rightarrow K > -1$

$$\begin{cases} K > \varnothing & \text{STABILE} \\ K < -1 & \text{INSTABILE 1 Pd} \\ \varnothing > K > -1 & \text{INSTABILE 2 Pd} \end{cases}$$

$L(s) = (s^2+1) + K(10s+1) = s^2 + 1 + 10Ks + K = s^2 + 10Ks + K + 1$

$$\begin{array}{c|cc} n & & \\ 2 & 1 & K+1 \\ 1 & 10K & \\ \varnothing & K+1 & \end{array}$$

$$\begin{cases} 10K > \varnothing \\ K+1 > \varnothing \end{cases} \qquad \begin{cases} K > \varnothing \\ K > -1 \end{cases}$$

ES. 4

$$G(s) = \frac{s+1}{s(s-1)(s+10)} = -\frac{1}{10} \cdot \frac{(1+s)}{(1-s)\left(1+\frac{s}{10}\right)\cdot s}$$

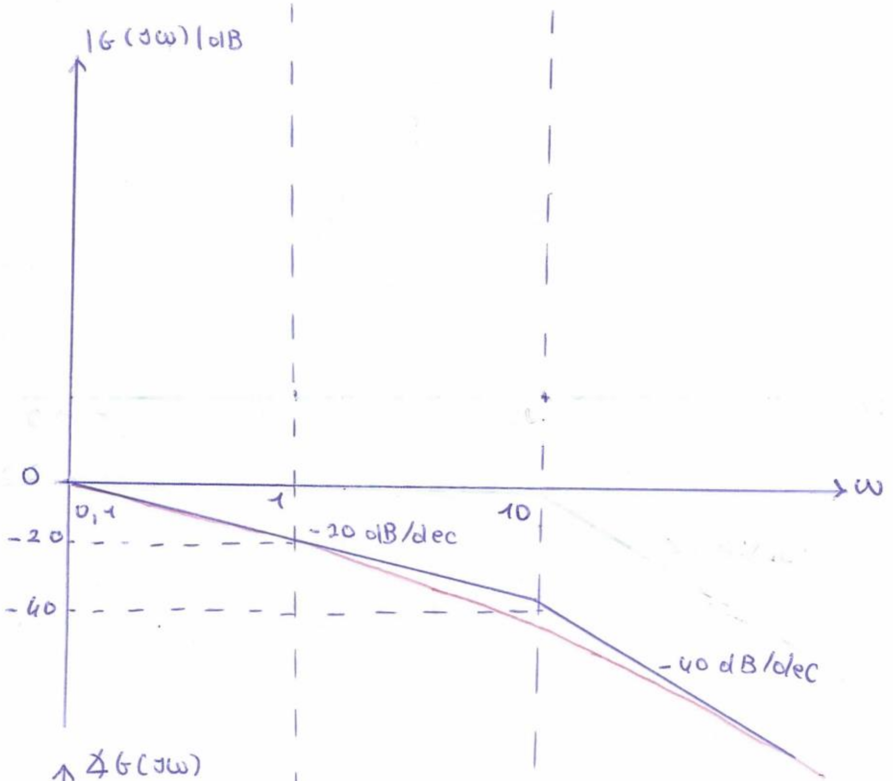
$$\begin{cases} K_B = -\frac{1}{10} \\ z_1 = -1 \\ p_1 = 1 \\ p_2 = -10 \\ p_3 = \phi \end{cases}$$

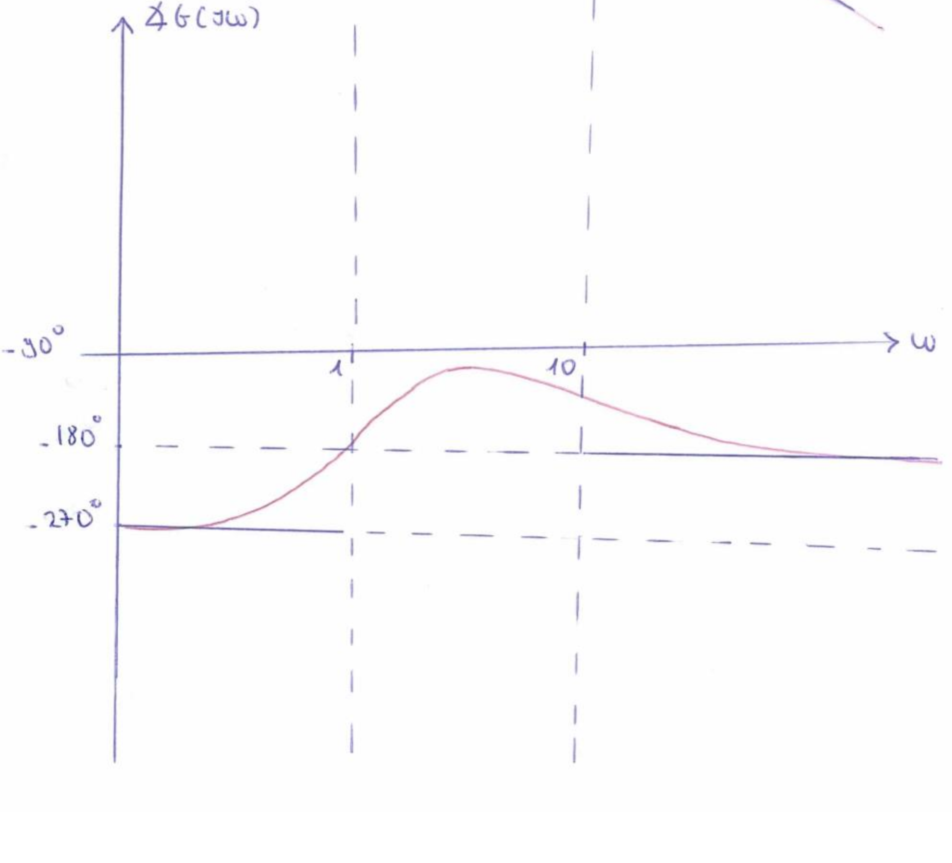
$$\left|-\frac{1}{10}\cdot\frac{1}{s}\right|_{s=j\omega} = \phi \text{ dB}$$

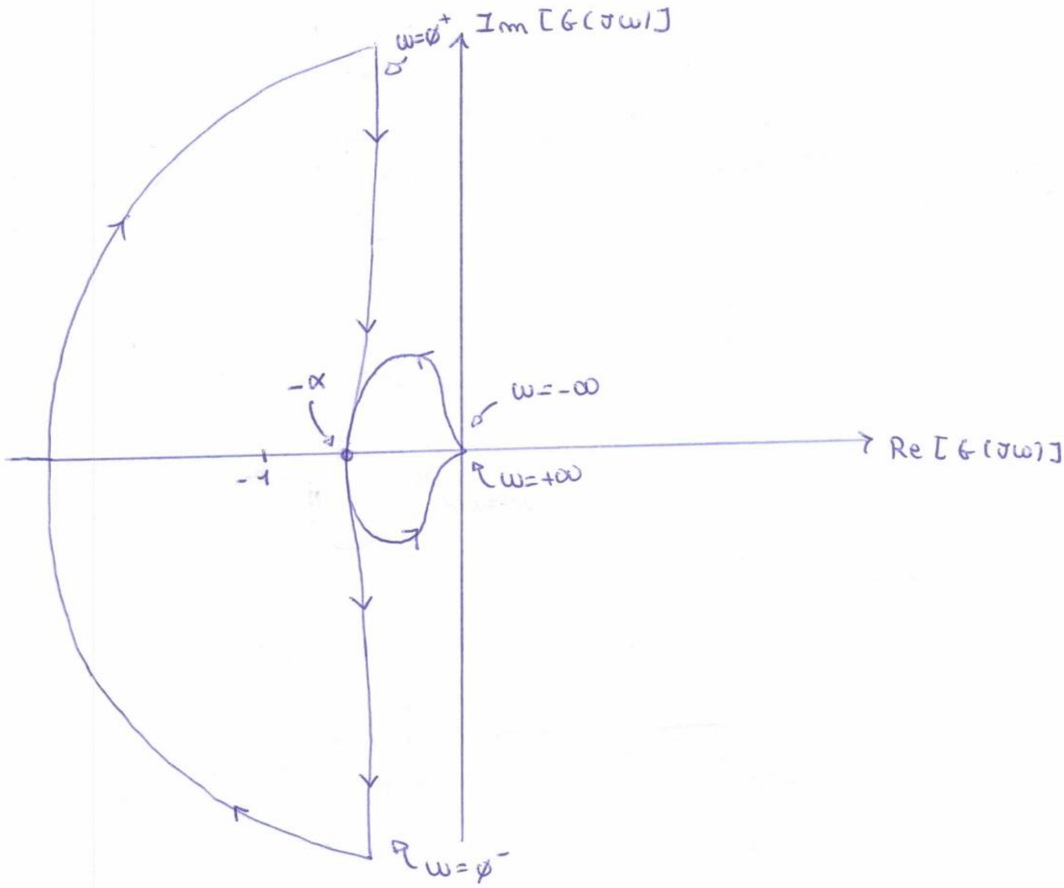
$$\left|-\frac{1}{10}\cdot\frac{1}{j\omega}\right| = 1$$

$$\frac{1}{10}\cdot\frac{1}{\omega} = 1$$

$$\omega = \frac{1}{10} = 0{,}1$$







Im [G(jω)]
ω=0⁺
-α
ω=-∞
Re [G(jω)]
-1
ω=+∞
ω=0⁻

$N_{Pd}^{AC} = N_{Pd}^{AA} + N_{OR,-1}$

$N_{Pd}^{AA} = 1$

$N_{OR,-1} = 1$ $\Rightarrow N_{Pd}^{AC} = 2$ se sistema in retroazione non è stabile.

$K > \emptyset$:

- $\alpha = ? \rightarrow \frac{1}{\alpha} = ?$

  - $K > 10$: STABILE
  - $K < 10$: INSTABILE 2 Pd

$K < \emptyset$: INSTABILE 1 Pd

$$\begin{cases} K > \frac{1}{\alpha} & \text{STABILE} \\ \emptyset < K < \frac{1}{\alpha} & \text{INSTABILE 2 Pd} \\ K < \emptyset & \text{INSTABILE 1 Pd} \end{cases}$$

$$L(s) = s(s-1)(s+10) + K(s+1) = (s^2 - s)(s+10) + Ks + K$$
$$= s^3 + 10s^2 - s^2 - 10s + Ks + K = s^3 + 9s^2 + s(-10+K) + K$$

n		
3	1	$-10+K$
2	$+9$	K
1	$8K-90$	
$\emptyset$	K	

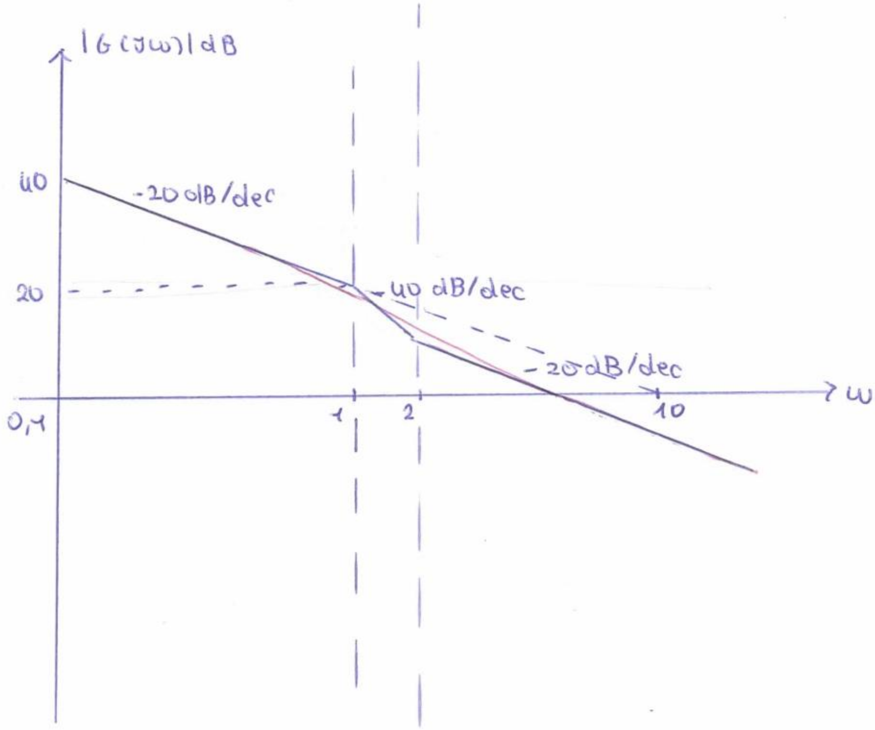
$$\begin{cases} 8K - 90 > \emptyset \\ K > \emptyset \end{cases} \rightarrow \begin{cases} K > \frac{90}{8} = \frac{45}{4} \\ K > \emptyset \end{cases}$$

$\alpha \Rightarrow \frac{4}{45}$.

