

LABORATORIO DI ELETTRONICA APPLICATA

[Appunti In Breve]

A CURA DI ALESSANDRO PAGHI

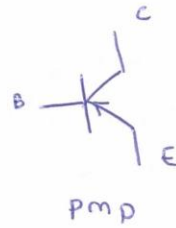
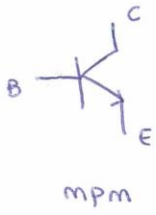
PROFESSORE: Valerio Vignoli (<http://www3.diism.unisi.it/people/person.php?id=58&aa=2015>)

LINK AL CORSO ANNO 2015/2016:

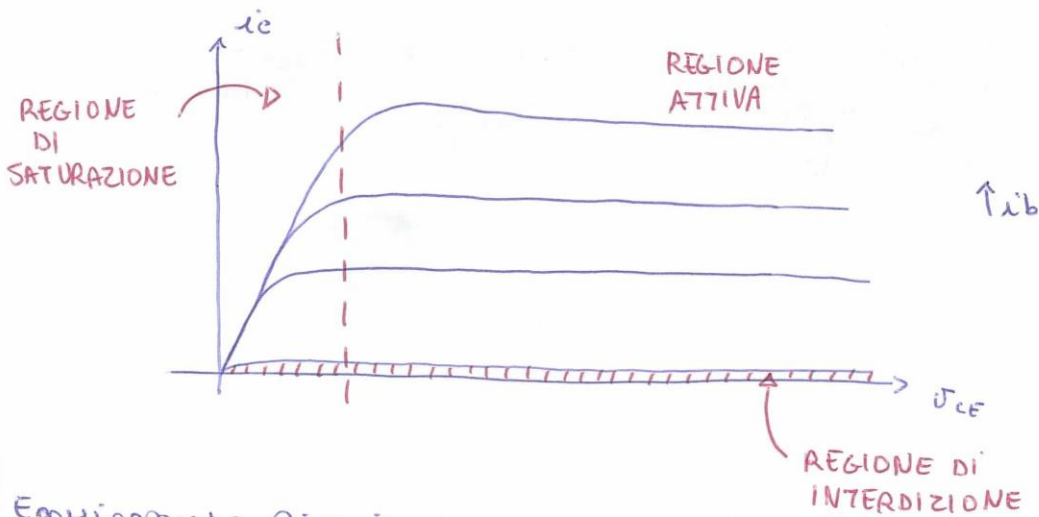
<http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=55145&aa=2015>

FREQUENTAZIONE: Consigliata.

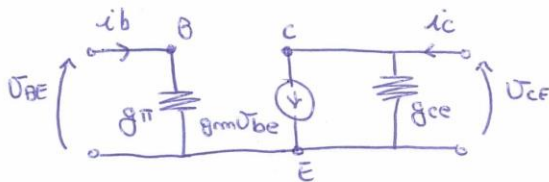
BJT



$$\begin{cases} i_c = I_s \left(e^{\frac{V_{BE}}{V_T}} - 1 \right) \left(1 + \frac{V_{CE}}{V_A} \right) \\ i_B = \frac{i_c}{\beta} \approx \frac{I_s}{\beta} \left(e^{\frac{V_{BE}}{V_T}} - 1 \right) \end{cases}$$

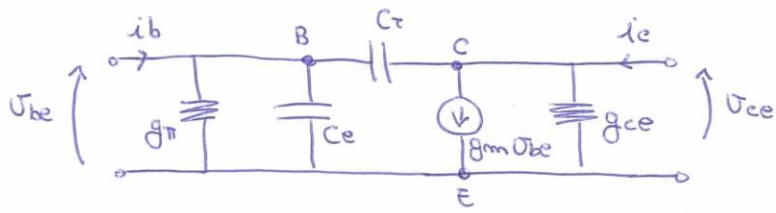


Equivalenti circuitale:

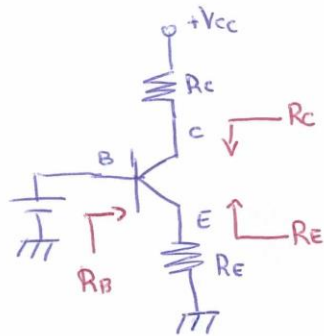


$$\begin{cases} g_m \approx \frac{I_c}{V_T} \\ g_{ce} \approx \frac{I_c}{V_A} \\ g_{\pi} \approx \frac{\beta g_m}{\beta} \end{cases}$$

Equivalent circuit in small frequency:



Resistances at terminals in case of small signals:



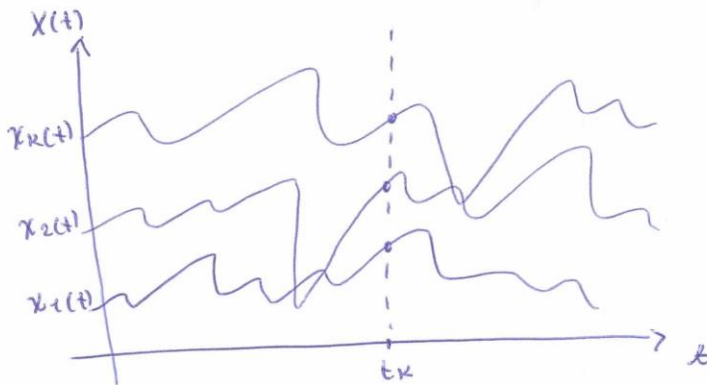
$$R_B = r_{\pi} + R_E (1 + \beta)$$

$$R_E = \frac{1}{g_m}$$

$$R_c = \begin{cases} g_m r_{ce} \cdot r_{\pi} \cdot \frac{R_E}{R_E + r_{\pi}} \\ R_E > \frac{r_{\pi}}{\beta} \end{cases}$$

PROCESSO STOCASTICO

È una n.a. indicizzata $X(t)$



Media di insieme: $\mu_x(t_k) = E[X(t_k)] = \int_{-\infty}^{+\infty} x \cdot f_{X(t_k)}(x) dx$

Correlazione per un processo stocastico:

$$R_{xx}(t_1, t_2) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} x_1 x_2 f_{X(t_1)X(t_2)}(x_1, x_2) dx_1 dx_2$$

Processo Stazionario in senso lato:

$$\mu_x(t) = \text{cost} \quad \forall t$$

$$R_{xx}(t_1, t_2) = R_x(\tau) \quad \text{con } \tau = t_2 - t_1$$

Media Temporale

$$\bar{x}(t) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) dt$$

Processo Regolare:

$$\bar{x}(t) = \text{cost} \quad \forall t$$

Processo Ergodico:

Se medie di insieme e temporali possono essere scambiate.

