

FONDAMENTI DI TELECOMUNICAZIONI A

[Fotocopie di Appunti]

A CURA DI ALESSANDRO PAGHI

PROFESSORE: Giuliano Benelli (<http://www3.diism.unisi.it/people/person.php?id=18>)

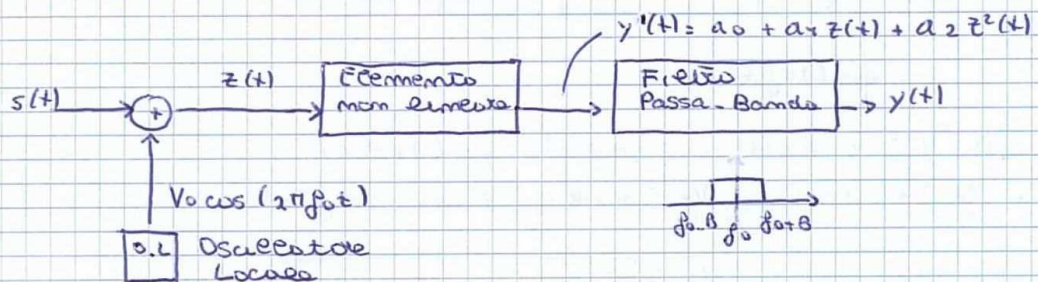
LINK AL CORSO ANNO 2014/2015:

<http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=54889&aa=2014>

FREQUENTAZIONE: Consigliata.

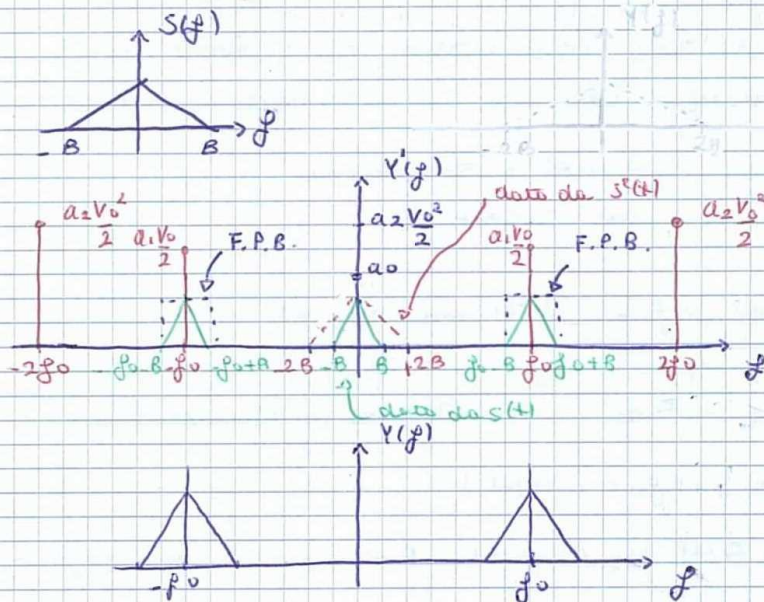
$$K = \max |s(t)| \leq 1$$

da maggiore parte della potenza è utilizzata per trasmettere la portante.



$$z(t) = s(t) + V_0 \cos(2\pi f_0 t)$$

$$y'(t) = a_0 + a_1 V_0 \cos(2\pi f_0 t) + a_1 s(t) + a_2 s^2(t) + a_2 V_0^2 \cos^2(2\pi f_0 t) + 2a_2 V_0 s(t) \cos(2\pi f_0 t)$$



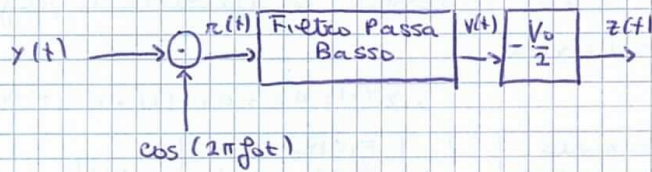
$$y(t) = a_1 V_0 \cos(2\pi f_0 t) + 2a_2 V_0 s(t) \cos(2\pi f_0 t)$$

$$= a_1 V_0 \left[1 + \frac{2a_2}{a_1} s(t) \right] \cos(2\pi f_0 t)$$

$\downarrow$
K

$$y(t) = V_0 [1 + K s(t) \cos(2\pi f_0 t)]$$

DEMODULATORE IN AMPIEZZA



$$\begin{aligned} r(t) &= V_0 [1 + K s(t) \cos(2\pi f_0 t)] \cos(2\pi f_0 t) \\ &= \frac{V_0}{2} [1 + K s(t)] + \frac{V_0}{2} [1 + K s(t)] \cos(4\pi f_0 t) \end{aligned}$$

$\nearrow$ $2f_0$

$$v(t) = \frac{V_0}{2} [1 + K s(t)]$$

$$z(t) = \frac{V_0}{2} K s(t)$$

Il cos utilizzato dal trasmettitore deve avere fase e frequenza uguale a quello del cos utilizzato dal ricevitore!

Demodulazione Costante o Sincrona

$$\cos_{transm} = \cos_{ricev.}$$

$$y(t) = \underbrace{a(t) \cos(2\pi f_0 t)}_{\text{Coma in Fase}} + \underbrace{b(t) \sin(2\pi f_0 t)}_{\text{Coma in Quadratura.}}$$

$$= R(t) \cos[\theta(t) - \theta(t)]$$

$$= R(t) \cos \theta(t) \cos(2\pi f_0 t) + R(t) \sin \theta(t) \sin(2\pi f_0 t)$$

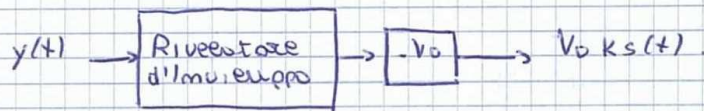
$$\rightarrow a(t) = R(t) \cos \theta(t)$$

$$b(t) = R(t) \sin \theta(t)$$

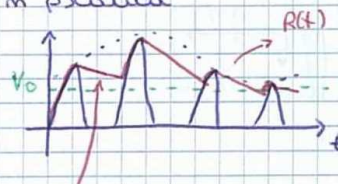
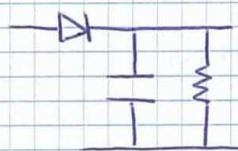
$$\begin{cases} R(t) = \sqrt{a^2(t) + b^2(t)} & \rightarrow \text{Inviluppo del segnale} \\ \tan \theta(t) = \frac{b(t)}{a(t)} & \rightarrow \text{Fase del segnale} \end{cases}$$

meo nostro caso:

$$R(t) = V_0 [1 + K s(t)]$$



Rivelatore d'increspatura in pratica



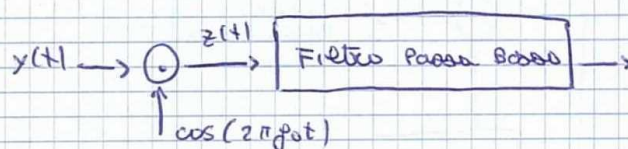
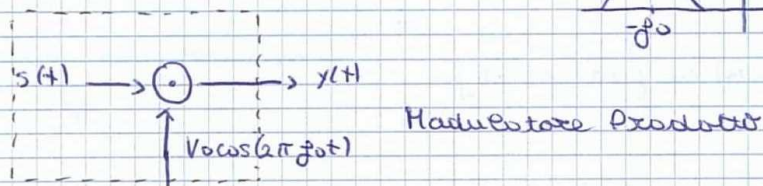
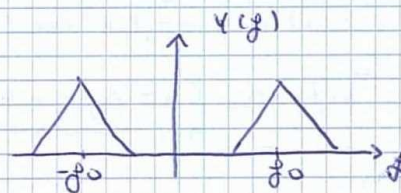
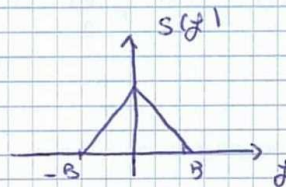
C si scarica su R.

Se tengo a $R(t)$, V_0 , ottengo $V_0 K s(t)$

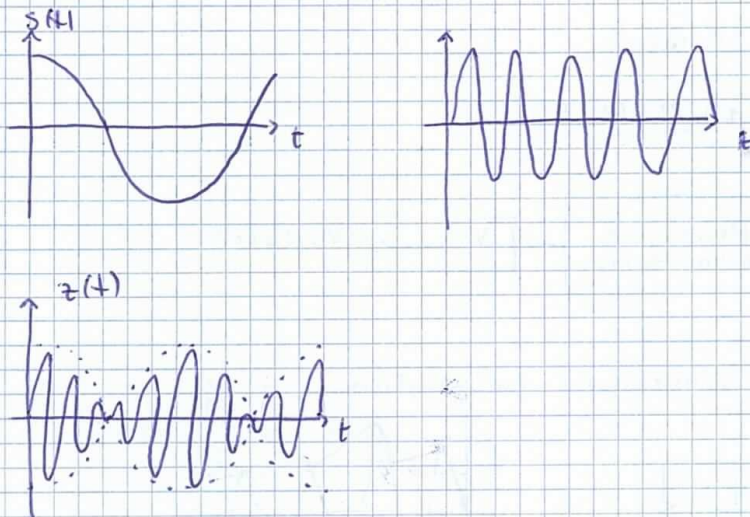
AM-DSB (Double Side Band)

$$V(t) = V_0 \cos(2\pi f_0 t), \quad s(t)$$

$$y(t) = V_0 s(t) \cos(2\pi f_0 t)$$



$$\begin{aligned} z(t) &= V_0 s(t) \cos(2\pi f_0 t) \cos(2\pi f_0 t) \\ &= \frac{V_0}{2} s(t) + \frac{V_0}{2} \cos(4\pi f_0 t) \end{aligned}$$



Se per ricostruire usò: $\cos(2\pi f_0 t + \theta)$

$$z(t) = V_0 s(t) \cos(2\pi f_0 t) \cos(2\pi f_0 t + \theta)$$

$$= \frac{V_0}{2} s(t) \cos \theta + \frac{V_0}{2} s(t) \cos(4\pi f_0 t + \theta)$$

Si annulla per le frequenze! $2f_0$

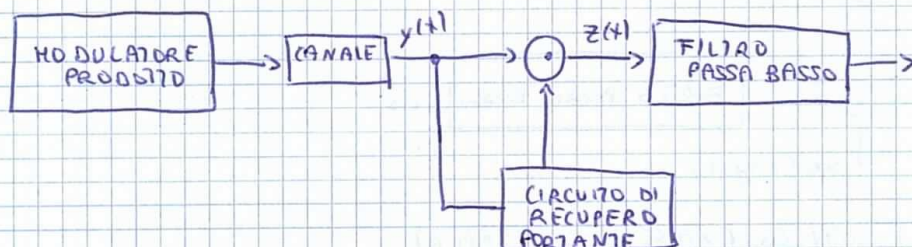
Se per ricostruire usò: $\cos[2\pi(f_0 + \Delta f)t + \theta]$

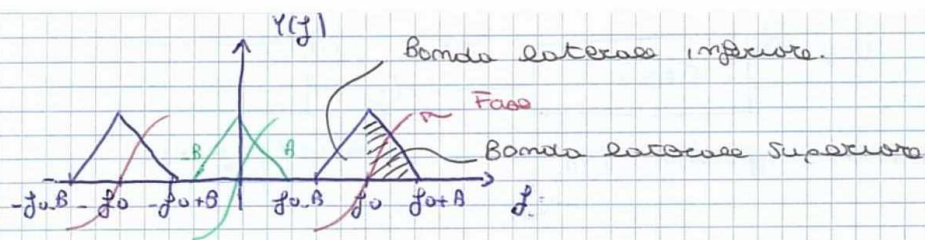
$$z(t) = V_0 s(t) \cos(2\pi f_0 t) \cos(2\pi(f_0 + \Delta f)t + \theta)$$

$$= \frac{V_0}{2} s(t) \cos[2\pi \Delta f t + \theta] + \frac{V_0}{2} s(t) \cos[4\pi f_0 t + 2\pi \Delta f t + \theta]$$