ES. 5

$$G(s) = 5 \cdot \frac{s+2}{s(s-1)} = -\frac{5 \cdot 2}{s} \frac{\left(1+\frac{s}{2}\right)}{(1-s)} = -\frac{10}{s} \cdot \frac{\left(1+\frac{s}{2}\right)}{(1-s)}$$

$$\begin{cases} K_B = -10 \\ P_1 = \emptyset \\ P_2 = 1 \\ Z_1 = -2 \end{cases}$$



$\left| -\frac{10}{s} \right|_{s=j\omega} = \emptyset \text{ dB}$

$\left| \frac{10}{j\omega} \right| = 1$

$\frac{10}{\omega} = 1$

$\omega = 10$

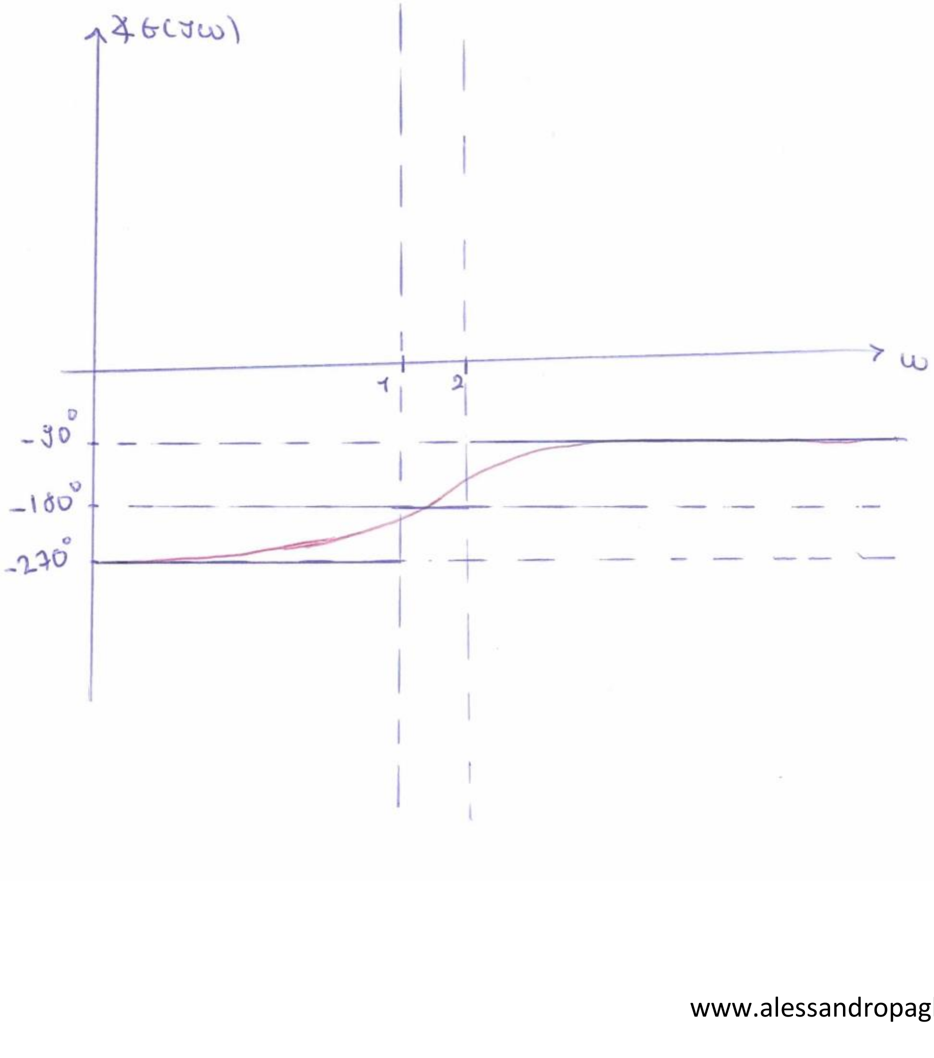
$\frac{10}{\omega} = x_{lim}$

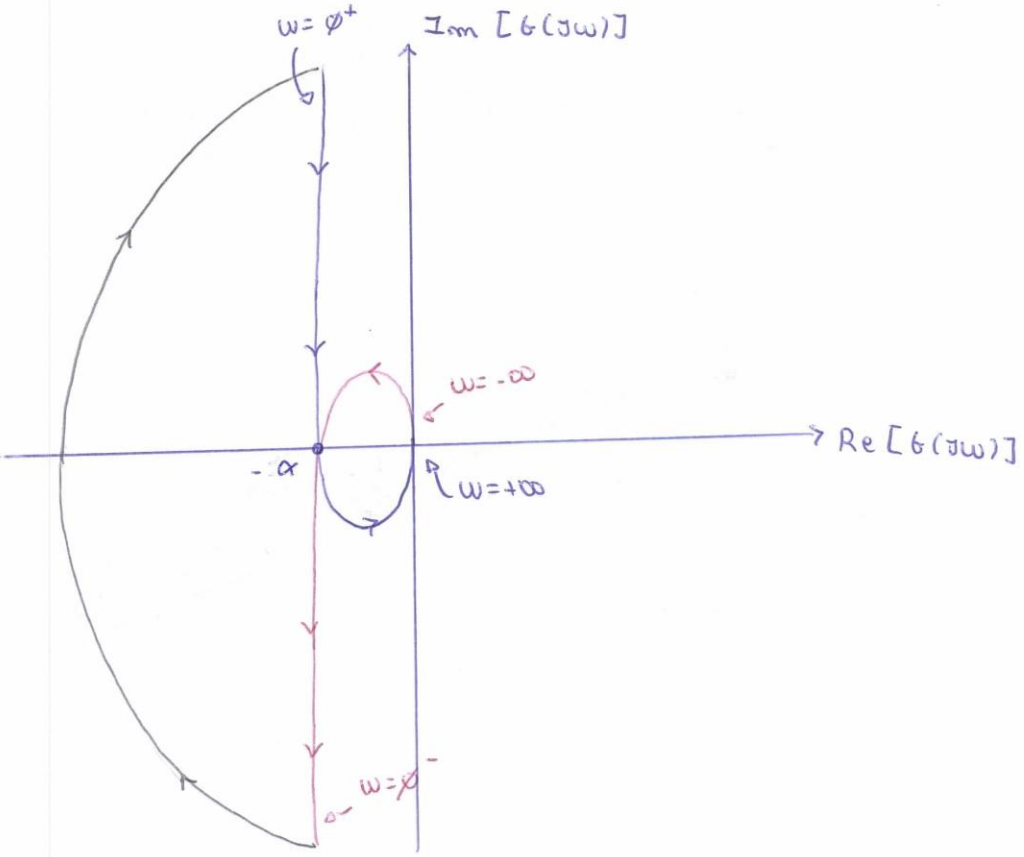
$\omega = 0,1$

$\Rightarrow \frac{10}{0,1} = x_{lim}$

$x_{lim} = 100$

$x_{dB} = 40$





ω = ∅⁺
Im [G(jω)]
ω = -∞
Re [G(jω)]
-α
ω = +∞
ω = ∅⁻

$$N_{Pd}^{AC} = N_{Pd}^{AA} + N_{OR,-1}$$

$N_{Pd}^{AA} = 1$

$N_{OR,-1} = ?$, dipende dove si trova $-\alpha$.

$K > 0$

- $K > \frac{1}{\alpha}$: STABILE
- $K < \frac{1}{\alpha}$: INSTABILE 2Pd

$K < 0$: INSTABILE 1 Pd

$$\begin{cases} K > \frac{1}{\alpha} & \text{STABILE} \\ 0 < K < \frac{1}{\alpha} & \text{INSTABILE 2Pd} \\ K < 0 & \text{INSTABILE 1Pd} \end{cases}$$

$$l(s) = s(s-1) + 5K(s+2) = s^2 - s + s5K + 10K = s^2 + s(5K-1) + 10K$$

$$\begin{array}{c|cc} n & & \\ 2 & 1 & 10K \\ 1 & 5K-1 & \\ 0 & 10K & \end{array}$$

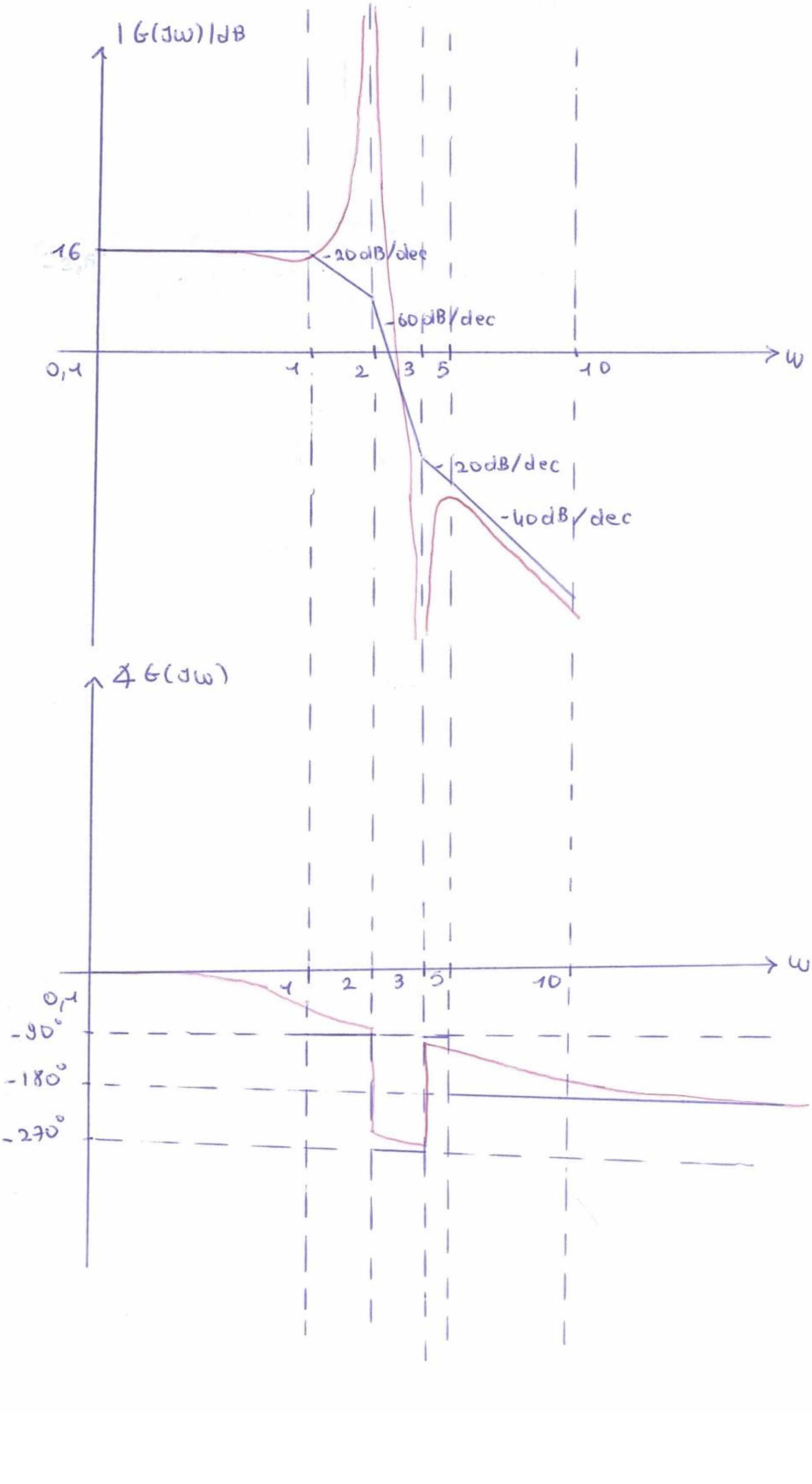
$$\begin{cases} 10K > 0 \\ 5K-1 > 0 \end{cases} \Rightarrow \begin{cases} K > 0 \\ K > \frac{1}{5} \end{cases}$$