Densità Spettrale di Potenza:

$$S(f) = \int_{-\infty}^{+\infty} R_x(\tau) e^{-j2\pi f\tau} d\tau$$

$$\rightarrow R_x(\tau) = \int_{-\infty}^{+\infty} S(f) e^{j2\pi f\tau} df$$

$$P_x = \int_{-\infty}^{+\infty} S(f) df = R_x(0)$$

Se il processo è ergodico:

$$P_x = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(\tau) d\tau$$

Se il processo è a media nulla:

$$P_x = \sigma_x^2$$

Valore Efficace:

$$x_{rms} = \sigma_x \quad \text{se } \mu_x = 0$$

$$x_{rms} = \sqrt{\lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(\tau) d\tau}$$

PROCESSO BIANCO

$$S(f) = \frac{N_0}{2} \quad ; \quad R_x(\tau) = \frac{N_0}{2} \cdot \delta(\tau)$$

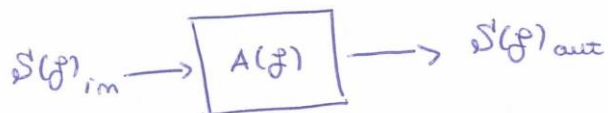
In una banda B:

$$S(f) = \begin{cases} N_0/2 & -B \leq f \leq B \\ \emptyset & \text{altrove} \end{cases}$$

$$\rightarrow P_x = N_0 \cdot B$$

$$R_x(\tau) = N_0 B \operatorname{sinc}(2\pi B\tau)$$

SISTEMA LTI



$$S(f)_{out} = S(f)_{in} \cdot |A(f)|^2$$

BANDA EQUIVALENTE DI RUMORE

Supponiamo un sistema LTI:

$$S(f)_{out} = |A(f)|^2 \cdot S(f)_{in}$$

Se inserisco in ingresso un rumore bianco:

$$S(f)_{out} = |A(f)|^2 \cdot \frac{N_0}{2}$$

$$P_1 = \int_{-\infty}^{+\infty} S(f)_{out} df = \int_{-\infty}^{+\infty} \frac{N_0}{2} |A(f)|^2 df$$

Supponiamo ora un filtro ideale con banda B_{ez} , con guadagno A_0 , ed inseriamo in ingresso un rumore bianco

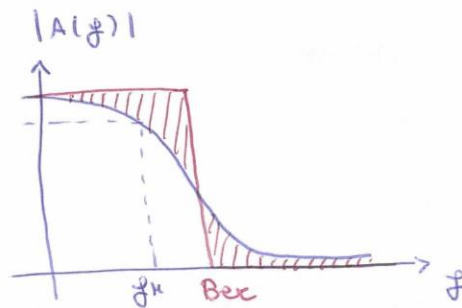
$$P_2 = S(f)_{out} = A_0^2 \cdot \frac{N_0}{2}$$

$$P_2 = \int_{-\infty}^{+\infty} S(f)_{out} df = A_0^2 \frac{N_0}{2} \cdot B_{ez}$$

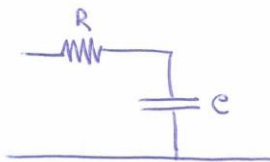
Imponiamo $P_1 = P_2$

$$\Rightarrow B_{ez} : \text{Banda equivalente di rumore} = \frac{1}{A_0^2} \int_{-\infty}^{+\infty} |A(f)|^2 df$$

La banda equivalente di rumore è la banda del filtro ideale che in uscita produce lo stesso guadagno del mio sistema LTI, che quando ha in ingresso un rumore bianco lascia passare la stessa potenza di rumore del filtro ideale con ingresso un rumore bianco.



Per un filtro RC 1° ordine:



$$A(f) = \frac{1}{1 + j2\pi fRC} = \frac{1}{1 + j \frac{f}{f_H}}$$

$$\text{con } f_H = \frac{1}{2\pi RC}$$

$$B_{eq} = \frac{1}{1} \cdot \int_0^{+\infty} \frac{1}{1 + (2\pi fRC)^2} \cdot df = f_H \cdot \frac{\pi}{2} = \frac{1}{2\pi RC} \cdot \frac{\pi}{2}$$

POTENZA DELLA SOMMA

$$Y(t) = X_1(t) + X_2(t) \quad , \quad X_1(t), X_2(t) \text{ n.a.}$$

$$\overline{Y(t)^2} = \overline{(X_1(t) + X_2(t))^2} = \overline{X_1(t)^2} + \overline{X_2(t)^2} + 2 \overline{X_1(t)X_2(t)}$$

Se il processo è ergodico:

$$\overline{Y(t)^2} = \overline{X_1(t)^2} + \overline{X_2(t)^2}$$

La potenza della somma è la somma delle potenze.

SPOT NOISE

$$SN(f) = \lim_{B \rightarrow \infty} \frac{|V_{\text{sig}}(f, B)|}{B^{-1/2}}$$

Per un rumore bianco: $SN(f) = \sqrt{\frac{N_0}{2}}$

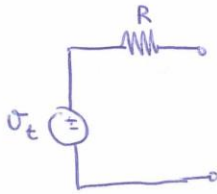
FONTE DI RUMORE

RUMORE TERMICO

È riconducibile ad un processo bianco a media nulla.

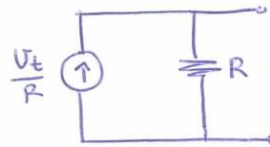
PSD = $4kT$

Equivalente di Thevenin:



$$PSD = 4kTR \left[\frac{V^2 \delta f^2}{Hz} \right]$$

Equivalente di Norton:



$$PSD = \frac{4kT}{R} \left[\frac{A^2 \delta f^2}{Hz} \right]$$

Se $T = 25^\circ C$

$$\begin{aligned} \rightarrow PSD &= 16 \cdot 10^{-18} \cdot R \\ &= 1,6 \cdot 10^{-20} \cdot R_{k\Omega} \end{aligned}$$

$$\rightarrow SN = 4 \cdot \sqrt{R_{k\Omega}} \cdot \frac{mV}{\sqrt{Hz}}$$

nel caso di un'impedenza generica risultata:

$$P = 4kT \int_{\beta}^{+\infty} \text{Re}(z) df$$

RUMORE SHOTCKY

È un processo bianco legato alla corrente di corrente continua, in particolare alla fluttuazione del valore medio.

$$PSD = 2q I_{DC}$$

$$\begin{aligned} \rightarrow PSD &= 3,2 \cdot 10^{-19} \cdot I_{DC} \\ &= 0,32 \cdot 10^{-24} \cdot I_{DC} \mu A \end{aligned}$$

$$\rightarrow SN = 0,57 \sqrt{I_{DC} \mu A} \cdot \frac{pA}{\sqrt{Hz}}$$

RUMORE FLICKER

Non è un processo bianco ma è a media nulla. È provocato da fluttuazioni del valore medio della corrente, ed è significativo quando fa variazioni di conducibilità.

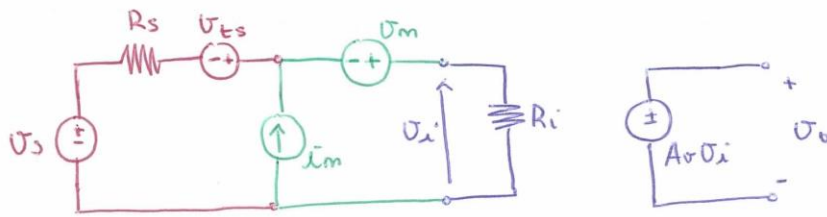
$$PSD(f) = \frac{K_f \cdot I_{DC}}{f}$$

$$\rightarrow P(f_1, f_2) = \int_{f_1}^{f_2} \frac{K_f \cdot I_{DC}}{f} df = K_f \cdot I_{DC} \cdot \ln\left(\frac{f_2}{f_1}\right)$$

$$\rightarrow SN = \lim_{f_2 \rightarrow f_1} \frac{|V_{eff}(f, B)|}{(f_2 - f_1)^{1/2}} ; V_{eff}(f, B) = \sqrt{K_f I_{DC} \ln\left(\frac{f_2}{f_1}\right)}$$

$$SN = \sqrt{\frac{K_f I_{DC}}{f_1}}$$

MODELLO U_m, I_m DELL'A.O.



$$SNR_{im} = \frac{P_s}{P_m} = \frac{\overline{V_s}^2}{4kT R_s B}$$

$$U_o = A_v \cdot U_i = A_v \cdot \left[\frac{R_i}{R_s + R_i} (U_s + U_n + U_m) + I_m \cdot \frac{R_i R_s}{R_i + R_s} \right]$$

$$= A_v \cdot \frac{R_i}{R_i + R_s} \left[U_s + U_n + U_m + I_m \cdot R_s \right]$$

$$= A_v \cdot \frac{R_i}{R_i + R_s} \left[U_s + U_{mi} \right]$$

con $U_{mi} = U_n + U_m + I_m \cdot R_s$: Nota della tensione di rumore equivalente in ingresso.