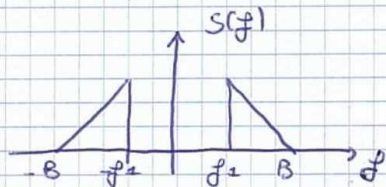
Si annulla per le frequenze $2f_0$

Quindi $z(t) = \frac{V_0}{2} s(t) \cos[2\pi \Delta f t + \theta]$

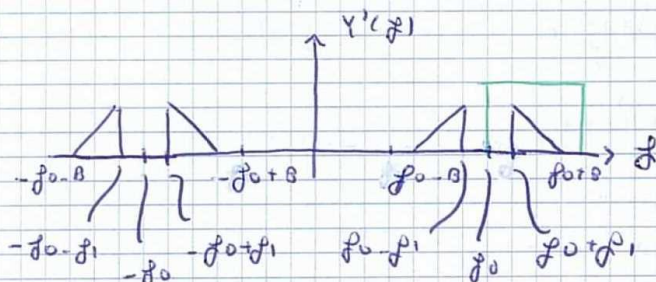




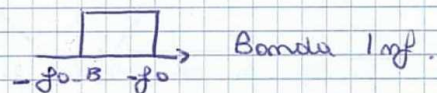
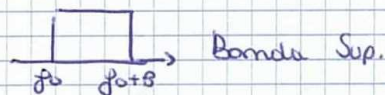
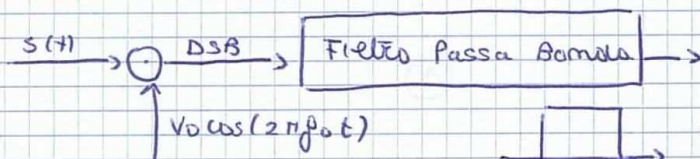
SSB (Single Side Band)



$$y'(t) = V_0 s(t) \cos(2\pi f_0 t) \rightarrow \text{DSB}$$

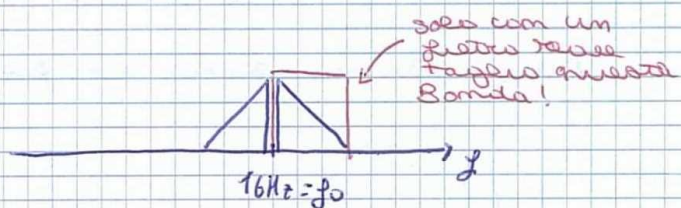
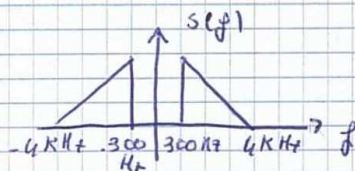


SSB

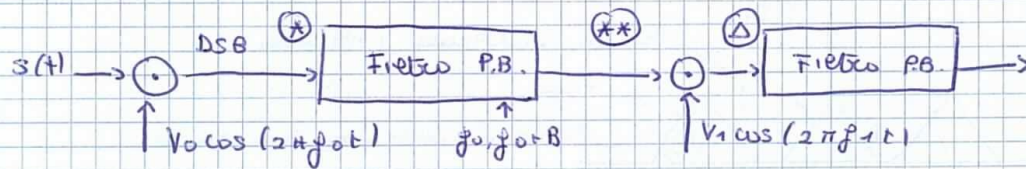


$$f_0 = 16 \text{ KHz}$$

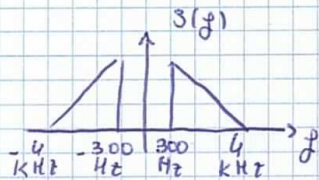
$$B = 4 \text{ KHz}$$



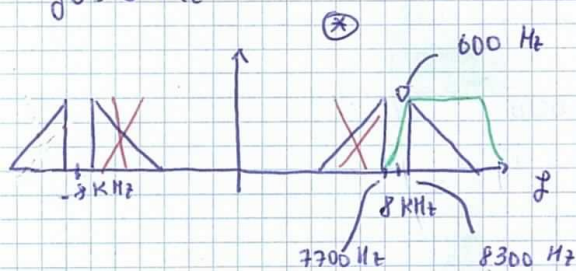
600 MHz di Spazio ad 1 GHz non sono nulla



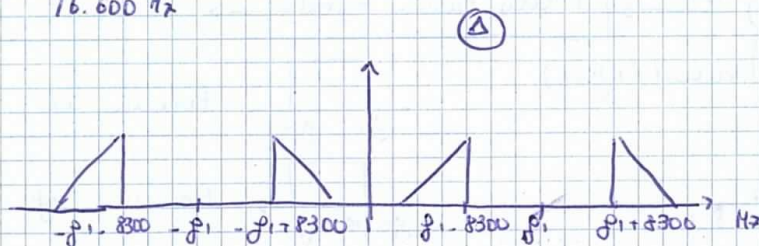
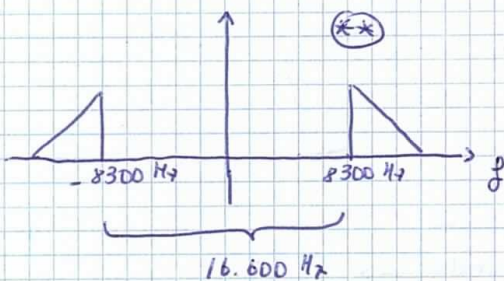
$$f_1 = 16 \text{ KHz}$$



$$f_0 = 8 \text{ KHz}$$



600 Hz di spazio a 8 KHz
è abbastanza per realizzare
un filtro.



Desidero più spazio per filtrare!

27/04/2015

AM

$$y(t) = V_0 [1 + K \cdot s(t)] \cdot \cos(2\pi f_0 t) \quad \rightarrow \text{Demodulatore di Inviluppo (incoerente)}$$

$$m = K \cdot \max |s(t)| \leq 1$$

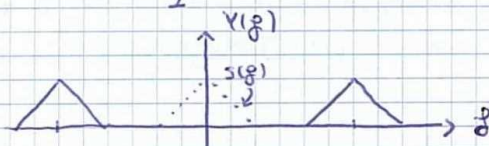
$$P_y = \frac{V_0^2}{2} [1 + K^2 \overline{s^2(t)}], \quad \overline{s(t)} = 0.$$

AM - DSB

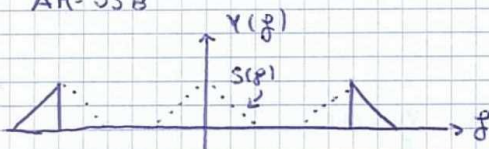
$$y(t) = V_0 s(t) \cos(2\pi f_0 t)$$

$\rightarrow$ Demodulatore Coerente

$$P_y = \frac{V_0^2}{2} \overline{s^2(t)}$$



AM-SSB



$$y(t) = V_0 s(t) \cdot \cos(2\pi f_0 t) \pm V_0 \hat{s}(t) \sin(2\pi f_0 t)$$

$$\hat{s}(t) \text{ trasg. di Hilbert di } s(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{s(\tau)}{t-\tau} d\tau = \frac{1}{\pi t} * s(t).$$