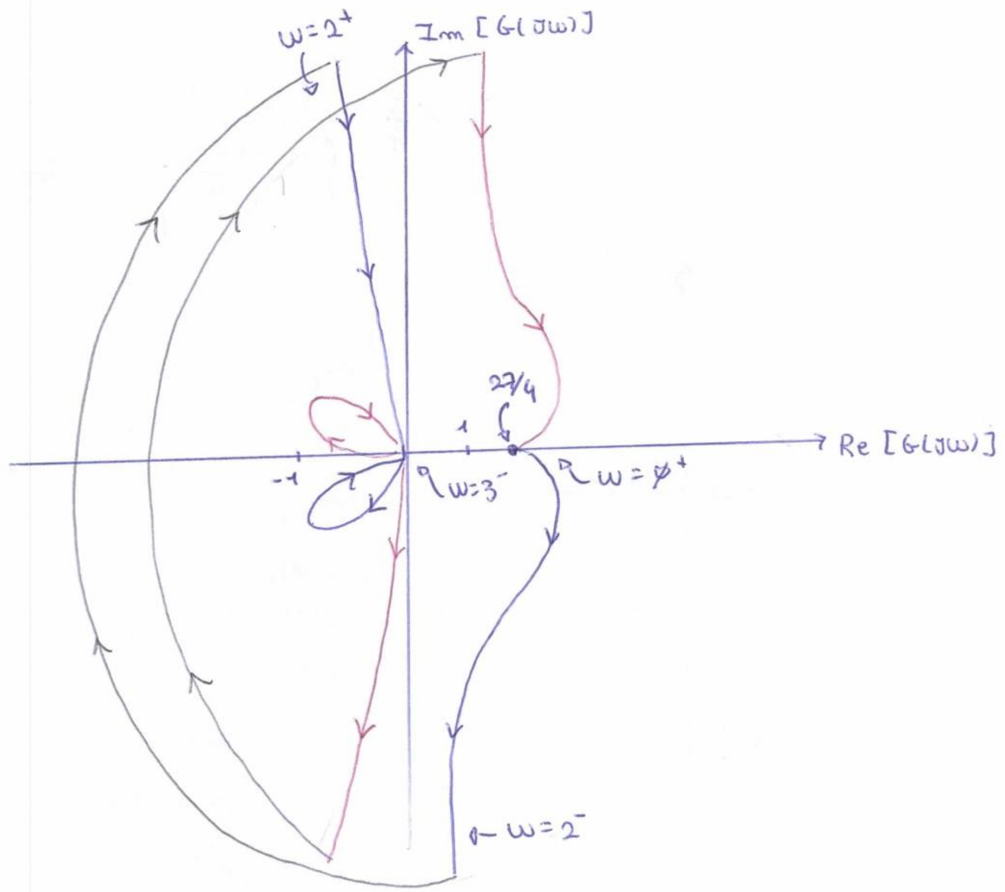
$$\underline{\underline{\alpha = 5}}$$

ES. 6

$$G(s) = \frac{3(s^2+9)}{(1+s)(s^2+4)(1+0,2s)} = 9 \cdot \frac{3}{4} \frac{\left(\frac{s^2}{9}+1\right)}{(1+s)\left(\frac{s^2}{4}+1\right)\left(1+\frac{s}{5}\right)}$$

$$\begin{cases} K_B = 27/4 \\ p_1 = -1 \\ p_2 = -5 \\ \omega_n = 2 \\ \zeta = \emptyset \\ \alpha_n = 3 \\ \xi = \emptyset \end{cases}$$





ω=2⁺
Im [G(jω)]
27/4
1
-1
Re [G(jω)]
ω=3⁻
ω=0⁺
ω=2⁻

$N_{Pd}^{AC} = \cancel{N_{Pd}^{AA}}^{\to \emptyset} + N_{OA,-1}$

$N_{Pd}^{AA} = \emptyset$

$N_{OA,-1} = 2$

se sistema in retroazione è instabile

$K > \emptyset$: INSTABILE 2Pd

$K < \emptyset$:

$\alpha = \frac{27}{4} \Rightarrow \frac{1}{\alpha} = \frac{4}{27}$

- $|K| > \frac{4}{27}$: INSTABILE 1 Pd $\to K < -\frac{4}{27}$
- $|K| < \frac{4}{27}$: STABILE $\to K > -\frac{4}{27}$

$$\begin{cases} K > \emptyset & \text{INSTABILE 2Pd} \\ K < -\frac{4}{27} & \text{INSTABILE 1Pd} \\ -\frac{4}{27} < K < \emptyset & \text{STABILE} \end{cases}$$

$$\begin{aligned}
L(s) &= (1+s)(s^2+4)\left(1+\frac{s}{5}\right) + 3k(s^2+9) \\
&= (s^2+4+s^3+4s)\left(1+\frac{s}{5}\right) + 3k(s^2+9) \\
&= s^2 + \frac{s^3}{5} + 4 + \frac{4}{5}s + s^3 + \frac{s^4}{5} + 4s + \frac{4}{5}s^2 + 3ks^2 + 27k \\
&= s^4\left(\frac{1}{5}\right) + s^3\left(\frac{1}{5}+1\right) + s^2\left(1 + \frac{4}{5} + 3k\right) + s\left(\frac{4}{5}+4\right) + 4+27k \\
&= \frac{1}{5}s^4 + \frac{6}{5}s^3 + s^2\left(\frac{9}{5}+3k\right) + \frac{24}{5}s + 4+27k
\end{aligned}$$

n			
4	$\frac{1}{5}$	$\frac{9}{5}+15k$	$20+135k$
3	$\frac{6}{5}$ 1	$\frac{24}{5}$ 4	
2	$1+3k$	$4+27k$	
1	$-\frac{15k}{1+3k}$		
$\emptyset$	$4+27k$		

$$\begin{cases} 1+3k > \emptyset \\ -\frac{15k}{1+3k} > \emptyset \\ 4+27k > \emptyset \end{cases} \Rightarrow \begin{cases} k > -\frac{1}{3} \\ k < \emptyset \\ k > -\frac{4}{27} \end{cases}$$

$-\frac{4}{27} < k < \emptyset$

ES. 7

$$G(s) = \frac{64\,(s+2)}{s\,(s+0{,}5)\,(s^2+3{,}2s+64)} = \frac{\cancel{64}\cdot 2}{s\cdot 64}\;\frac{\left(1+\frac{s}{2}\right)}{\left(1+\frac{s}{1/2}\right)\left(\frac{s^2}{64}+\frac{1}{20}s+1\right)}$$

$$= \frac{4}{s}\cdot\frac{1+\frac{s}{2}}{\left(1+\frac{s}{\frac{1}{2}}\right)\left(s^2+\frac{16}{5}s+64\right)}$$

$$\begin{cases} K_B = 4 \\ p_1 = \emptyset \\ z_1 = -2 \\ p_2 = -\frac{1}{2} \\ \omega_n = 8 \\ \xi = 1/5 \end{cases}$$