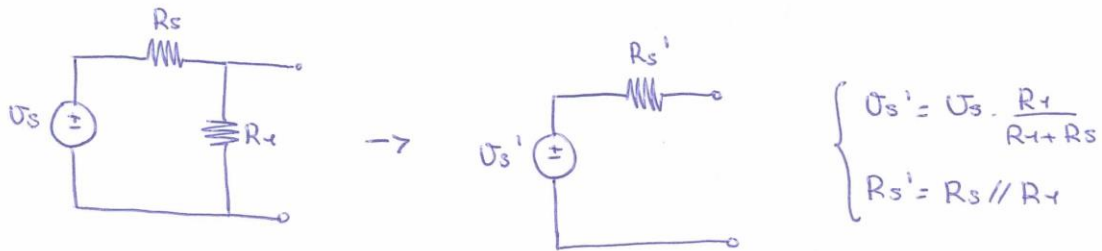
$$V_{mi}^2 = 4kT R_s B + V_m^2 + I_m^2 R_s^2 + 2\rho V_m I_m R_s$$

Impresso c.c. $\rightarrow U_o = A_v \cdot U_m$

Impresso c.a. $\rightarrow U_o = A_v \cdot R_i I_m$ con $R_i = \frac{R_i R_s}{R_i + R_s}$

$$SNR_{out} = \frac{P_s}{P_m} = \frac{V_s^2}{V_{mi}^2}$$

È possibile migliorare SNR_{out} usando una rete di accoppiamento di modo che $R_s \downarrow$?



$$U_o = A_v \cdot U_i = A_v \cdot \frac{R_i}{R_s' + R_i} \left[U_s' \frac{R_L}{R_L + R_s'} + U_{th}' + U_m + i_m \cdot R_s' \right]$$

$$= A_v \cdot \frac{R_i}{R_s // R_L + R_i} \left[U_s \cdot \frac{R_L}{R_L + R_s} + U_{th}' + U_m + i_m \cdot R_s' \right]$$

$$= A_v \cdot \frac{R_i}{R_s // R_L + R_i} \cdot \frac{R_L}{R_L + R_s} \left[U_s + \left(\frac{R_L + R_s}{R_L} \right) (U_{th}' + U_m) + i_m \cdot \frac{R_s R_L}{R_s + R_L} \cdot \frac{R_s + R_L}{R_L} \right]$$

$$= A_v \cdot \frac{R_i}{R_s // R_L + R_i} \cdot \frac{R_L}{R_L + R_s} \left[U_s + \left(\frac{R_L + R_s}{R_L} \right) (U_{th}' + U_m) + i_m \cdot R_s \right]$$

$$= A_v \cdot \frac{R_i}{R_s // R_L + R_i} \cdot \frac{R_L}{R_L + R_s} \left[U_s + U_{mi} \right]$$

$$\text{con } U_{mi} = \frac{R_L + R_s}{R_L} (U_{th}' + U_m) + i_m \cdot R_s$$

$$V_{mi}^2 = 4kTB \frac{R_s R_L}{R_s + R_L} \cdot \left(\frac{R_s + R_L}{R_L} \right)^2 + V_m^2 \left(\frac{R_L + R_s}{R_L} \right)^2 + I_m^2 R_s^2 + 2\rho V_m I_m R_s \cdot \frac{R_L + R_s}{R_L}$$

$$= 4kT \frac{R_s (R_s + R_L)}{R_L} B + V_m^2 \left(\frac{R_L + R_s}{R_L} \right)^2 + I_m^2 R_s^2 + 2\rho V_m I_m R_s \cdot \frac{R_L + R_s}{R_L}$$

la soluzione che minimizza si ha per $R_L = \infty \Omega$

$$\rightarrow R_L = c.a.$$

CIFRA DI RUMORE

$$NF_{dB} = 10 \log_{10} \frac{P_{out} \text{ con } \sigma_s = \phi}{P_{out} \text{ con } \sigma_m \text{ c.c. e } \sigma_i \text{ c.a.}}$$

Per le morbelle σ_m, σ_i dall'A.O. risulta

$$NF_{dB} = 10 \log_{10} \frac{4kTR_s B + V_m^2 + I_m^2 R_s^2 + 2\rho V_m I_m R_s}{4kTR_s B}$$

$$= 10 \log_{10} \left(1 + \frac{V_m^2 + I_m^2 R_s^2 + 2\rho V_m I_m R_s}{4kTR_s B} \right)$$

Quanto minimizzato SNF_{dB} ?

$$\frac{d}{dR_s} \left(\frac{V_m^2 + I_m^2 R_s^2 + 2\rho V_m I_m R_s}{4kTR_s B} \right) =$$

$$= \frac{[2I_m^2 R_s + 2\rho V_m I_m] \cdot R_s - [V_m^2 + I_m^2 R_s^2 + 2\rho V_m I_m R_s]}{R_s^2} = \phi$$

$$\Rightarrow I_m^2 R_s^2 - V_m^2 = \phi \quad \rightarrow R_s = \frac{V_m}{I_m}$$

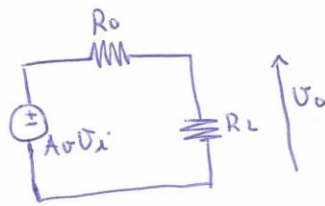
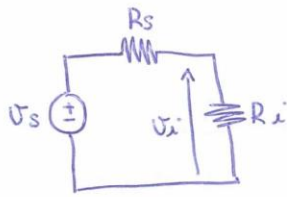
NF_{dB} si minimizza per $R_s = \frac{V_m}{I_m}$.

$$SNR_{out, dB} = \left(\frac{\bar{V}_s^2}{V_{mi}^2} \right)_{dB} = \left(\frac{\bar{V}_s^2}{4kTR_s B} \right)_{dB} + \left(\frac{4kTR_s B}{V_{mi}^2} \right)_{dB}$$

$$= SNR_{in, dB} - NF_{dB}$$

$$\rightarrow NF_{dB} = SNR_{in, dB} - SNR_{out, dB} = \frac{SNR_{in}}{SNR_{out}}$$

A.O. IN CASCATA



$$U_o = A_v \cdot U_i \cdot \frac{R_L}{R_o + R_L} = A_v \cdot U_s \cdot \frac{R_i}{R_s + R_i} \cdot \frac{R_L}{R_o + R_L}$$

$$A_{v \text{ c.a.}} = A_v \cdot \frac{R_i}{R_s + R_i}$$

$$A_{v \text{ tot}} = A_{v \text{ c.a.}} \cdot \frac{R_L}{R_o + R_L}$$

Se ho 2 A.O. in cascata:

$$A_{v \text{ tot}} = A_{v1 \text{ c.a.}} \cdot A_{v2 \text{ c.a.}} \cdot \frac{R_L}{R_{o2} + R_L}$$

Se ho n A.O. in cascata:

$$A_{v \text{ tot c.a.}} = \prod_{j=1}^n A_{v_j \text{ c.a.}}$$

$$\text{con } A_{v_j \text{ c.a.}} = A_{v_j} \cdot \frac{R_{i j}}{R_{o_{j-1}} + R_{i j}}$$

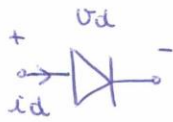
$$R_{o_j} = R_s$$

$$A_{v \text{ tot}} = A_{v \text{ tot c.a.}} \cdot \frac{R_L}{R_{o_N} + R_L}$$

Il primo A.D. dal punto di vista del numero è il più importante:

$$\begin{aligned}
 U_{\text{mult}} &= (U_{m1} + U_{k1}) A_{U_{T01}} + U_{m2} \frac{A_{U_{T01}}}{A_{U_{k1 c.a.}}} + \dots + U_{mN} \frac{A_{U_{T01}}}{\prod_{k=1}^{N-1} A_{U_{k c.a.}}} \\
 &= A_{U_{T01}} \left(U_{k1} + U_{m1} + \sum_{k=2}^N \frac{U_{m k}}{\prod_{k=1}^{k-1} A_{U_{k c.a.}}} \right)
 \end{aligned}$$