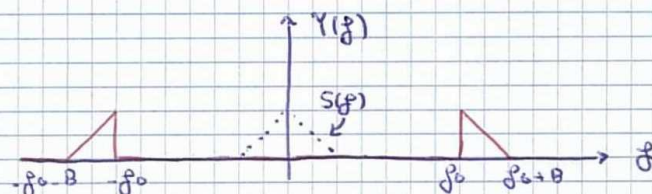
$$\hat{S}(f) = -j \operatorname{segm}(f) \cdot S(f).$$

$$Y(f) = \frac{V_0}{2} [S(f+f_0) + S(f-f_0)] - \frac{V_0}{2j} [\hat{S}(f-f_0) - \hat{S}(f+f_0)]$$

$$= \frac{V_0}{2} [S(f+f_0) + S(f-f_0)] - \frac{V_0}{2j} [-j \operatorname{segm}(f-f_0) S(f-f_0) + j \operatorname{segm}(f+f_0) S(f+f_0)]$$

$$= \frac{V_0}{2} \left\{ S(f+f_0) \left[1 - \frac{j}{j} \operatorname{segm}(f+f_0) \right] + S(f-f_0) \left[1 + \frac{j}{j} \operatorname{segm}(f-f_0) \right] \right\}$$

$$= \frac{V_0}{2} \left\{ S(f+f_0) [1 - \operatorname{segm}(f+f_0)] + S(f-f_0) [1 + \operatorname{segm}(f-f_0)] \right\}$$



$$\begin{aligned} f < -f_0 &\rightarrow V_0 S(f+f_0), & -f_0 < f < f_0 &\rightarrow 0 \\ 0 < f < f_0 &\rightarrow 0, & f > f_0 &\rightarrow V_0 S(f-f_0) \end{aligned}$$

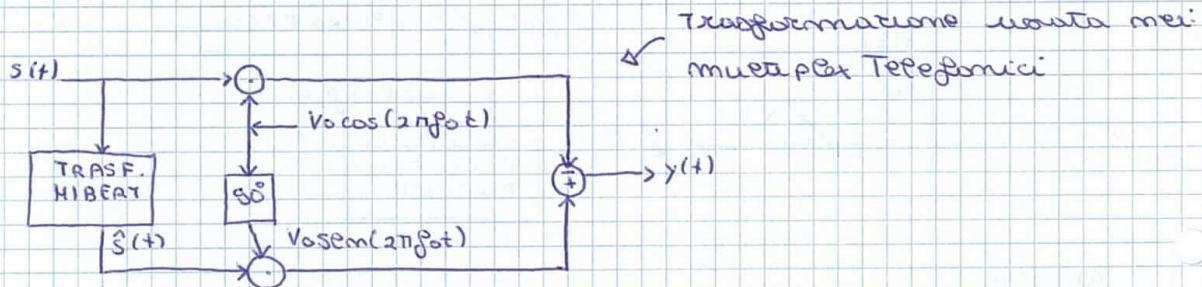
$$y(t) = V_0 s(t) \cos(2\pi f_0 t) \pm V_0 \hat{s}(t) \sin(2\pi f_0 t)$$

$$\hat{S}(f) = -j \operatorname{sgn}(f) \cdot S(f)$$

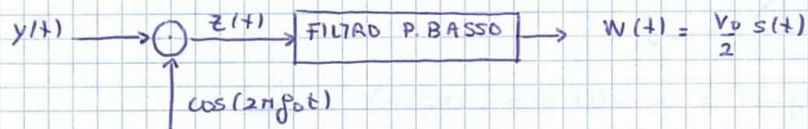
la potenza di $\hat{S}(f)$ è uguale alla potenza di $S(f)$.

$$P_y = \frac{V_0^2}{2} \overline{s^2(t)} = V_0^2 \overline{s^2(t)}$$

modulatore



Demodulatore



$$\begin{aligned} w(t) &= V_0 s(t) \cos(2\pi f_0 t) \cos(2\pi f_0 t) - V_0 \hat{s}(t) \sin(2\pi f_0 t) \cos(2\pi f_0 t) \\ &= \frac{V_0}{2} s(t) + \frac{V_0}{2} s(t) \cos(4\pi f_0 t) - \frac{V_0}{2} \hat{s}(t) \sin(\phi) - \frac{V_0}{2} \hat{s}(t) \sin(4\pi f_0 t) \end{aligned}$$

$\swarrow$ eliminato dal filtro $\searrow$ ϕ $\swarrow$ eliminato dal filtro

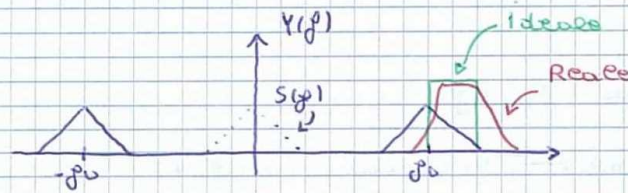
$$= \frac{V_0}{2} s(t)$$

In generale: $a(t) \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t)$

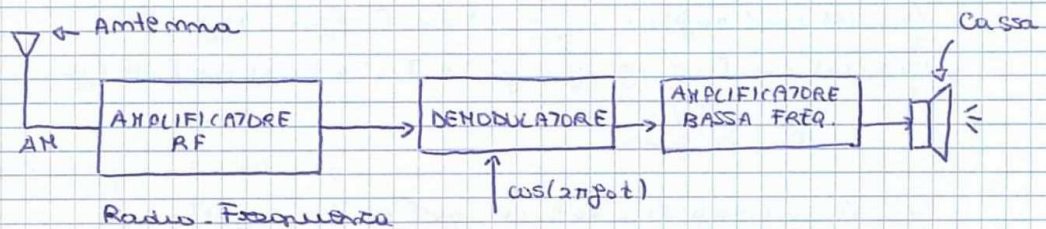
se moltiplico per $\cos(\cdot)$ $\rightarrow$ recupero $a(t)$, componente in Fase

se moltiplico per $\sin(\cdot)$ $\rightarrow$ recupero $b(t)$, componente in Quadratura.