$$\left|\frac{4}{s}\right|_{s=j\omega} = \emptyset\ dB$$

$$\left|\frac{4}{j\omega}\right| = 1$$

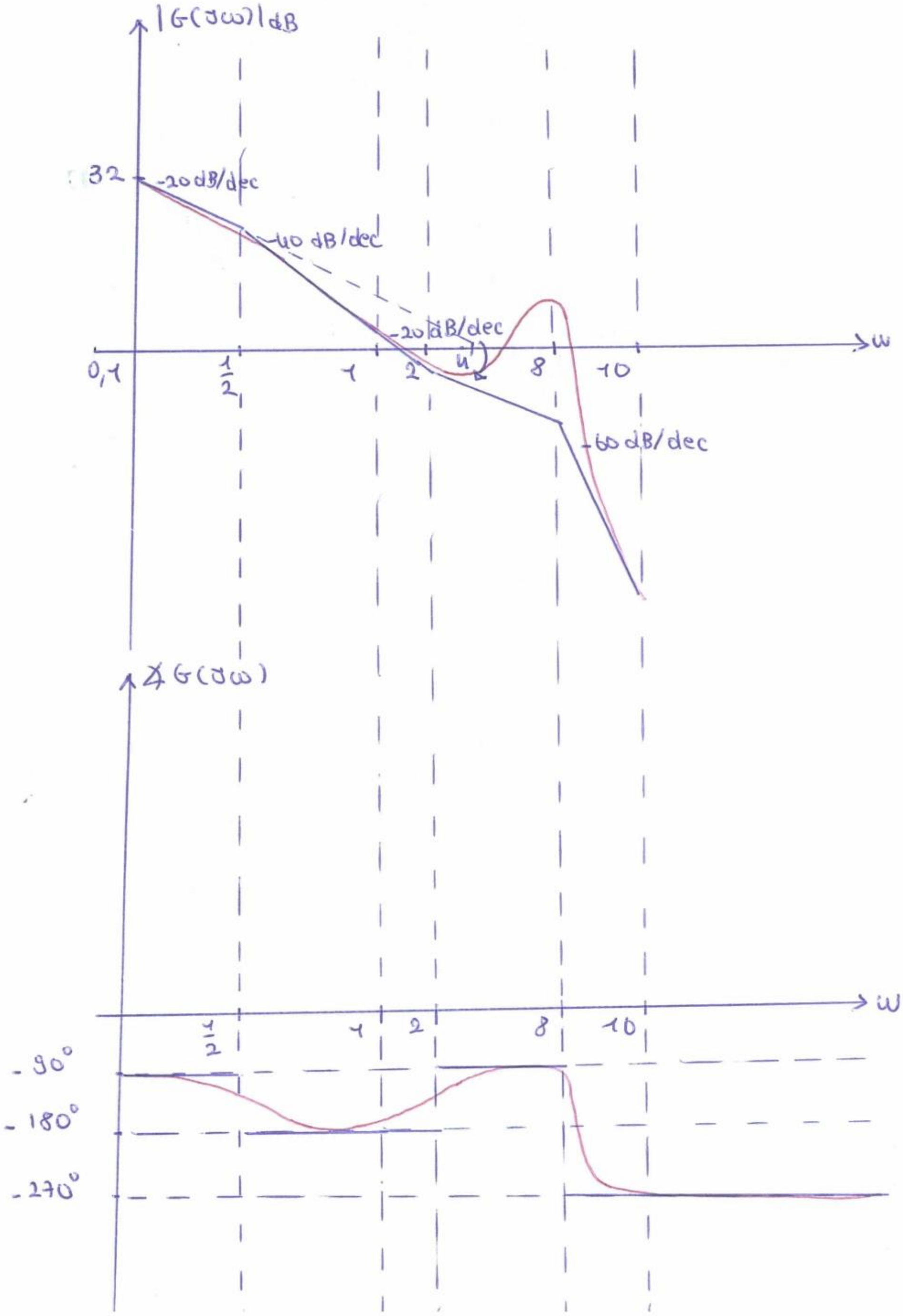
$$\frac{4}{\omega} = 1$$

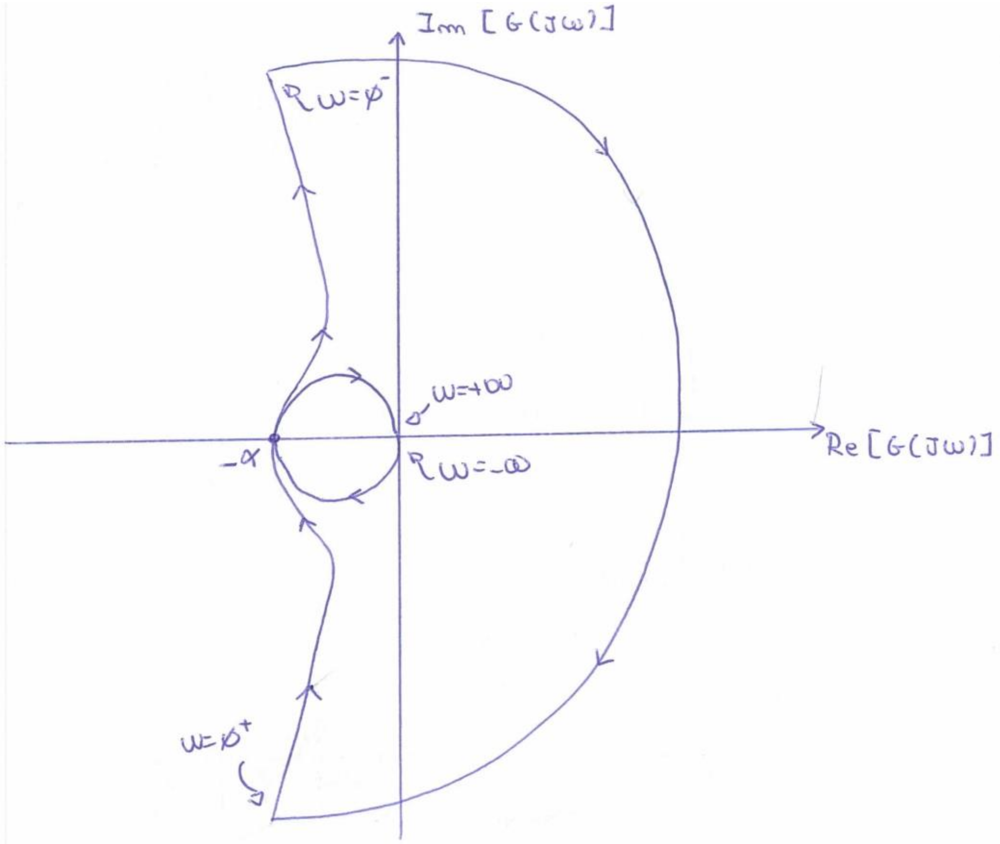
$$\Rightarrow \omega = 4$$

$$\frac{4}{0{,}1} = x_{lim}$$

$$x_{lim} = 40$$

$$x_{dB} = 32$$




Im [G(jω)]
ω=0⁻
ω=+∞
-α
ω=-∞
Re [G(jω)]
ω=0⁺

$$N_{Pd}^{AC} = \cancel{N_{Pd}^{AA}} - N_{OR,-1}$$

$K > 0$:

$|K| > \frac{1}{\alpha}$: INSTABILE 2Pd

$K < \frac{1}{\alpha}$: STABILE

$K < 0$: INSTABILE 1 Pd

$$L(s) = s\left(s+\frac{1}{2}\right)\left(s^2+\frac{16}{5}s+64\right) + 64\cdot K(s+2)$$

$$= \left(s^2+\frac{s}{2}\right)\left(s^2+\frac{16}{5}s+64\right) + 64K(s+2)$$

$$= s^4 + \frac{16}{5}s^3 + 64s^2 + \frac{s^3}{2} + \frac{8}{5}s^2 + 32s + 64K\cdot s + 128K$$

$$= s^4 + \frac{37}{10}s^3 + \frac{328}{5}s^2 + s\left(32+64K\right) + 128K$$

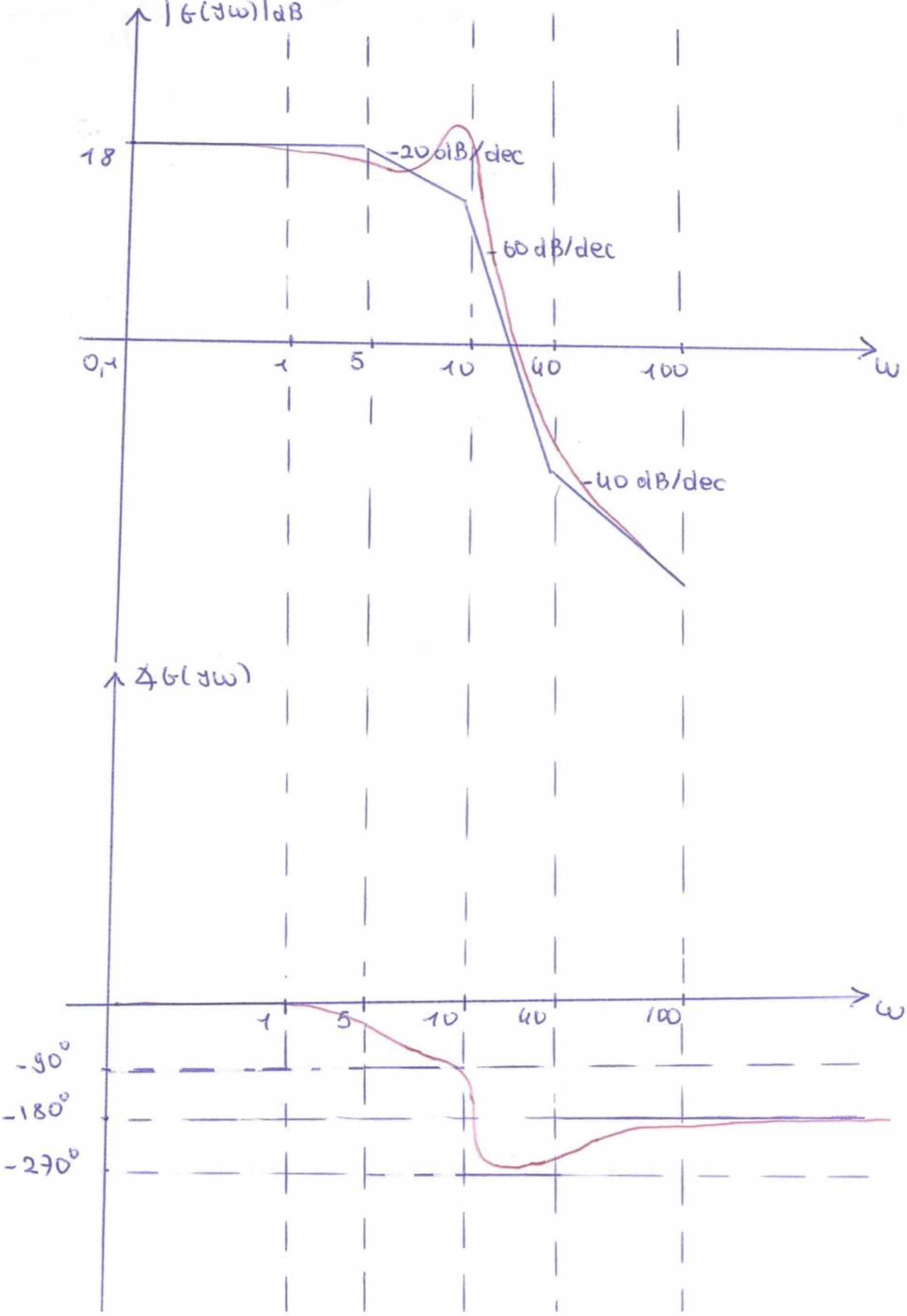
...

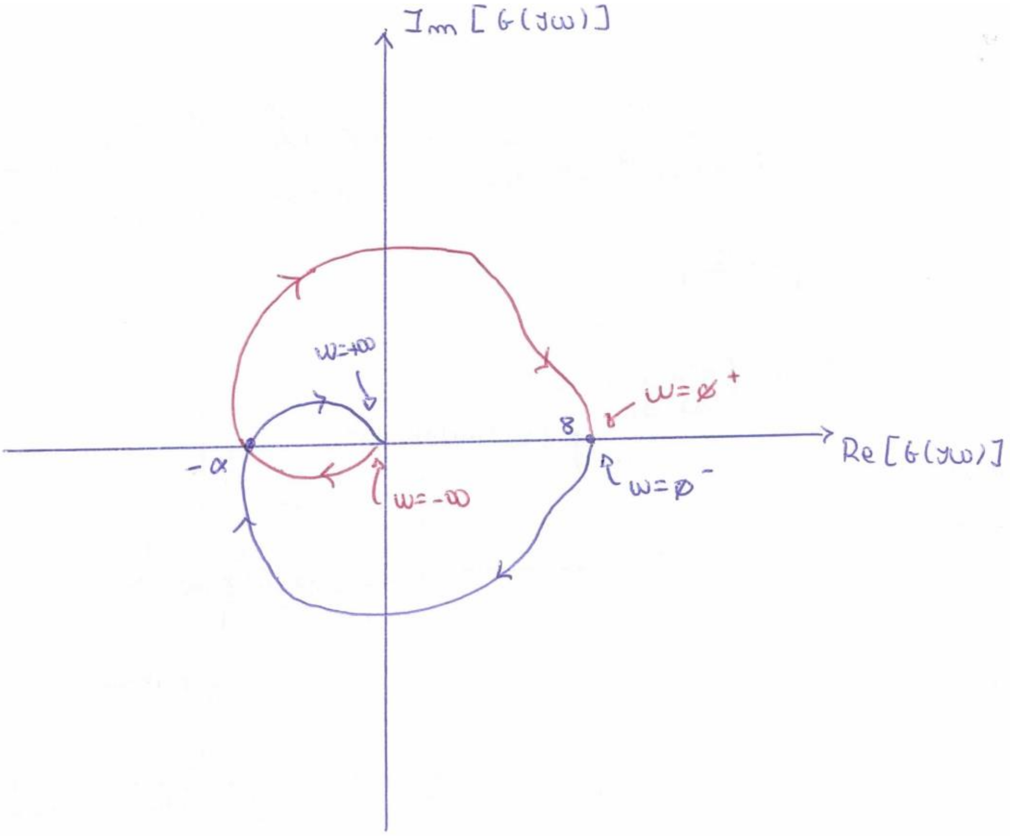
ES. 8

$$G(s) = 100 \cdot \frac{s+40}{(s+5)(s^2+2s+100)} = \frac{\overset{8}{\cancel{100} \cdot \cancel{40}}}{\cancel{5} \cdot \cancel{100}} \frac{\left(1+\frac{s}{40}\right)}{\left(1+\frac{s}{5}\right)\left(\frac{s^2}{100}+\frac{2s}{100}+1\right)}$$

$$= 8 \cdot \frac{\left(1+\frac{s}{40}\right)}{\left(1+\frac{s}{5}\right)\left(\frac{s^2}{100}+\frac{1}{50}s+1\right)}$$

$$\begin{cases} K_B = 8 \\ z_1 = -40 \\ p_1 = -5 \\ \omega_n = 10 \\ \zeta = 1/10 \end{cases}$$





Im [G(jω)]
Re [G(jω)]
ω=+∞
ω=0⁺
8
-α
ω=-∞
ω=0⁻

$N_{Pd}^{AC} = N_{Pd}^{AA} + N_{OR,-1}$

$K > \emptyset$:

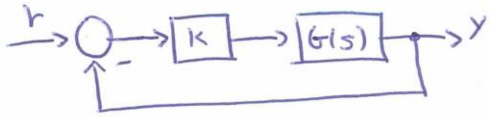
- $K > \frac{1}{\alpha}$: INSTABILE 2 Pd
- $K < \frac{1}{\alpha}$: STABILE

$K < \emptyset$:

- $|K| > \frac{1}{8}$: INSTABILE 1 Pd
- $|K| < \frac{1}{8}$: STABILE

... zauth

LUOGO DELLE RADICI

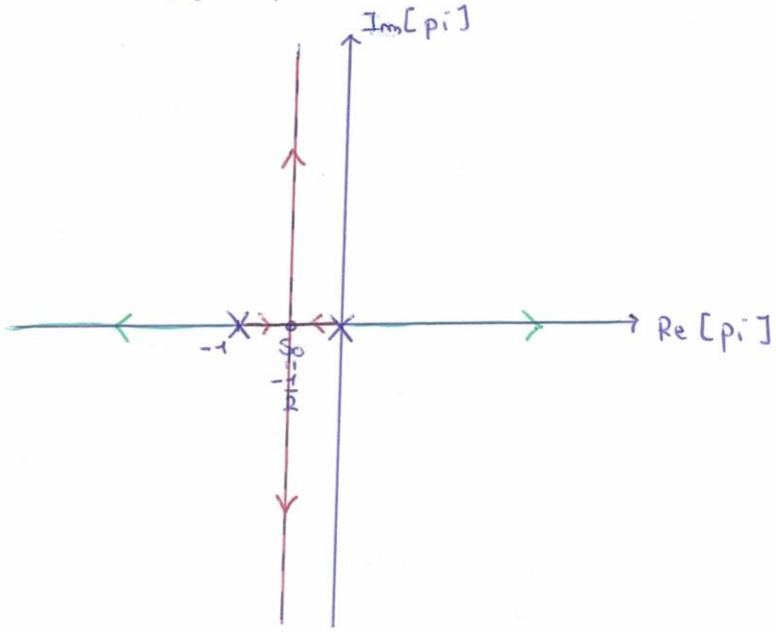


ES. 1

$$G(s) = \frac{1}{s(s+1)} \quad , \quad L(s) = \frac{k}{s(s+1)}$$

Polo in ϕ;

Polo in -1;



$m - m = 2$ asintoti

$$S_0 = \frac{\sum p_j - \sum z_i}{m - m} = \frac{\phi - 1}{2} = -\frac{1}{2}$$

$$\begin{cases} \frac{(2h+1)\pi}{m-m} = \left\{ \frac{1}{2}\pi \; ; \; \frac{3}{2}\pi \right\} & \text{luogo diretto} \\ \frac{2h\pi}{m-m} = \left\{ \phi , \pi \right\} & \text{luogo inverso} \end{cases}$$

$K > \phi$: STABILE

$K < \phi$: INSTABILE 1 Pol

$$\bar{G}(s) = \frac{K}{s^2 + s + K} \; ; \quad d(s) = s^2 + s + K \qquad S_{1/2} = \frac{-1 \pm \sqrt{1 - 4K}}{2}$$

m		
2	1	K
1	1	
ϕ	K	

$K > \phi$: STABILE!

$K < \phi$: INSTABILE 1 Pol

$G(s) = \frac{1}{S} \cdot \frac{1}{(1+s)}$

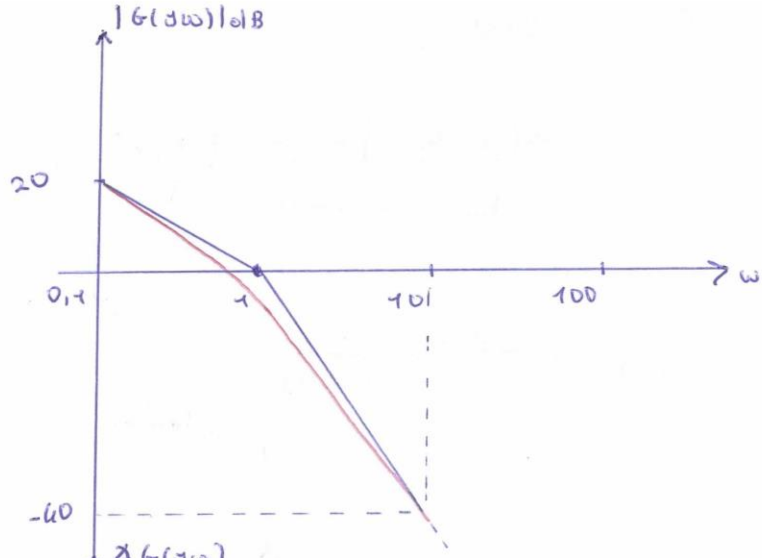
$K_B = 1$

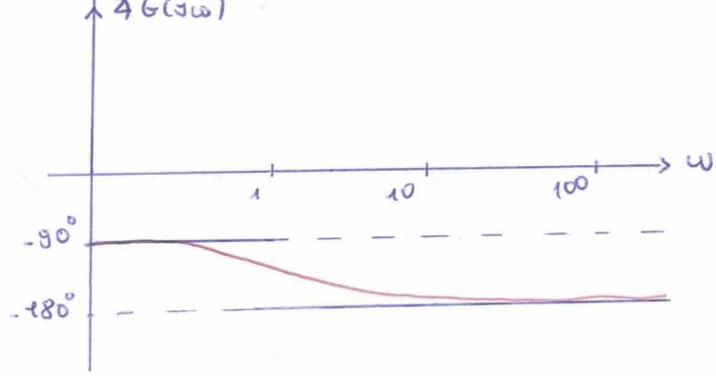
$p_1 = \emptyset$

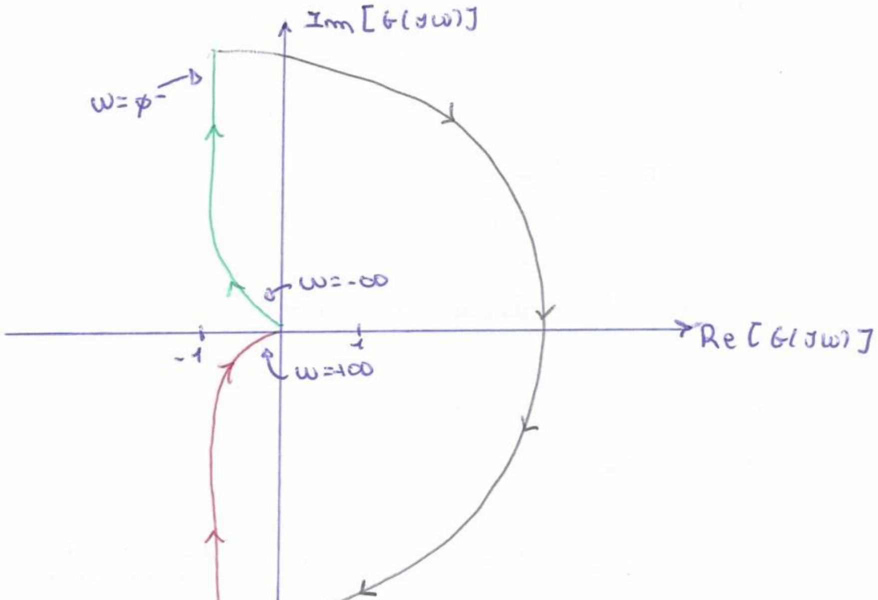
$p_2 = -1$

$\left| \frac{1}{j\omega} \right| = \emptyset \text{ dB}$

$\frac{1}{\omega} = 1 \Rightarrow \omega = 1$







K > ∅ : STABILE

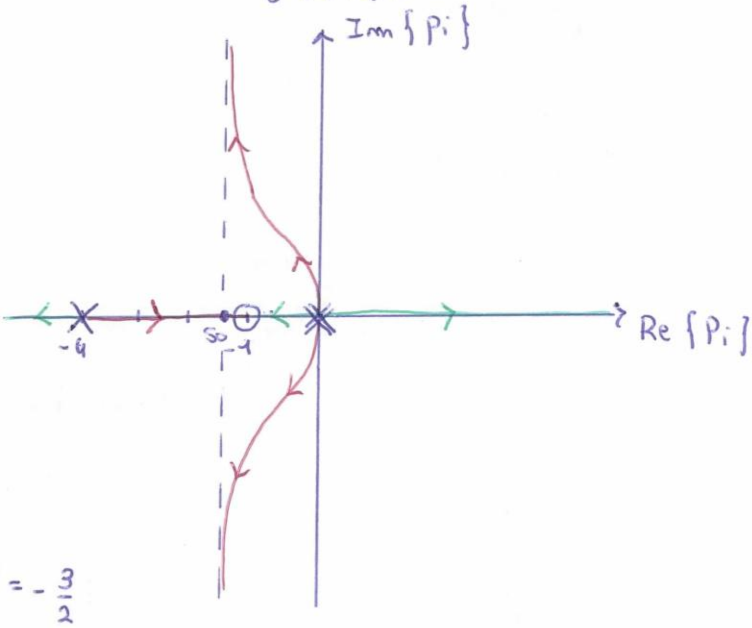
K < ∅ : INSTABILE
1 Pd

$N_{Pd}^{AC} = \cancel{N_{Pd}^{AA}} + N_{OR,-1}$

ES. 2

$$G(s) = \frac{s+1}{s^2(s+4)} \quad ; \quad L(s) = K\frac{(s+1)}{s^2(s+4)} \qquad n-m = 2$$

$$\begin{cases} z_1 = -1 \\ p_{1/2} = \emptyset \\ p_3 = -4 \end{cases}$$



$$S_0 = \frac{\sum p_j - \sum z_i}{n-m}$$

$$= \frac{\emptyset + \emptyset - 4 - (-1)}{2} = -\frac{3}{2}$$

$$\begin{cases} \frac{(2h+1)\pi}{n-m} = \left\{\frac{1}{2}\pi \, ; \, \frac{3}{2}\pi\right\} & \text{luogo diretto;} \\ \frac{2h\pi}{n-m} = \left\{\emptyset \, ; \, \pi\right\} & \text{luogo inverso;} \end{cases}$$

$K > \emptyset$: STABILE

$K < \emptyset$: INSTABILE 1 Pd

$$d(s) = s^2(s+4) + K(s+1) = s^3 + 4s^2 + Ks + K = \emptyset$$

n		
3	1	K
2	4	K
1	3K	
$\emptyset$	K	

$K > \emptyset$: STABILE

$K < \emptyset$: INSTABILE 1 Pd

$$G(s) = \frac{1}{4s^2} \frac{(1+s)}{\left(1+\frac{s}{4}\right)}$$

$K_B = 4$

$p_{1,2} = \varnothing$

$p_3 = -4$

$z_1 = -1$

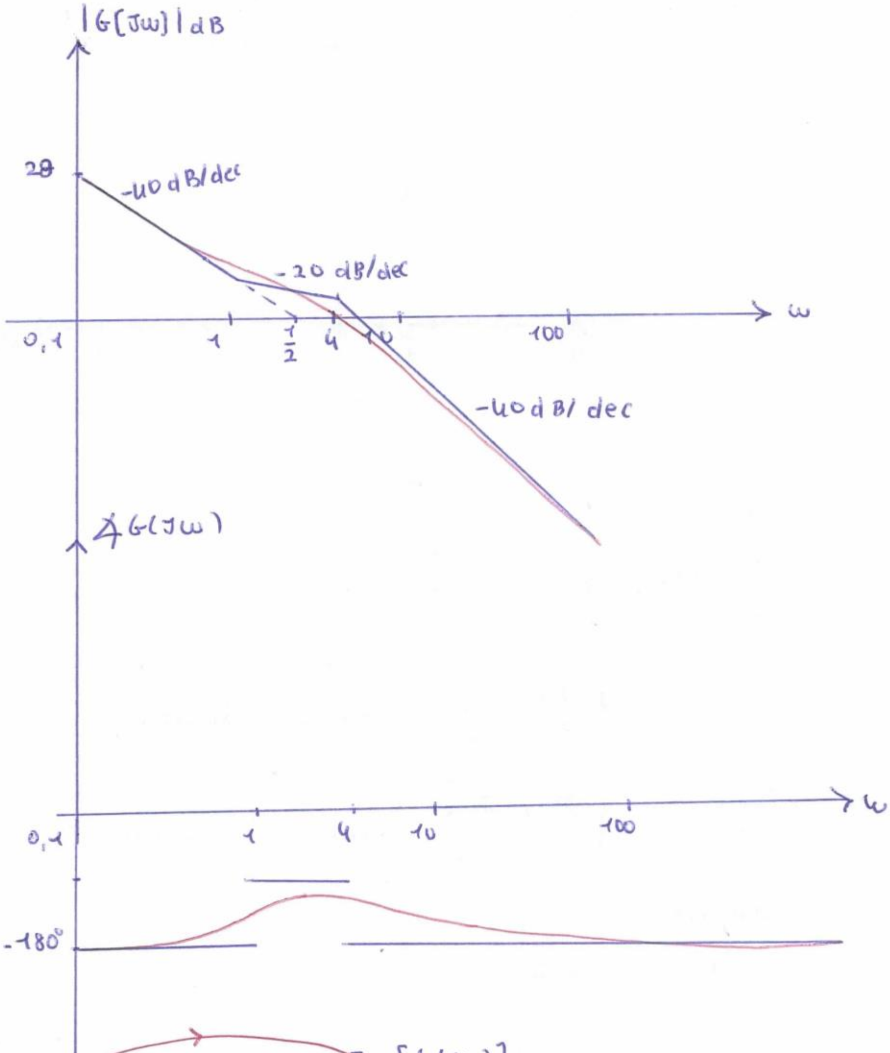
$$\left|\frac{1}{4\cdot(j\omega^2)}\right| = 1$$