RUMORE NELLE GIUNZIONI PN



$$\begin{aligned}
 i_d &= I_s \left(e^{\frac{v_d}{V_T}} - 1 \right) \\
 &= I_s e^{\frac{v_d}{V_T}} - I_s \\
 &= I_F - I_s \quad \text{con } I_F = I_s e^{\frac{v_d}{V_T}}
 \end{aligned}$$

Niente contributo rumore shot:

$$PSD = 2q(I_F + I_s) = 2qI_0$$

Niente contributo rumore flicker:

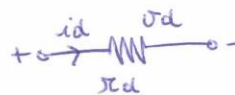
$$PSD = \frac{K_f I_D}{f}$$

In totale:

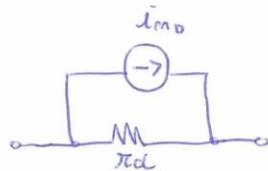
$$PSD = 2qI_0 + \frac{K_f I_D}{f}$$

Per piccoli segnali, il diodo può essere considerato come:

$$r_d = \left(\frac{\partial i_d}{\partial v_d} \right)^{-1} = \frac{V_T}{I_D}$$

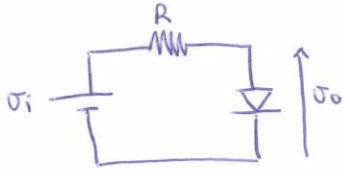


Considerando anche il rumore termico:



Supponiamo solo il Rumore Shot:

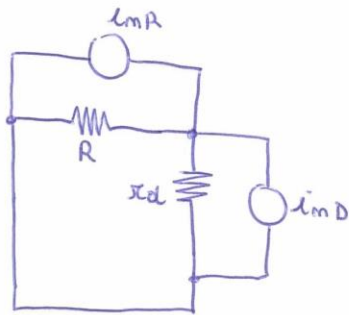
$$SN = 0,57 \sqrt{I_{DC} \mu A}$$



non considerando \$V_f\$:

$$\rightarrow I_{DC} = I_D = \frac{V_i}{R}$$

Per piccoli segnali:



$$r_d = \frac{V_T}{I_D}$$

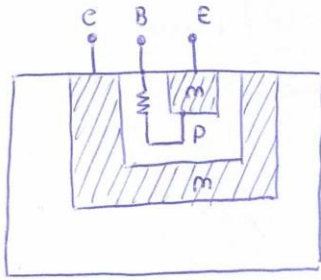
$$PSD(i_{mR}) = \frac{4kT}{R}$$

$$PSD(i_{mD}) = 2qI_D$$

Rumore totale in uscita:

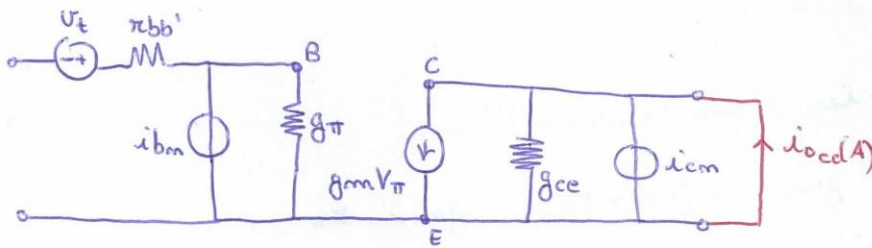
$$V_{out} = (i_{mD} + i_{mR}) \cdot r_d // R$$

RUMORE NEI BJT

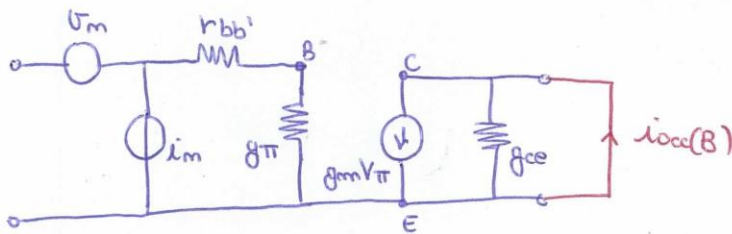


È presente un fenomeno resistivo che si oppone allo accorciamento di corrente fra Base ed Emittitore.

Modello con Rumore:



Per studiare questo modello:



$R_s = 0 \Omega \rightarrow$ Pradua effetto solo V_m

$R_s = \infty \Omega \rightarrow$ Pradua effetto solo i_m

$$r_{bb'} \approx 10 \Omega \rightarrow r_{bb'} \ll r_{\pi} = \frac{1}{g_{\pi}} \approx 1 \div 10 \text{ k}\Omega$$

$$R_s = \phi \approx \infty$$

$$i_{occ}(A) = g_m V_{\pi} + i_{cm} = g_m \left[\cancel{V_t \cdot \frac{\pi_{\pi}}{r_{bb'} + \pi_{\pi}}} + \underbrace{i_{bm} \cdot r_{\pi} // r_{bb'}}_{\text{transcursibile}} \right] + i_{cm}$$

$$i_{occ}(B) = g_m V_{\pi} = g_m V_m \cdot \frac{r_{\pi}}{r_{bb'} + r_{\pi}} = g_m V_m$$

$$g_m V_m \approx g_m V_t + i_{cm}$$

$$\rightarrow V_m = V_t + \frac{i_{cm}}{g_m}$$

rumore shot

$$R_s = \infty \Omega$$

rumore termico

$$i_{oca}(A) = g_m V_{\pi} + i_{cm} = g_m (i_{bm} \cdot \pi_{\pi}) + i_{cm}$$

$$i_{oca}(B) = g_m V_{\pi} = g_m (i_m (r_{bb'} + \pi_{\pi})) = g_m i_m \pi_{\pi}$$

$$\rightarrow i_m = i_{bm} + \frac{i_{cm}}{g_m \pi_{\pi}}$$

rumore shot

rumore shot + flicker

$$PSD(V_m) = 4kT \pi_{bb'} + \frac{1}{g_m^2} (2qI_c)$$

$$g_m = \frac{I_c}{V_t} = \frac{I_c}{kT} \cdot q$$

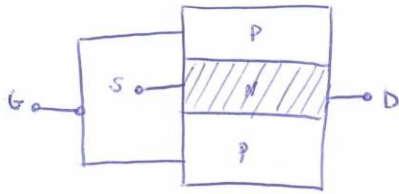
$$\rightarrow PSD(V_m) = 4kT \pi_{bb'} + \frac{1}{g_m} \frac{2qI_c}{I_c \cdot q} \cdot kT \cdot \frac{1}{2}$$

$$= 4kT \left[\pi_{bb'} + \frac{1}{2g_m} \right]$$

Nota con piacere che $PSD(V_m)$ dipende da I_c , ovvero dal punto di lavoro.