VSB (Vestigial side Band)



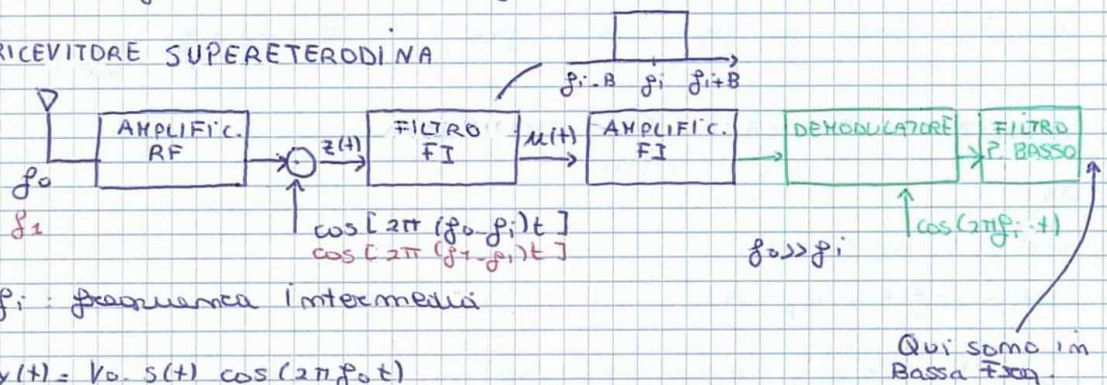
Ricevitore Generico



Per prima cosa amplifico le segnali in quanto lo ricevo molto debole

e l'amplificatore RF amplifica tutta la Banda

RICEVITORE SUPERETERODINA

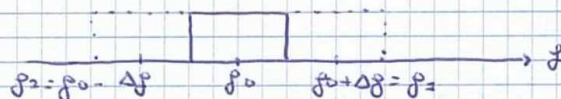


f_i : frequenza intermedia

$$y(t) = V_0 s(t) \cos(2\pi f_0 t)$$

$$z(t) = V_0 s(t) \cos(2\pi f_0 t) \cos[2\pi(f_0 - f_i)t] = V_0 (*)$$

Generalmente nella Radio AM:



$$(*) = \frac{V_0}{2} s(t) \cos(2\pi f_i t) + \frac{V_0}{2} s(t) \cos[4\pi f_0 t - 2\pi f_i t]$$

$$u(t) = \frac{V_0}{2} s(t) \cos(2\pi f_i t)$$

$$u(t) = \frac{V_0}{2} s(t) \cos(2\pi f_i t)$$

→ Entrambe centrate su f_i

$$f_0$$

$$f_1 = f_0 + \Delta f = f_0 + 2B$$

Trasmetto insieme f_0 e f_1 .

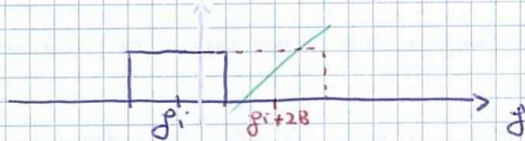
$$y(t) = V_0 s_0(t) \cos(2\pi f_0 t) + V_0 s_1(t) \cos(2\pi f_1 t)$$

$$\begin{aligned} u(t) &= V_0 s_0(t) \cos(2\pi f_0 t) \cos[2\pi(f_0 + f_i)t] + \\ &\quad + V_0 s_1(t) \cos(2\pi f_1 t) \cos[2\pi(f_0 + f_i)t] \\ &= \frac{V_0}{2} s_0(t) \cos(2\pi f_i t) + \frac{V_0}{2} s_0(t) \cos[4\pi f_0 t - 2\pi f_i t] + \\ &\quad + \frac{V_0}{2} s_1(t) \cos[2\pi(f_1 + f_0 - f_i)t] + \frac{V_0}{2} s_1(t) \cos[2\pi(f_1 - f_0 + f_i)t] \end{aligned}$$

Per le frequenze

$$= \frac{V_0}{2} s(t) \cos(2\pi f_i t) + \frac{V_0}{2} s_1(t) \cos[2\pi(f_1 - \Delta f)t]$$

Non la sento per via del filtro.



Assento solo le frequenze fra $f_i - B \div f_i + B$

MODULAZIONI ANGOLARI (FM, PM)

PM

$$y(t) = V_0 \cos(\theta(t))$$

$$\theta(t) = 2\pi f_0 t + K_p s(t)$$

$$\rightarrow y(t) = V_0 \cos(2\pi f_0 t + K_p s(t))$$

FM

$$f = f_0 + K_f s(t)$$

$$s(t) = V_m \cos(2\pi f_m t)$$

$$f = f_0 + K_f V_m \cos(2\pi f_m t)$$

$$\rightarrow f_0 - K_f V_m \leq f \leq f_0 + K_f V_m$$

$K_f V_m = \Delta f_{\max}$: massima Deviazione di Frequenza.

$$K_f \cdot \max |s(t)| = \Delta f_{\max}$$

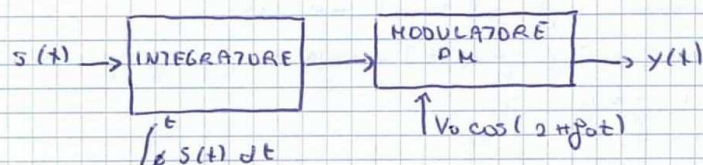
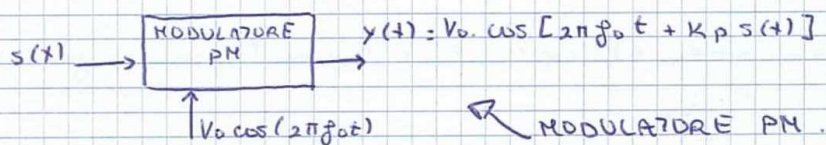
$$f = f_0 + K_f s(t)$$

$$\omega = 2\pi f = 2\pi f_0 + 2\pi K_f s(t)$$

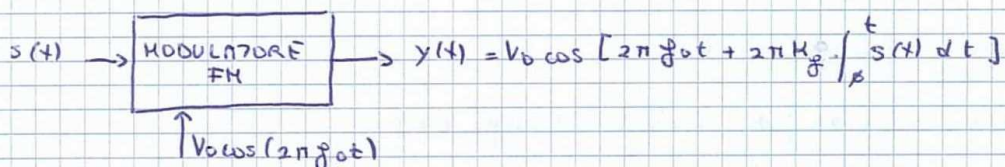
$$\theta(t) = \int_{-\infty}^t \omega dt = 2\pi f_0 t + 2\pi K_f \int_{-\infty}^t s(t) dt$$

$$y(t) = V_0 \cos \left[2\pi f_0 t + 2\pi K_f \int_{-\infty}^t s(t) dt \right] \quad \text{FM}$$

$$y(t) = V_0 \cos [2\pi f_0 t + K_p s(t)] \quad \text{PM}$$



MODULATORE FM



MODULATORE PM

FM

$$s(t) = V_m \cos(2\pi f_m t)$$

$$y(t) = V_0 \cos \left[2\pi f_0 t + 2\pi K_f \int_{-\infty}^t s(t) dt \right]$$

$$= V_0 \cos \left[2\pi f_0 t + 2\pi K_f V_m \cdot \frac{1}{2\pi f_m} \cdot \sin(2\pi f_m t) \right]$$

$$= V_0 \cos \left[2\pi f_0 t + \frac{K_f V_m}{f_m} \cdot \sin(2\pi f_m t) \right]$$