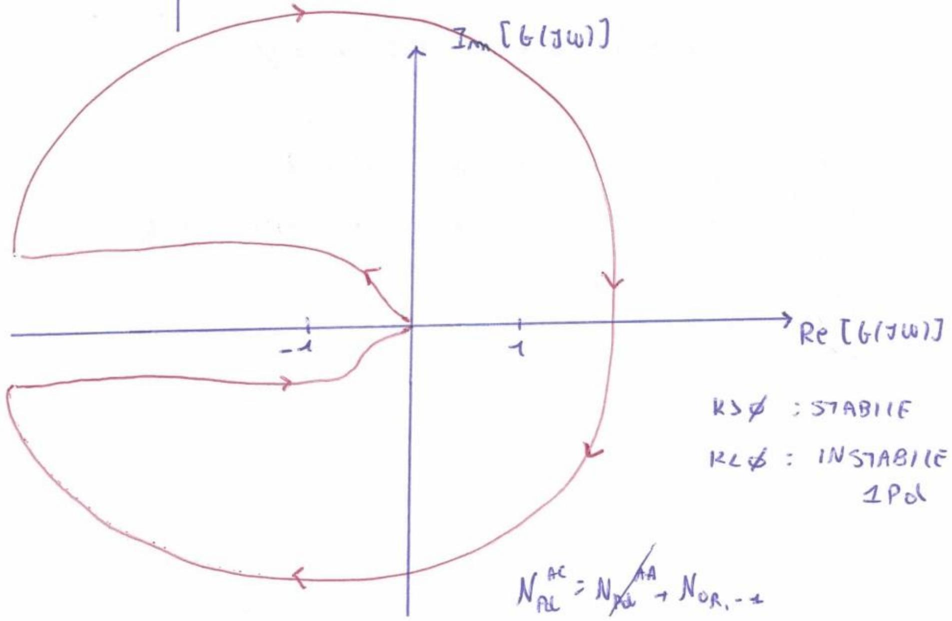
$$\frac{1}{4\omega^2} = 1$$

$$\omega = \frac{1}{2}$$

$$\frac{1}{4\cdot(0,1)^2} = x_{lim} = 25$$

$x_{dB} =$





$K>\varnothing$: STABILE

$K<\varnothing$: INSTABILE
1 Pd

$$N_{PL}^{AC} = \cancel{N_{PL}^{AA}} + N_{OR,-1}$$

ES. 3

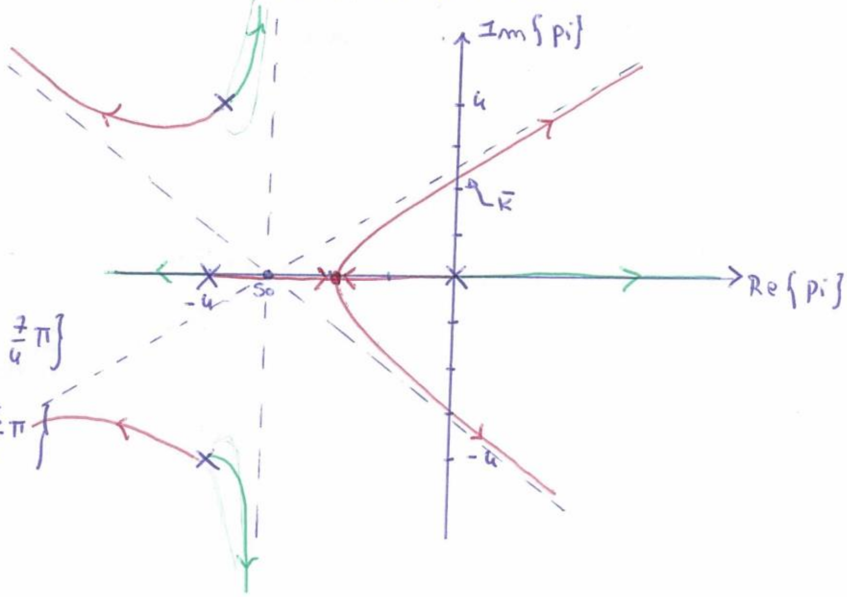
$$G(s) = \frac{1}{s(s+4)(s^2+8s+32)} \quad , \quad L(s) = \frac{k}{s(s+4)(s^2+8s+32)} \quad , \; n-m = 4$$

$p_1 = -4$

$p_2 = \emptyset$

$p_{3/4} = -4 \pm j4$

$$S_0 = \frac{-4-4+\cancel{j4}-4-\cancel{j4}}{4} = -3$$



$$\begin{cases} \dfrac{(2h+1)\pi}{n-m} = \left\{ \frac{1}{4}\pi ; \frac{3}{4}\pi ; \frac{5}{4}\pi ; \frac{7}{4}\pi \right\} \\ \dfrac{2h\pi}{n-m} = \left\{ \emptyset ; \frac{2}{4}\pi ; \frac{4}{4}\pi ; \frac{6}{4}\pi \right\} \end{cases}$$

$k > \emptyset$:

$\emptyset < k < \bar{k}$: STABILE

$k > \bar{k}$: INSTABILE 2Pd

$k < \emptyset$: INSTABILE 1Pd

$$d(s) = s(s+4)(s^2+8s+32) + k = (s^2+4s)(s^2+8s+32) + k$$
$$= s^4 + 8s^3 + 32s^2 + 4s^3 + 32s^2 + 128s + k$$
$$= s^4 + 12s^3 + 64s^2 + 128s + k = \emptyset$$

n			
4	1	64	k
3	3	32	
2	160	3k	
1	-9k+5120		
$\emptyset$	3k		

$$\begin{cases} -9k + 5120 > \emptyset \\ 3k > \emptyset \end{cases} \rightarrow \begin{cases} k < \frac{5120}{9} \approx 569 \\ k > \emptyset \end{cases}$$

$\emptyset < k < 569$

$\bar{k} = 569$

$$G(s) = \frac{1}{s} \cdot \frac{1}{\left(1+\frac{s}{4}\right)\cdot 4 \cdot 32\left(1+\frac{8}{32}s+\frac{s^2}{32}\right)} = \frac{1}{128 \cdot s} \cdot \frac{1}{\left(1+\frac{s}{4}\right)\left(1+\frac{1}{4}s+\frac{1}{32}s^2\right)}$$

$K_B = \frac{1}{128}$

$P_1 = \varnothing$

$P_2 = -4$

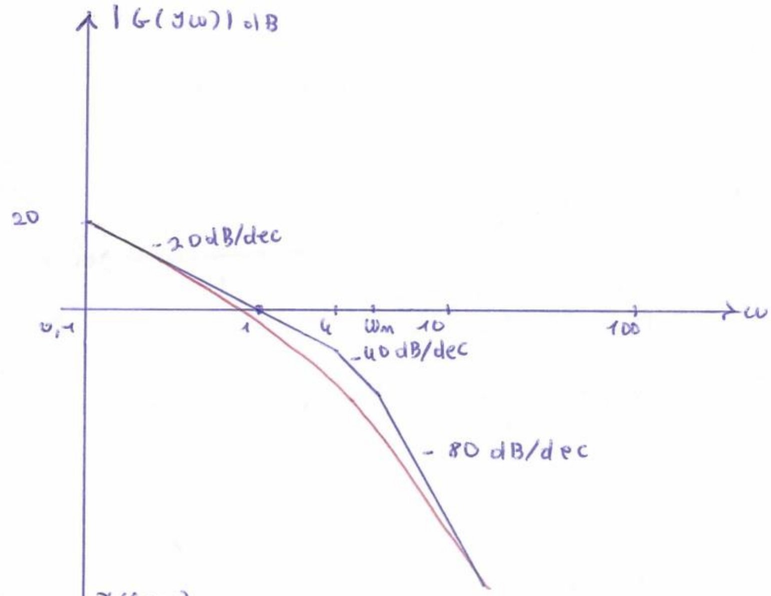
$1 + \frac{2}{\omega_m}\xi \cdot s + \frac{1}{\omega_m^2} \cdot s^2$

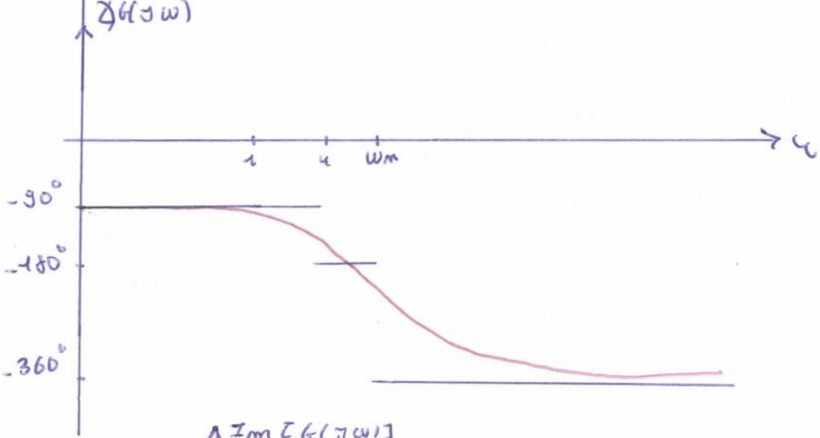
$\omega_m = \sqrt{32} = 5{,}65$

$\frac{\cancel{2}}{\cancel{\sqrt{32}}} \cdot \xi = \frac{1}{\cancel{4}2} = \frac{\sqrt{32}}{2} \approx 2{,}82$

$\left|\frac{1}{\omega}\right| = 1$

$\Rightarrow \omega = 1$



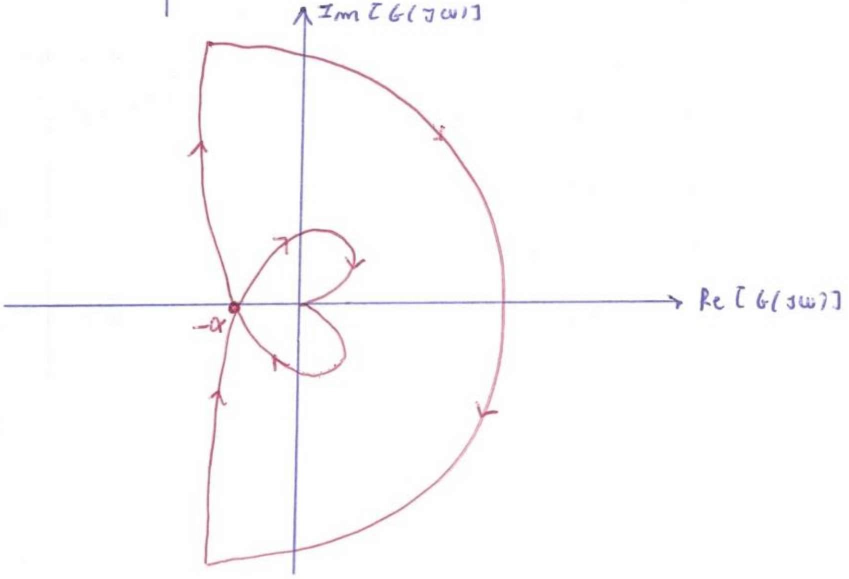


$K > \varnothing$:

$K > \frac{1}{\alpha}$: INST. 2 Pd

$K < \frac{1}{\alpha}$: STAB.