RUMORE NEI FET

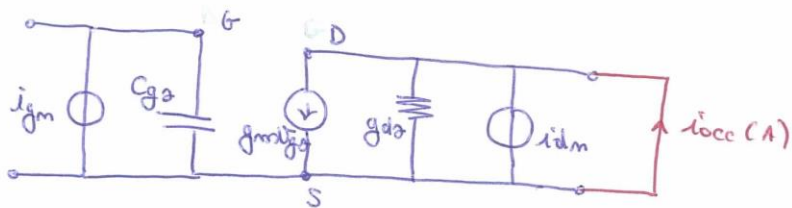
JFET



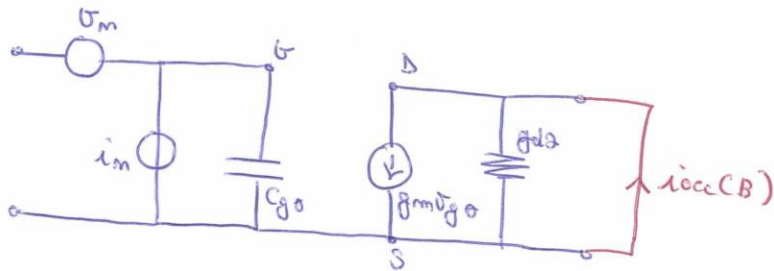
Se $V_{GS} > \phi$, cessa la regione di carica spaziale e si riduce il canale.

La differenza del MOSFET i_g non è trascurabile.

Modello circuitale di un FET:



Taglio internamente questo modello:



$$R_s = 0 \Omega$$

$$i_{occ(A)} = g_m v_{gs} + i_{dm} = i_{dm}$$

$$i_{occ(B)} = g_m v_{gs} = g_m v_m$$

$$\rightarrow v_m = \frac{i_{dm}}{g_m} \leftarrow \text{rumore termico + flicker}$$

$$R_s = \infty \Omega$$

$$i_{oca}(A) = g_m \bar{v}_{gs} + i_{dm} = g_m \left(i_{gm} \cdot \frac{1}{\omega C_{gs}} \right) + i_{dm}$$

$$i_{oca}(B) = g_m \bar{v}_{gs} = g_m \left(i_{im} \cdot \frac{1}{\omega C_{gs}} \right)$$

$$\rightarrow i_{im} = i_{gm} + \frac{\omega C_{gs}}{g_m} \cdot i_{dm} \leftarrow \text{summae sPlot} \quad \leftarrow \text{summae termies + flicker}$$

$$\text{PSD}(\bar{v}_m) = \left[4kT \frac{2}{3} g_m + \frac{K_f I_D}{f} \right] \cdot \frac{1}{g_m^2}$$

$$= 4kT \frac{2}{3} \cdot \frac{1}{g_m} + \frac{K_f \cdot I_D}{f} \cdot \frac{1}{g_m^2}$$

$$g_m = \sqrt{2kI_D}$$

$$\text{PSD}(\bar{v}_m) = 4kT \frac{2}{3} \cdot \frac{1}{g_m} + \frac{K_f I_D}{f \cdot 2kI_D}$$

$$k = \frac{1}{2} \mu C_{ox} \frac{W}{L}$$

$$\text{PSD}(\bar{v}_m) = 4kT \frac{2}{3} \frac{1}{g_m} + \frac{K_f}{\mu C_{ox} \frac{W}{L} \cdot f}$$

$$\text{PSD}(i_m) = 2qI_G + \frac{\omega^2 C_{gs}^2}{g_m^2} \left[4kT \frac{2}{3} g_m + \frac{K_f I_D}{f} \right]$$

$$\text{Per } I_D \approx 1 \text{ mA} \rightarrow g_m^{-1} \approx (100 \Omega)^{-1}$$

$$4kT \frac{2}{3} \frac{1}{g_m} \approx 1,03 \text{ mV}/\sqrt{\text{Hz}}$$

$$\text{Suppongo } f: \frac{K_f I_D}{f} \text{ sia trascurabile.}$$

$$\rightarrow \text{PSD}(i_m) \approx 2qI_G$$

$$R_s = \frac{\bar{v}_m}{i_m} = \frac{\text{SN}(\bar{v}_m) \cdot B}{\text{SN}(i_m) \cdot B} = \frac{\text{SN}(\bar{v}_m)}{\text{SN}(i_m)}$$



FILTRO PASSA BASSO DI ORDINE m

$$|A(s)| = \frac{A_0}{1 + a_1 s + \dots + a_m s^m} \quad a_i \in \mathbb{R}^+$$

1 POLO :

$$f_d t = \frac{1}{1 + j\omega RC} = \frac{1}{1 + j \frac{\omega}{\omega_H}} \quad \text{con } \omega_H = \frac{1}{RC}$$

$$\rightarrow f_d t = \frac{1}{1 + j \frac{\omega}{\omega_H}} = \frac{1}{1 + j^y} \quad \text{con } j^y = \frac{\omega}{\omega_H}$$

$$\rightarrow f_d t = \frac{1}{1 + j \frac{\omega}{\omega_H}} = \frac{1}{1 + j \Omega} \quad \text{con } \Omega = \frac{\omega}{\omega_H}$$

$$\Omega_H : \frac{1}{\sqrt{1 + \Omega_H^2}} = \frac{1}{\sqrt{2}}$$

POLO MULTIPLIO :

$$f_d t = \frac{1}{(1 + j\omega RC)^m}$$

$$|A|^2 = \frac{1}{[1 + (\omega RC)^2]^m}$$

$$\omega_{HTOT} : |A(\omega_{HTOT})| = \frac{1}{\sqrt{2}} \quad \rightarrow \omega_{HTOT} = \omega_H \cdot \sqrt[m]{\sqrt{2} - 1}$$

POLE COMPLESSI:

$$A(j\omega) = \frac{1}{1 + j\omega CR - \omega^2 LC} = \frac{1}{1 + \alpha s + \beta s^2} = \frac{1}{1 + \alpha \omega_0 \frac{s}{\omega_0} + \frac{s^2}{\omega_0^2}}$$

$$\omega_0^2 = \frac{1}{\beta}$$

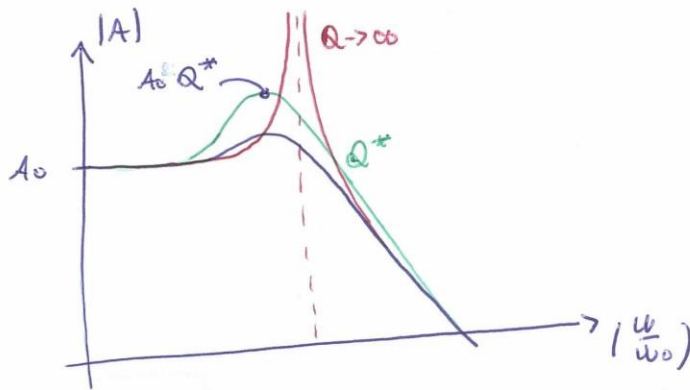
$$\alpha \omega_0 = \frac{1}{Q} = 2 \zeta$$

$$|A(j\omega)|^2 = \frac{1}{\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right]^2 + \frac{1}{Q^2} \left(\frac{\omega}{\omega_0}\right)^2}$$

$$\frac{d}{d\left(\frac{\omega}{\omega_0}\right)^2} = 0 \rightarrow \left(\frac{\omega}{\omega_0}\right)^2 = \sqrt{1 - \frac{1}{2Q^2}}$$

$$\left(\frac{\omega}{\omega_0}\right) \in \mathbb{R} \text{ ssc } Q^2 > \frac{1}{2}, Q > \frac{1}{\sqrt{2}}$$

→ Poles complessi solo per radici complesse e coniugate.



$$\left| A(j\omega) \right|^2 \Big|_{\left(\frac{\omega}{\omega_0}\right)^2 = 1 - \frac{1}{2Q^2}} \approx A_0^2 Q^2$$

FILTRO L.P. DI BUTTERWORTH

-> Massima piatezza in banda passante.

$$|A(j\Omega)|^2 = \frac{A_0^2}{1 + K_2 \Omega^2 + K_4 \Omega^4 + \dots + K_{2m} \Omega^{2m}} \quad \text{con } \Omega = \left(\frac{\omega}{\omega_c} \right)$$

BUTTERWORTH CONDITION:

$$\begin{cases} K_{2 \cdot i} = 0 & i = 1, m-1 \\ K_{2 \cdot m} \neq 0 \end{cases}$$

$$\rightarrow |A(j\Omega)|^2 \Big|_{\text{Butterworth}} = \frac{A_0^2}{1 + K_{2m} \Omega^{2m}}$$

Per $\omega = \omega_c \rightarrow \Omega = 1 \rightarrow K_{2m} = 1$.