$$K_f V_m = \Delta f_{\max} ; \quad \frac{K_f V_m}{f_m} = m_f : \text{indice di modulazione in frequenza.}$$

mezza Banda FM:

$$\Delta f_{\max} = 75 \text{ KHz}$$

$$B = f_m = 15 \text{ KHz}$$

$$m_f = 5$$

Im AK:

$$B = 4,5 \text{ KHz}$$

QUI
↓

$$m_f = m$$

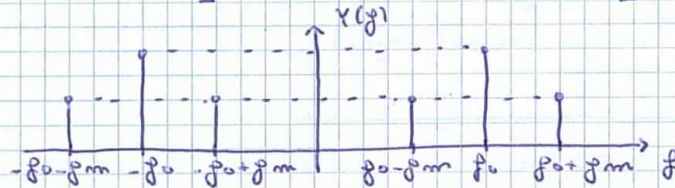
$$y(t) = V_0 \cos [2\pi f_0 t + m \cdot \sin(2\pi f_m t)]$$

$$= V_0 \cdot \underbrace{\cos[m \sin(2\pi f_m t)]}_{\approx 1} \cos(2\pi f_0 t) - V_0 \underbrace{\sin[m \sin(2\pi f_m t)]}_{\approx 0} \sin(2\pi f_0 t)$$

$$m \ll 1$$

$$\approx V_0 \cos(2\pi f_0 t) - V_0 m \sin(2\pi f_m t) \sin(2\pi f_0 t)$$

$$= V_0 \cos(2\pi f_0 t) - \frac{V_0 m}{2} \cos[2\pi(f_0 - f_m)t] + \frac{V_0 m}{2} \cos[2\pi(f_0 + f_m)t]$$



$m \gg 1$. (NO ORACLE)

$$y(t) = V_0 \cos [2\pi f_0 t + m \sin(2\pi f_m t)]$$

$$= \operatorname{Re} \left\{ V_0 e^{j m \sin(2\pi f_m t)} e^{j 2\pi f_0 t} \right\}$$

$$\tilde{y}(t) = V_0 e^{j m \sin(2\pi f_m t)} \quad \text{periodico con } T = \frac{1}{f_m}$$

$$\tilde{y}(t) = \sum_{n=-\infty}^{+\infty} c_n e^{j 2\pi n f_m t}$$

$$c_n = f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} \tilde{y}(t) e^{-j 2\pi n f_m t} dt$$

$$= f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} V_0 e^{j m \sin(2\pi f_m t)} e^{-j 2\pi n f_m t} dt$$

$$= f_m \int_{-\frac{1}{2f_m}}^{\frac{1}{2f_m}} V_0 e^{j m \sin(2\pi f_m t) - j 2\pi n f_m t} dt \quad \cdot \frac{2\pi}{2\pi}$$

$$J_m(m) = \frac{1}{2\pi} \int_{-\pi}^{\pi} e^{j m \sin x - m x} dx \quad \text{Funzione di Bessel di Ordine } m.$$

$$\lambda = 2\pi f_m t, \quad d\lambda = 2\pi f_m dt$$

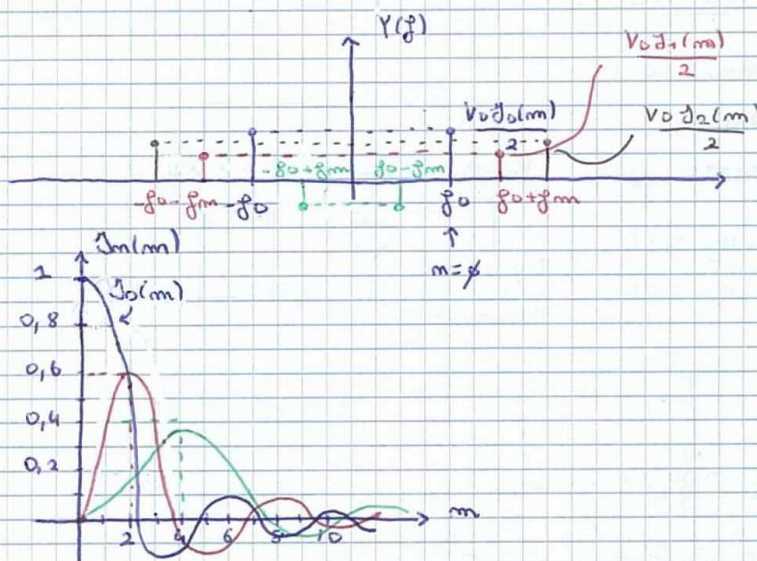
$$c_m = \frac{V_0}{2\pi} \int_{-\pi}^{\pi} e^{j[m \sin x - m\lambda]} dx = V_0 J_m(m)$$

$$\tilde{y}(t) = \sum_{m=-\infty}^{+\infty} c_m e^{j2\pi m f_m t} = V_0 \sum_{m=-\infty}^{+\infty} J_m(m) e^{j2\pi m f_m t}$$

$$y(t) = \operatorname{Re} \left\{ V_0 \sum_{m=-\infty}^{+\infty} J_m(m) e^{j2\pi m f_m t} \cdot e^{j2\pi f_0 t} \right\}$$

$$= V_0 \sum_{m=-\infty}^{+\infty} J_m(m) \cos [2\pi (f_0 + m f_m) t]$$

$$\begin{cases} J_{-m}(m) = (-1)^m J_m(m) \\ \sum_{m=-\infty}^{+\infty} J_m^2(m) = 1 \end{cases}$$



$$\text{Carson: } B_T \approx 2(m+1)B$$

$$\text{FM: } B_T = 2(6) \cdot 15 \text{ KHz} = 180 \text{ KHz}$$

28/04/2015

FM

$$s(t), v(t) = V_0 \cos(2\pi f_0 t)$$

$$y(t) = V_0 \cos \left[2\pi f_0 t + 2\pi k_f \int_{-\infty}^t s(t) dt \right]$$

$$s(t) = V_m \cos(2\pi f_m t)$$

$$y(t) = \sum_{m=-\infty}^{+\infty} V_0 J_m(m) \cos[2\pi(f_0 + m f_m)t]$$

$$B_t = 2(m+1)B \quad \text{Formula di Carson}$$

$$\begin{cases} J_{-m}(m) = (-1)^m J_m(m) \\ \sum_{m=-\infty}^{+\infty} J_m^2(m) = 1 \end{cases}$$