$K < \varnothing$: INST. 1 Pd



$N_{Pd}^{AC} = N_{Pd}^{\cancel{AA}} - N_{OR,-1}$

ES. 4

$$G(s) = \frac{s+2}{s(s-1)} \quad , \quad L(s) = \frac{k(s+2)}{s(s-1)}$$

$n - m = 2 - 1 = 1$

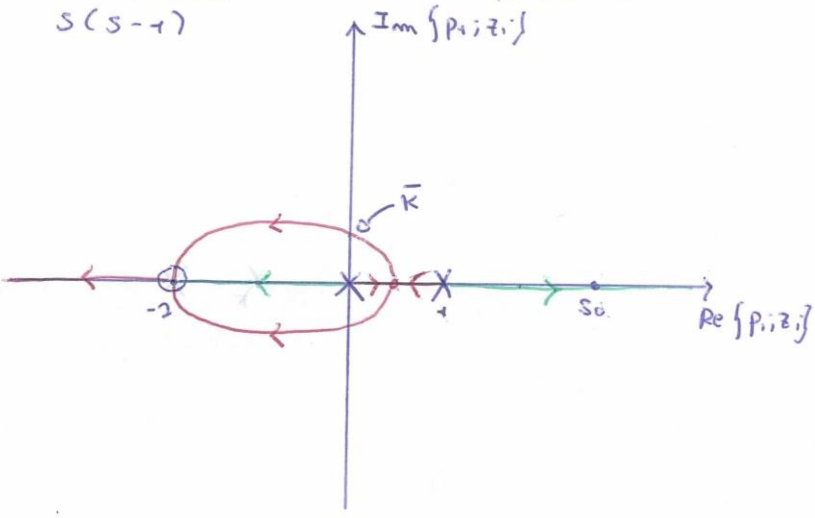
$p_1 = \emptyset$

$p_2 = 1$

$z_1 = -2$

$$S_0 = \frac{\emptyset + 1 - (-2)}{1} = 3$$

$$\begin{cases} \dfrac{(2h+1)\pi}{1} = \{\pi\} \\ \dfrac{2h\pi}{1} = \{\emptyset\} \end{cases}$$



$k > \emptyset$

$k > \bar{k}$: STABILE

$k < \bar{k}$: INSTABILE 2Pol

$k < \emptyset$: INSTABILE

$$d(s) = s(s-1) + k(s+2) = s^2 - s + ks + 2k = s^2 + s(k-1) + 2k = \emptyset$$

$$\begin{array}{c|cc} n & & \\ 2 & 1 & 2k \\ 1 & k-1 & \\ \emptyset & 2k & \end{array} \qquad \begin{cases} k-1 > \emptyset \\ 2k > \emptyset \end{cases} \quad \begin{cases} k > 1 \\ k > \emptyset \end{cases}$$

$k > 1$ STABILE

$\bar{k} = 1$

$$G(s) = \frac{1}{s} \cdot \frac{\left(1+\frac{s}{2}\right)\cdot 2}{(1-s)(-1)} = -\frac{2}{s} \cdot \frac{1+\frac{s}{2}}{1-s}$$

$K = -2$

$P_1 = \emptyset$

$P_2 = 1$

$Z_1 = -2$

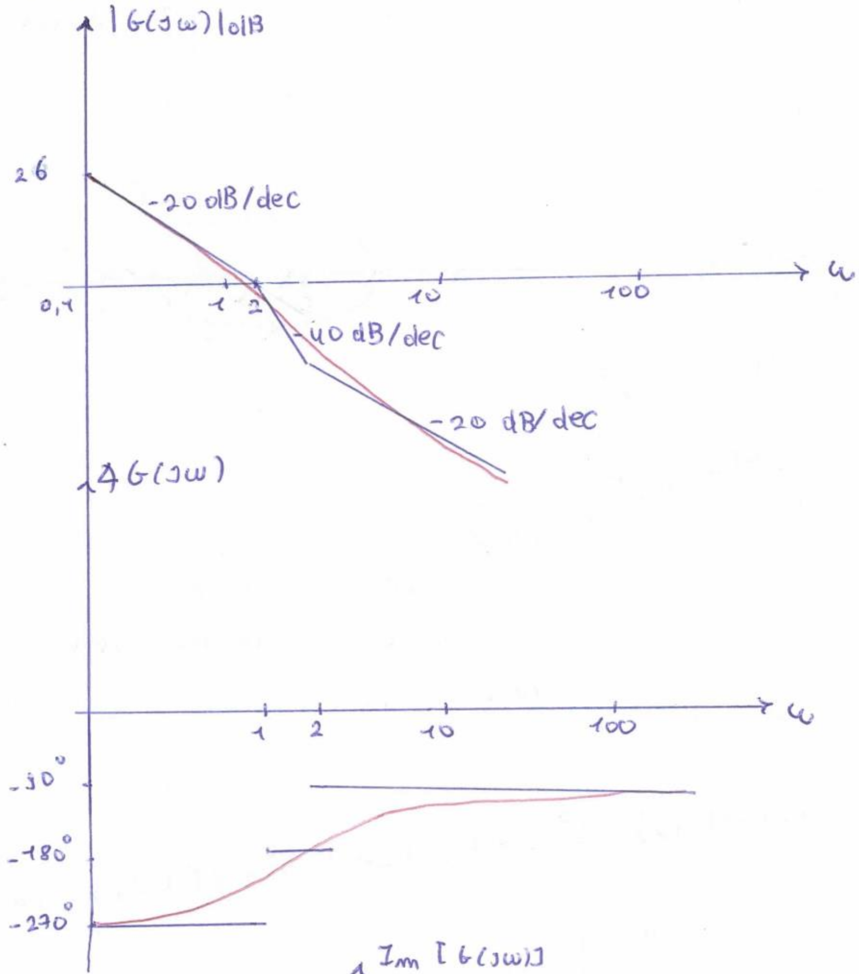
$\frac{2}{\omega} = 1$

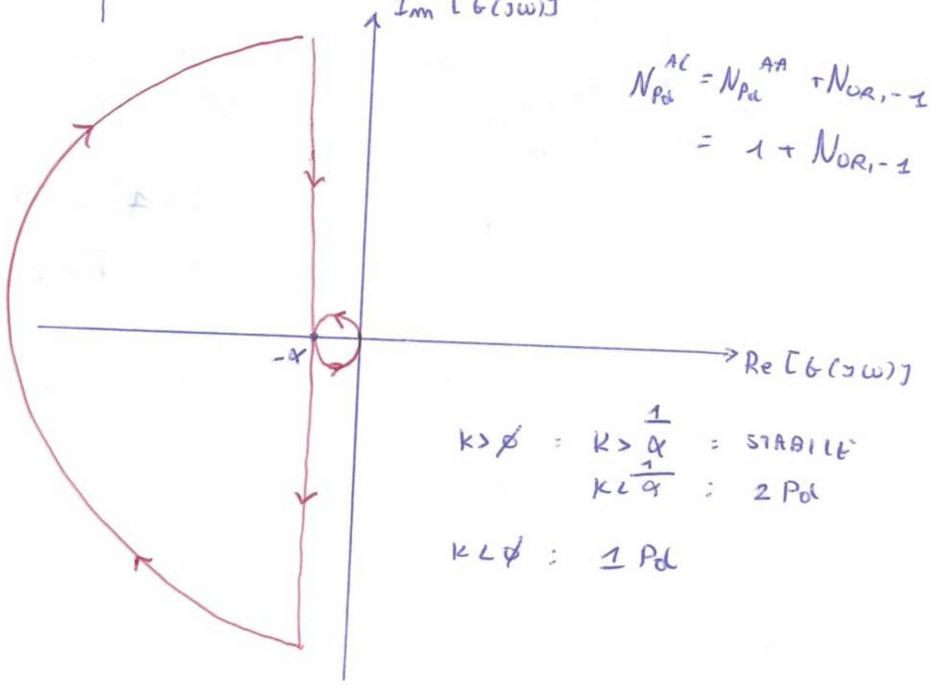
$\Rightarrow \omega = 2$

$\frac{2}{0,1} = X_{lim}$

$X_{lim} = 20$

$X_{dB} =$





$$N_{Pd}^{AC} = N_{Pd}^{AA} + N_{OR,-1}$$
$$= 1 + N_{OR,-1}$$

$K > \emptyset$: $K > \frac{1}{\alpha}$: STABILE

$K < \frac{1}{\alpha}$: 2 Pd

$K < \emptyset$: 1 Pd

ES. 6

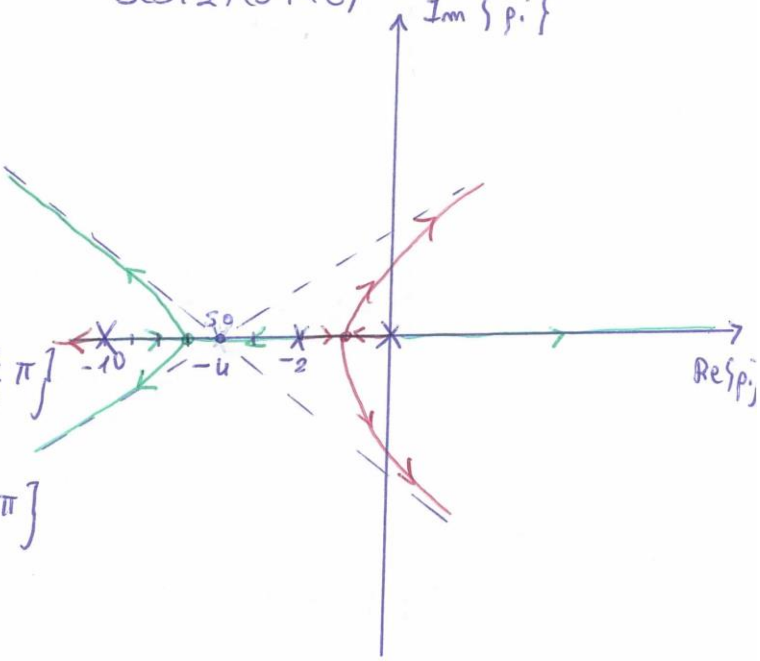
$$G(s) = \frac{1}{s(s+2)(s+10)} \quad ; \quad L(s) = \frac{k}{s(s+2)(s+10)} \qquad n - m = 3$$

$p_1 = 0$

$p_2 = -2$

$p_3 = -10$

$$S_0 = \frac{-12}{3} = -4$$



$$\begin{cases} \dfrac{(2h+1)\pi}{4} = \left\{ \dfrac{\pi}{4} ; \dfrac{3}{4}\pi ; \dfrac{5}{4}\pi ; \dfrac{7}{4}\pi \right\} \\ \dfrac{2h\pi}{4} = \left\{ 0 ; \dfrac{2}{4}\pi ; \dfrac{4}{4}\pi ; \dfrac{6}{4}\pi \right\} \end{cases}$$

$k > 0$:

$k < \bar{k}$: STABILE

$k > \bar{k}$: INSTABILE 2Pd

$k < 0$: INSTABILE 1Pd

$$d(s) = s(s+2)(s+10) + k = (s^2 + 2s)(s+10) + k = s^3 + 10s^2 + 2s^2 + 20s + k$$

$$= s^3 + 12s^2 + 20s + k = 0$$

n		
3	1	20
2	12	k
1	-k+240	
0	k	

$$\begin{cases} -k + 240 > 0 \rightarrow k < 240 \\ k > 0 \qquad\qquad k > 0 \end{cases}$$

$$0 < k < 240$$

$$\bar{k} = 240$$

$$G(s) = \frac{1}{s} \cdot \frac{1}{2\cdot\left(1+\frac{s}{2}\right)\cdot 10\left(1+\frac{s}{10}\right)} = \frac{1}{20\cdot s} \cdot \frac{1}{\left(1+\frac{s}{2}\right)\left(1+\frac{s}{10}\right)}$$