FILTRO L.P. DI CHEBISHEV

→ Ammette Ripple in banda passante ma raggiunge
i -40 dB/dec con pendenza iniziale maggiore.

$$|A(j\omega)|^2 = \frac{KA_0^2}{1 + \varepsilon^2 C_m^2(x)}$$

$$x = \frac{\omega}{\omega_c}$$

ω_c : pulsazione di uscita
della banda di RIPPLE.

$$C_m(x) = \begin{cases} \cos(m \cos^{-1}(x)) & |x| \leq 1 \\ \cosh(m \cosh^{-1}(x)) & |x| > 1 \end{cases}$$

$C_m(x)$: polinomio di
Chebyshev di ordine m .

$$x = \cos \alpha \rightarrow C_m(x) = \cos(m\alpha) \quad \text{per } |x| \leq 1$$

$$\cos a \cdot \cos b = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$$

$$C_1(x) = x$$

$$C_2(x) = \cos(2\alpha) = 2\cos^2\alpha - 1 = 2x^2 - 1$$

$$C_3(x) = 4x^3 - 3x$$

m dispari: manca il termine noto.

m pari: termine noto ± 1 .

K : fattore di normalizzazione

$$x = \phi \rightarrow |A(j\omega)|^2 = A_0^2$$

$$m \text{ dispari} : K = 1$$

$$m \text{ pari} : K = (1 + \varepsilon^2)$$

RIPPLE:

$$\frac{|A(j\omega)|_{\text{MAX}}^2}{|A(j\omega)|_{\text{MIN}}^2} \rightarrow \text{RIPPLE} = 20 \log_{10} \sqrt{1 + \varepsilon^2}$$

FILTRO L.P. DI BESSELL

→ Ritardo di gruppo massimamente costante in banda.

$$A(s) = \frac{1}{1+as+bs^2} = \frac{1}{1+a\omega - b\omega^2} \quad \text{con } s = j\omega$$
$$\omega = \frac{\omega}{\omega_H}$$

$$\phi = -\arctg\left(\frac{a\omega}{1-b\omega^2}\right)$$

$$\tau_{gr} = -\frac{d}{d\omega} \phi, \quad \text{ritardo di gruppo}$$

$$T_{gr} = \frac{\tau_{gr}}{T_H}, \quad \text{ritardo di gruppo normalizzato}$$

$$\text{con } T_H = \frac{1}{\omega_H}$$

$$T_{gr} = -\frac{d}{d\omega} \phi \cdot \frac{\omega_H}{2\pi} = -\frac{1}{2\pi} \cdot \frac{d}{d\omega} \phi$$

$$T_{gr} \approx \frac{1}{2\pi} \cdot \frac{a(1+b\omega^2)}{1+(a^2-2b)\omega^2} = \text{cost}$$

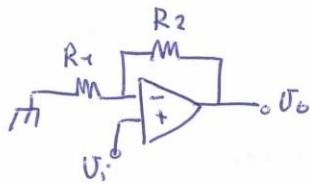
$$\rightarrow 1+b\omega^2 = 1+(a^2-2b)\omega^2$$

$$\rightarrow b = a^2 - 2b$$

$$|A(j\omega)|^2 = \frac{1}{(1-b\omega^2)^2 + a^2\omega^2} \Big|_{\omega=1} = \frac{1}{2}$$

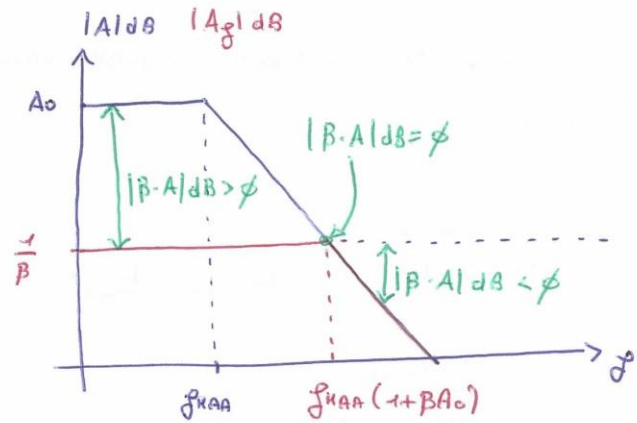
$$\rightarrow (1-b)^2 + a^2 = 2$$

A.O. NON INVERTENTE



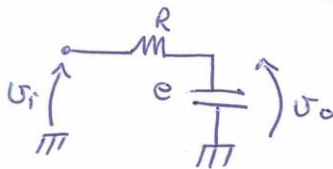
$$\frac{U_o}{U_i} = \frac{1}{\beta} \cdot \frac{\beta \cdot A}{1 + \beta \cdot A}$$

$$= \frac{1}{\beta} \quad \text{sc} \quad \beta \cdot A \gg 1.$$



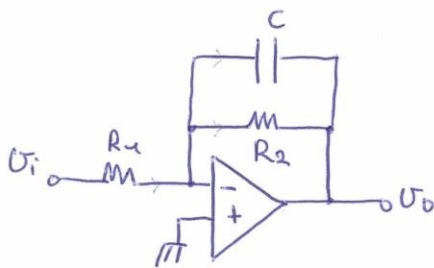
$$|\beta \cdot A|_{dB} = |\beta|_{dB} + |A|_{dB} = |A|_{dB} - \left| \frac{1}{\beta} \right|_{dB}$$

FILTRO 1° ORDINE PASSIVO



$$\frac{U_o}{U_i} = \frac{1}{1 + sCR}$$

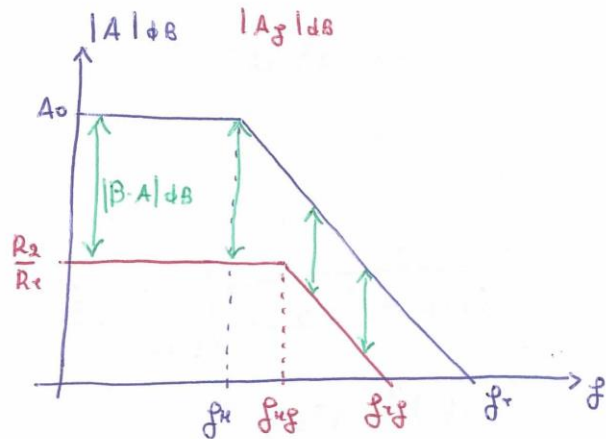
FILTRO 1° ORDINE ATTIVO



$$R_2 // C = Z_2$$

$$\frac{U_o}{U_i} = - \frac{R_2}{R_1} \cdot \frac{1}{1 + j\omega C R_2} = - \frac{Z_2}{R_1}$$