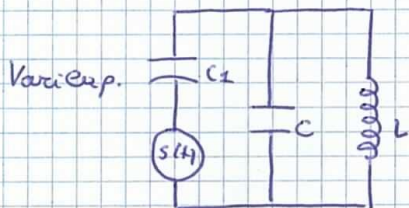
$$P_y = \sum_{m=-\infty}^{+\infty} \frac{V_0^2 J_m^2(m)}{2} = \frac{V_0^2}{2}$$

Metodo di Modulazione:

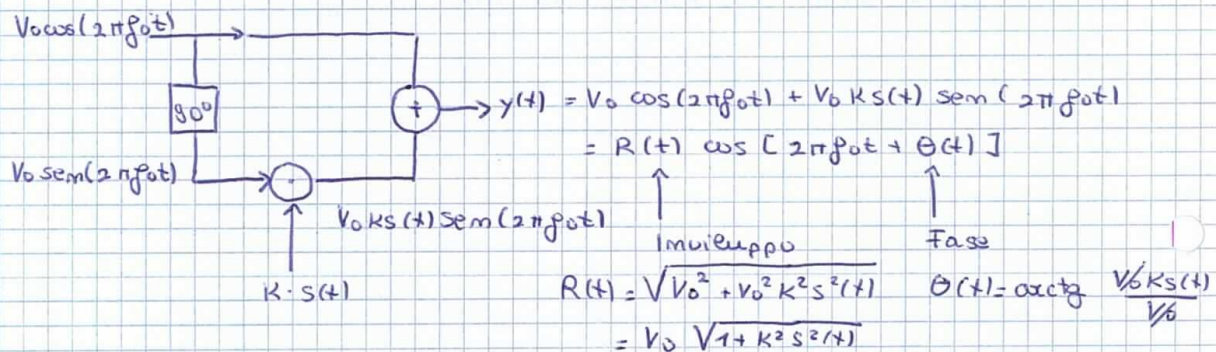
1° Metodo Diretto;

2° Metodo Indiretto;

$$\begin{aligned} 1) \quad s(t) &\rightarrow f = f_0 + k_f \cdot s(t) \\ f - f_0 &= k_f \cdot s(t) \end{aligned}$$



2) Modulatore di Armstrong



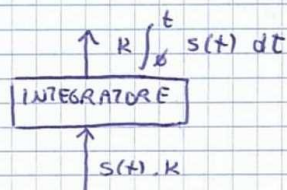
Im un segnale di questo tipo che sia una modulazione di ampiezza che una modulazione di Fase.

$$K \max |s(t)| \ll 1$$

$$\rightarrow R(t) \approx V_0, \quad \text{tg } x \approx x \rightarrow \theta(t) = K s(t)$$

Questo è un modulatore di Fase.

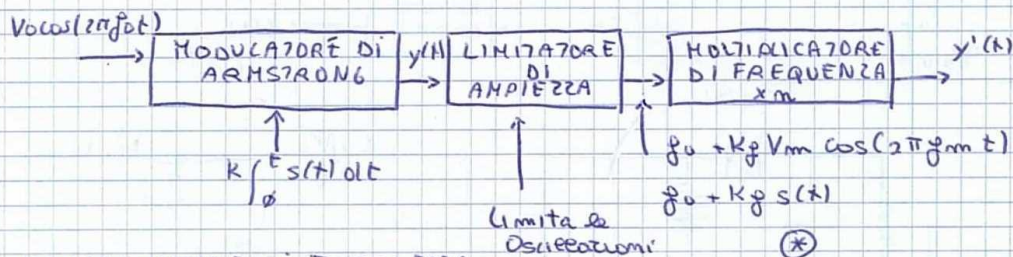
Se al posto di $R \cdot s(t)$ io inserisco questo blocco:



Ha un modulatore di Frequenza.

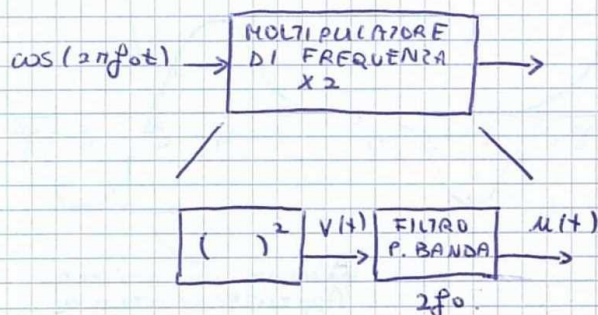
$$K \cdot \max |s(t)| \ll 1$$

$$s(t) = V_m \cdot \cos(2\pi f_m t) \rightarrow K V_m \ll 1 \rightarrow m = \frac{K V_m}{f_m}$$



MOLTIPLICATORE DI FREQUENZA

$$f_0 \rightarrow \boxed{} \rightarrow m f_0$$



$$\cos(2\pi f_0 t)$$

$$V(t) = \cos^2(2\pi f_0 t) = \frac{1 + \cos(4\pi f_0 t)}{2}$$

$$u(t) = \frac{1}{2} \cos(4\pi f_0 t)$$

$$f = f_0 + K_f V_m \cos(2\pi f_m t) \quad (*)$$

$$m f = m f_0 + m K_f V_m \cos(2\pi f_m t)$$

$$y'(t) = V_0 \cos \left[2\pi m f_0 t + \frac{2\pi m K_f V_m}{2\pi f_m} \sin(2\pi f_m t) \right]$$

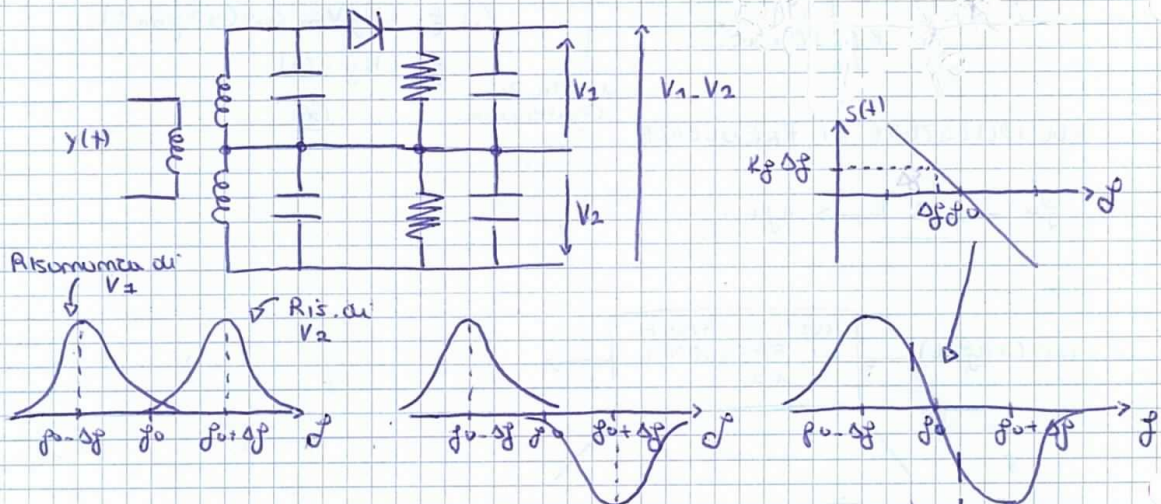
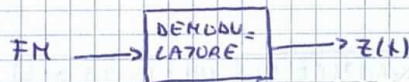
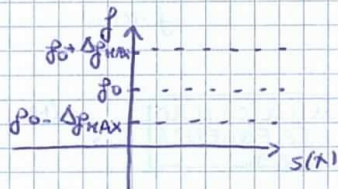
$$f_0' = m f_0$$