$K_B = \frac{1}{20}$

$p_1 = \emptyset$

$p_2 = -2$

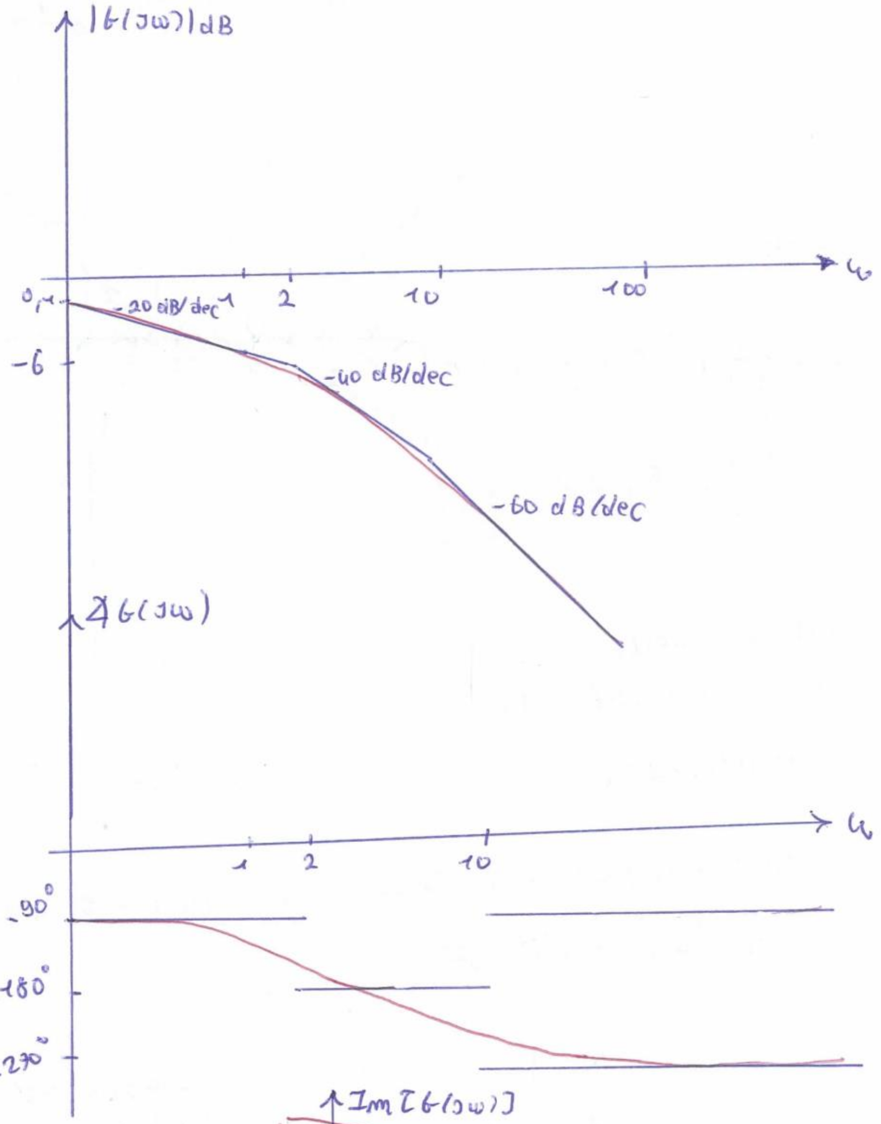
$p_3 = -10$

$\frac{1}{20\omega} = 1$

$\Rightarrow \omega = \frac{1}{20}$

$\frac{1}{20\cdot 0{,}1} = X_{lin} = \frac{1}{2}$

$X_{dB} = -6$

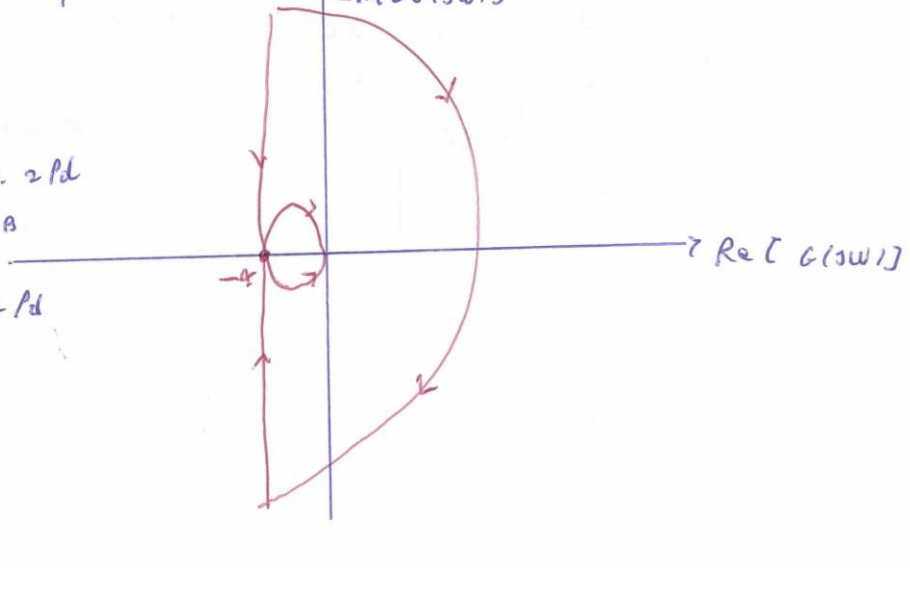


$K > \emptyset$:

$K > \alpha$: INST. 2 Pd

$K < \alpha$: STAB

$K < \emptyset$: INST. 1 Pd



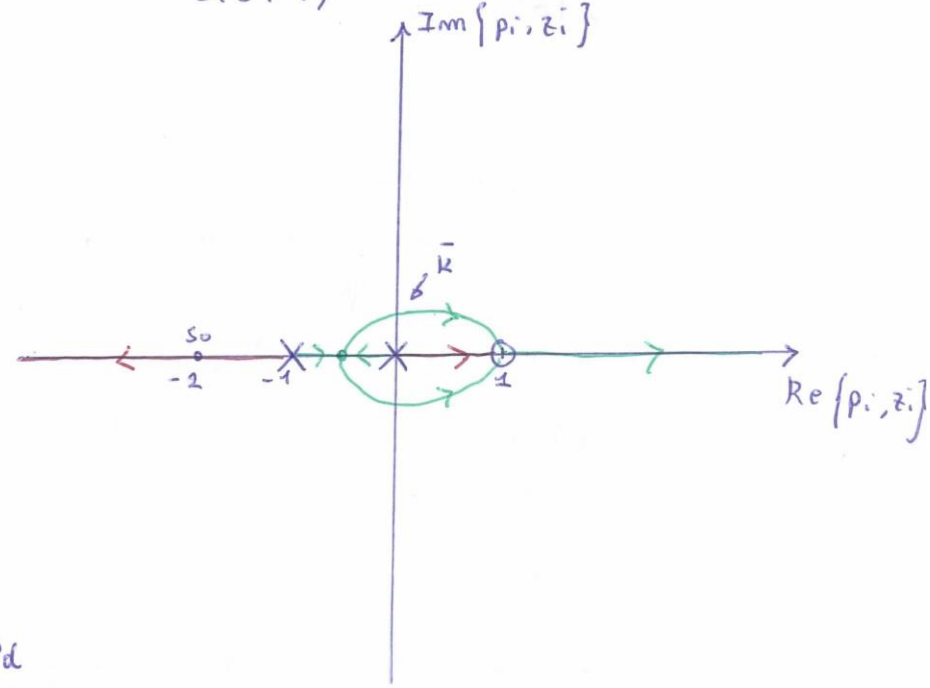
ES. 5

$G(s) = \frac{s-1}{s(s+1)}$; $L(s) = \frac{k(s-1)}{s(s+1)}$ $n - m = 1$

$z_1 = 1$
$p_1 = \emptyset$
$p_2 = -1$

$S_0 = \frac{-1-1}{1} = -2$

$$\begin{cases} \frac{(2h+1)\pi}{1} : \{\pi\} \\ \frac{2h\pi}{1} : \{\emptyset\} \end{cases}$$



$k > \emptyset$: INSTABILE 1 Pd

$k < \emptyset$: $|k| < \bar{k}$: STABILE $\Rightarrow k > -\bar{k}$

$|k| > \bar{k}$: INSTABILE 2 Pd $\Rightarrow k < -\bar{k}$

$d(s) = s(s+1) + k(s-1) = s^2 + s + ks - k = s^2 + s(k+1) - k = \emptyset$

n	
2	$1 \quad -k$
1	$k+1$
$\emptyset$	$-k$

$$\begin{cases} k+1 > \emptyset \\ -k > \emptyset \end{cases} \Rightarrow \begin{cases} k > -1 \\ k < \emptyset \end{cases}$$

$-1 < k < \emptyset$

$\Rightarrow \bar{k} = 1$

$$G(s) = -\frac{(1-s)}{s(1+s)}$$

$K_B = -1$

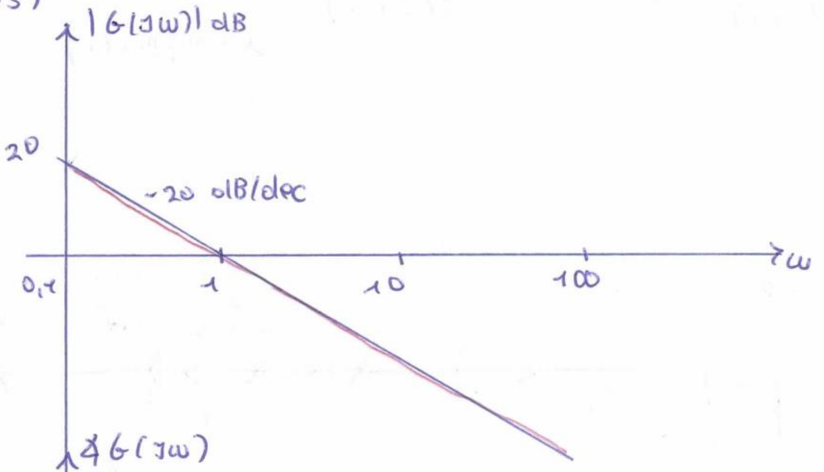
$z_1 = +1$

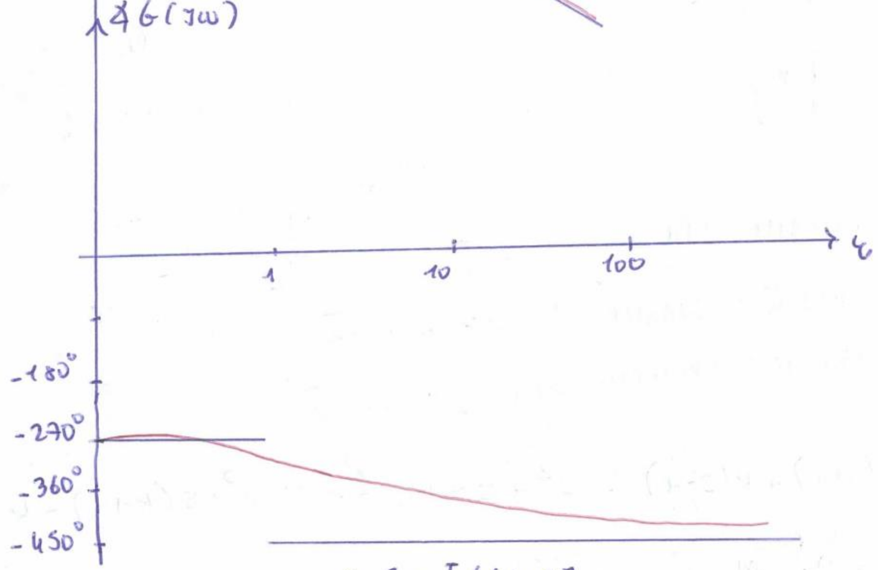
$p_1 = \emptyset$

$p_2 = -1$

$\left|\frac{1}{j\omega}\right| = 1$

$\rightarrow \omega = 1$





$N_{Pd}^{AC} = \cancel{N_{Pd}^{AA}} + N_{OR,-1}$

$K > \emptyset$: INST. 1 Pd

$K < \emptyset$:

$|K| > \alpha$: INST. 2 Pd

$|K| < \alpha$: STAB.