$$f_H = \frac{1}{2\pi R_2 C}$$

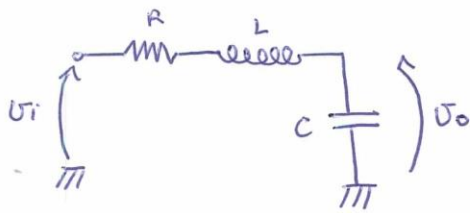


Per fase necessaria c.c.n.r. : $\beta \cdot A \gg 1$

$$\rightarrow f_{Tg} = f_{UG} \cdot A_g \ll f_T.$$

$$100 \cdot f_{Tg} \ll f_T.$$

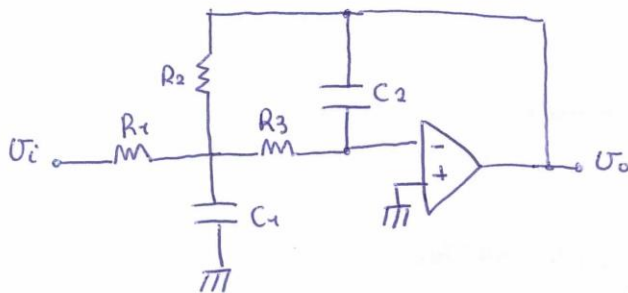
FILTRO 2° ORDINE PASSIVO



$$\frac{U_o}{U_i} = \frac{1}{1 + j\omega RC - \omega^2 LC}$$

FILTRO 2° ORDINE ATTIVO

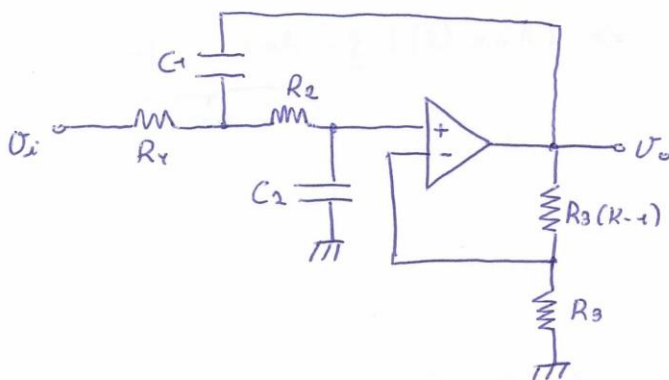
CELLA MULTIPLE FEEDBACK



$$\frac{U_o}{U_i} = \frac{-R_2/R_1}{1 + \alpha_{MFB} \cdot s + \beta_{MFB} \cdot s^2}$$

$$\begin{cases} \alpha_{MFB} = f(R_1, R_2, R_3, C_1, C_2) \\ \beta_{MFB} = f(R_1, R_2, R_3, C_1, C_2) \end{cases}$$

CELLA SOLLEN KEY



$$\frac{U_o}{U_i} = \frac{k}{1 + \alpha_{SK} \cdot s + \beta_{SK} \cdot s^2}$$

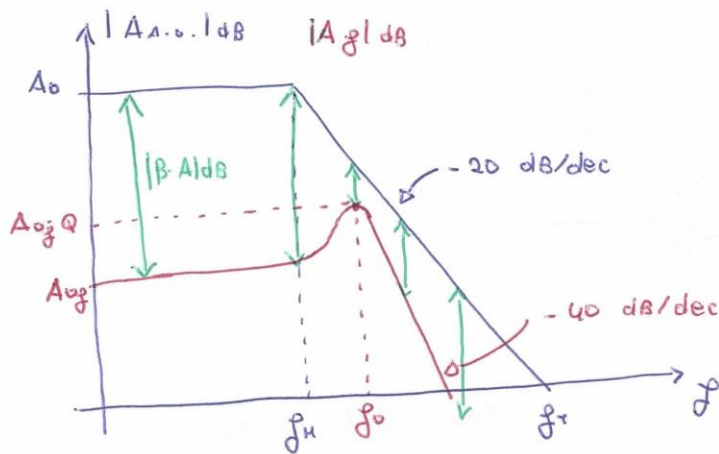
$$\begin{cases} \alpha_{SK} = f(R_1, R_2, C_1, C_2) \\ \beta_{SK} = f(R_1, R_2, C_1, C_2) \end{cases}$$

Sia per KFB che per SK è vero che:

$$\begin{cases} \sum_{R_i, C_i}^Q < 1 \\ \sum_{R_i, C_i} \omega_0 < 1 \end{cases}$$

$$A_f(s) = \frac{A_0}{1 + \frac{1}{Q} s + s^2}$$

$$A_{A.O.}(f) = \frac{A_0 A.O.}{1 + \beta \cdot \frac{f}{f_H}}$$



Per fare nostra c.c.v: $\beta \cdot A \gg 1$

$$f_T = A_0 \cdot f_H = \text{GBW}$$

$$\text{Per } f > f_H \rightarrow |A_{A.O.}(f)| = \frac{A_0 A.O.}{\frac{f}{f_H}} \Rightarrow |A_{A.O.}(f)| \cdot f = \underbrace{A_0 A.O. \cdot f_H}_{\text{GBW}}$$

$$\text{Quindi } |A_{A.O.}(f_0)| = \frac{\text{GBW}}{f_0}$$

$$|A_f(f_0)| = A_0 Q$$

$$A_0 Q \ll \frac{\text{GBW}}{f_0} \rightarrow \text{GBW} \geq 100 \cdot A_0 f_0 \cdot Q$$

in quanto:

$$|A_{A.O.}(f_0)| \gg |A_f(f_0)|$$

FILTRO HP

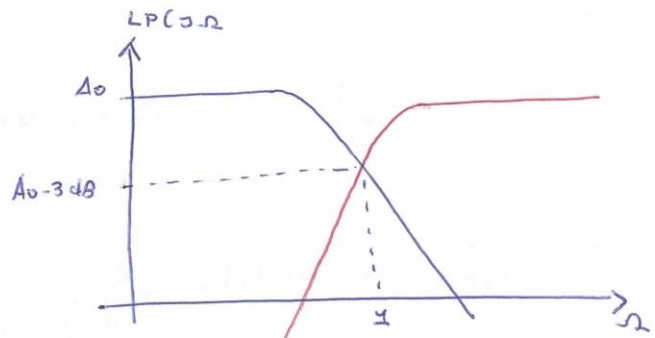
FILTRO HP DA LP

LP → HP

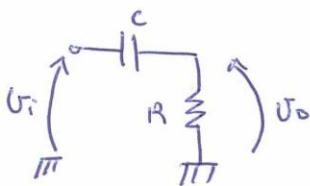
$s \rightarrow \frac{1}{s}$

$$LP(s) = \frac{A_0}{1+s} \rightarrow HP(s) = \frac{A_0 \cdot s}{1+s}$$

$$HP(j\omega) = \frac{A_0 \cdot j\omega}{1+j\omega}$$

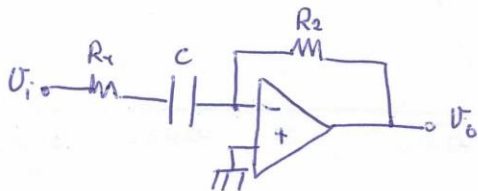


FILTRO HP 1° ORDINE PASSIVO



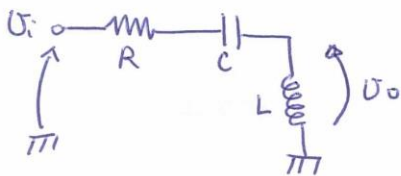
$$HP(s) = \frac{sCR}{1+sCR}$$

FILTRO HP 1° ORDINE ATTIVO



$$HP(s) = -\frac{R_2}{R_1 + \frac{1}{j\omega C}} = -\frac{j\omega C R_2}{1 + j\omega C R_1}$$

FILTRO HP 2° ORDINE PASSIVO



$$HP(s) = \frac{j\omega L}{R + j\omega L + \frac{1}{j\omega C}} = \frac{-\omega^2 LC}{1 + j\omega CR - \omega^2 LC}$$

FILTRO HP 2° ORDINE ATTIVO

Cella MFB e SK con C, R scambiati.

FILTRO BP

FILTRO BP DA LP

LP → BP

$$s \rightarrow \frac{1}{\Delta\omega} \left(s + \frac{1}{s} \right)$$

$$\Delta\omega = \omega_{MAX} - \omega_{MIN}$$

$$\omega_{MAX} = \frac{1}{\omega_{MIN}}$$

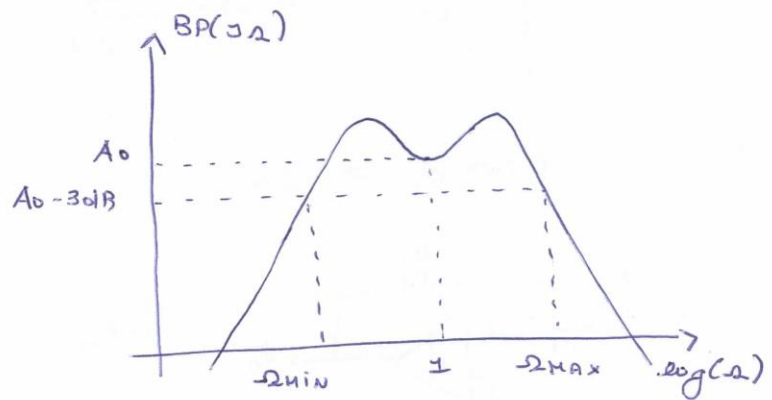
$$LP(s) = \frac{A_0}{1+s} \rightarrow BP(s) = \frac{A_0}{1 + \frac{1}{\Delta\omega} \left(s + \frac{1}{s} \right)} = \frac{A_0 s \Delta\omega}{1 + s\Delta\omega + s^2} \quad \left(Q = \frac{1}{\Delta\omega} \right)$$

$$s \rightarrow j\omega \quad \text{com } \omega = \frac{\omega}{\omega_r}$$

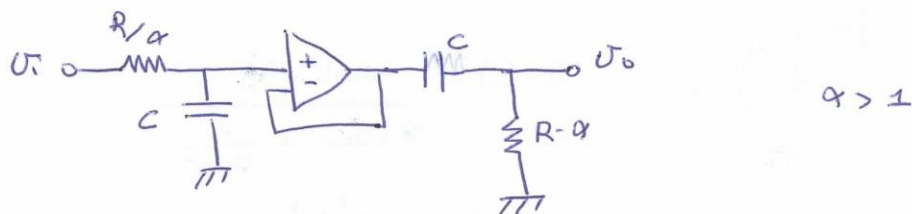
ω_r : Wanzweerde

$$|A|^2 = \frac{A_0^2 \omega^2 \Delta\omega^2}{(1 - \omega^2)^2 + \omega^2 \Delta\omega^2}$$

Se $\omega = 1 \rightarrow |A| = A_0$.



FILTRO BP CON CASCATA DI LP e HP



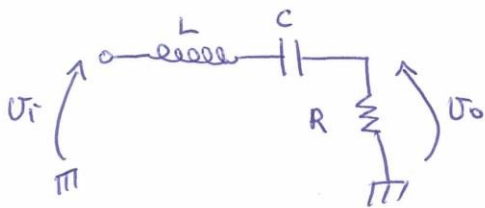
$$BP(s) = \frac{1}{1 + \frac{SCR}{\alpha}} \cdot \frac{SCR\alpha}{1 + SCR\alpha} = \frac{SCR\alpha}{1 + SCR \left(\alpha + \frac{1}{\alpha} \right) + s^2 C^2 R^2}$$

$$\omega_r = \frac{1}{CR}$$

$$BP(s') = \frac{s' \alpha}{1 + s' \left(\frac{Q^2 + 1}{\alpha} \right) + s'^2} \quad \text{con } s' = \frac{s}{\omega_r}$$

Per ottenere un BP a banda stretta uso due BP in cascata.

FILTRO BP 2° ORDINE PASSIVO



$$\begin{aligned} \frac{U_o}{U_i} &= \frac{R}{R + j\omega L + \frac{1}{j\omega C}} \\ &= \frac{j\omega C R}{1 + j\omega C R - \omega^2 L C} \end{aligned}$$

FILTRO BR

FILTRO BR DA LP

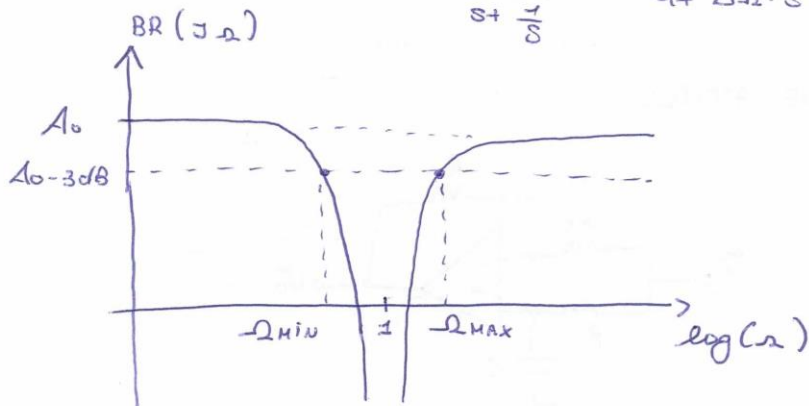
LP \rightarrow BR

$$s \rightarrow \frac{\Delta\omega}{s + \frac{1}{s}}$$

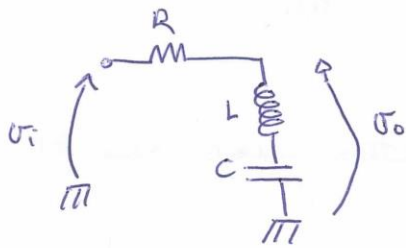
$$\Delta\omega = \omega_{MAX} - \omega_{MIN}$$

$$\omega_{MAX} = \frac{1}{\omega_{MIN}}$$

$$LP(s) = \frac{A_0}{1+s} \rightarrow BR(s) = \frac{A_0}{1 + \frac{\Delta\omega}{s + \frac{1}{s}}} = \frac{A_0(s^2 + 1)}{1 + \Delta\omega \cdot s + s^2}$$



FILTRO BR 2° ORDINE PASSIVO



$$BR(j\omega) = \frac{j\omega L + \frac{1}{j\omega C}}{R + j\omega L + \frac{1}{j\omega C}}$$

$$= \frac{\omega^2 LC - 1}{1 + j\omega RC - \omega^2 LC}$$

FILTRO AP

FILTRO AP DA LP

LP → AP

$$A(s) = \frac{\prod_{i=1}^{m/2} (1 - a_i s + b_i s^2)}{\prod_{i=1}^{m/2} (1 + a_i s + b_i s^2)}$$

$$A(j\Omega) = \frac{\prod_{i=1}^{m/2} [(1 - b_i \Omega^2)^2 + a_i^2 \Omega^2]^{1/2} e^{-j\alpha}}{\prod_{i=1}^{m/2} [(1 - b_i \Omega^2)^2 + a_i^2 \Omega^2]^{1/2} e^{j\alpha}}$$

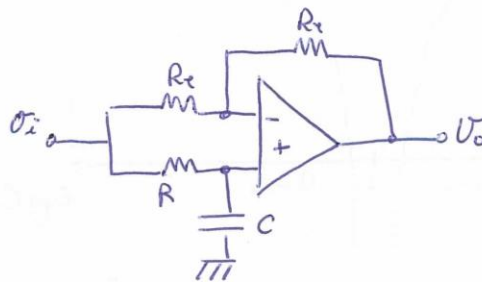
$$\Omega = \frac{\omega}{\omega_m}$$

$$\text{con } \alpha = \sum_{i=1}^{m/2} \arctan \frac{a_i \Omega}{1 - b_i \Omega^2}$$

$$\rightarrow A(j\Omega) = e^{-2j\alpha}$$

FILTRO AP 1° ORDINE ATTIVO

$$AP(s) = \frac{1 - as}{1 + as}$$



$$\rightarrow \frac{U_o}{U_i} = \frac{1 - j\omega RC}{1 + j\omega RC}$$