$$m' = \frac{m K_f V_m}{f_m} = m \cdot m$$

$$\text{con } s(t) = V_m \cos(2\pi f_m t)$$

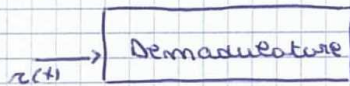
$$y(t) = V_0 \cos [2\pi f_0 t + m \sin(2\pi f_m t)] \quad , m = \frac{K_f V_m}{f_m}$$

DEMODULAZIONE (Varia l'ampiezza di $s(t)$ a seconda di Δf)

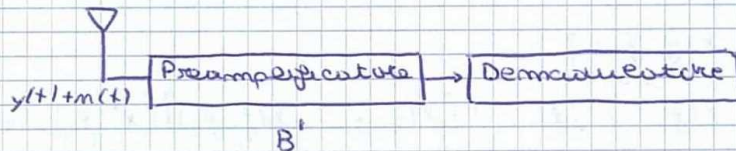


Lineare in prima Approssimazione

Teorema del Rumore in Banda Stretta.

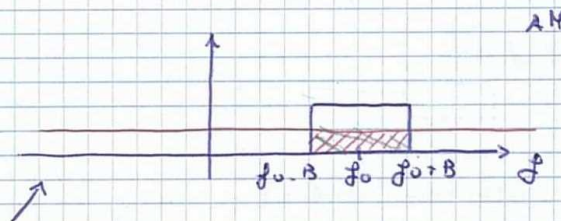


$$x(t) = y(t) + n(t)$$



$s(t)$ ha max frequenza B

$$f_0 \gg B$$



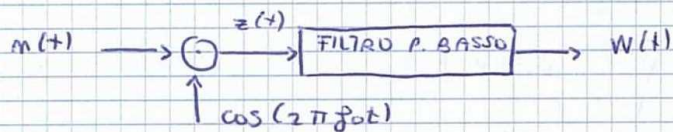
AM

$$FH: B_t = 2(m+1)B$$

Rumore a Banda Stretta ($f_0 \gg B$).

$$n(t) = R(t) \cdot \cos(2\pi f_0 t + \theta(t)) = a(t) \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t)$$

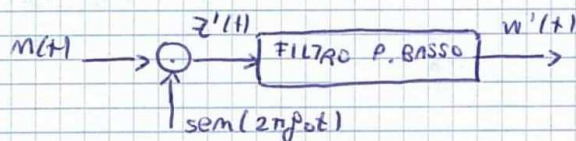
Rumore Additivo, Bianco, Gaussiano AWGN



$$\begin{aligned} z(t) &= a(t) \cos(2\pi f_0 t) \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t) \cos(2\pi f_0 t) \\ &= \frac{a(t)}{2} + \frac{a(t)}{2} \cos(4\pi f_0 t) - \frac{b(t)}{2} \sin(4\pi f_0 t) \end{aligned}$$

$$w(t) = \frac{a(t)}{2} = \frac{n(t) \cos(2\pi f_0 t)}{2}$$

$a(t)$ e $b(t)$ sono in
Bassa Frequenza.



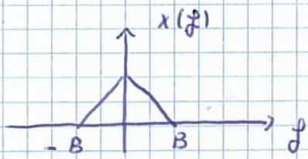
$$\begin{aligned} z'(t) &= a(t) \cos(2\pi f_0 t) \sin(2\pi f_0 t) - b(t) \sin(2\pi f_0 t) \sin(2\pi f_0 t) \\ &= \frac{a(t)}{2} \sin(4\pi f_0 t) - \frac{b(t)}{2} + \frac{b(t)}{2} \cos(4\pi f_0 t) \end{aligned}$$

$$w'(t) = -\frac{b(t)}{2}$$

Esercizio

$$x(t) \rightarrow \psi_x(f)$$

$$y(t) = x(t) \cos(2\pi f_0 t)$$



$$\Psi(f) = \frac{1}{2} [X(f+f_0) + X(f-f_0)]$$

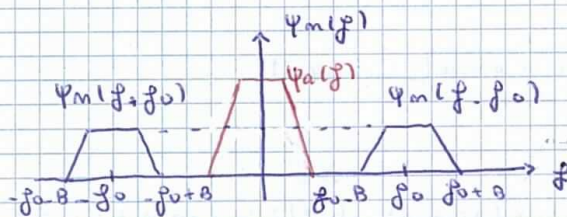
$$\begin{aligned} \Psi_y(f) &= |\Psi(f)|^2 = \Psi(f) \cdot \Psi^*(f) = \frac{1}{4} [X(f+f_0) + X(f-f_0)] [X^*(f+f_0) + X^*(f-f_0)] \\ &= \frac{1}{4} \left\{ \underbrace{X(f+f_0) X^*(f+f_0)}_{|X(f+f_0)|^2} + \cancel{X(f+f_0) X^*(f-f_0)} + \cancel{X(f-f_0) X^*(f+f_0)} + \underbrace{X(f-f_0) X^*(f-f_0)}_{|X(f-f_0)|^2} \right\} \\ &= \frac{1}{4} \left\{ \psi_x(f+f_0) + \psi_x(f-f_0) \right\} \end{aligned}$$

$$m(t) = a(t) \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t)$$

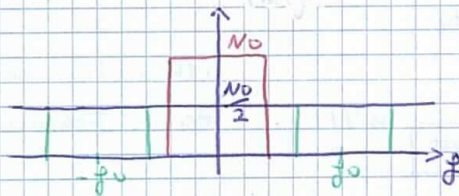
$$\frac{1}{2} a(t) = m(t) \cos(2\pi f_0 t) \rightarrow a(t) = 2 m(t) \cos(2\pi f_0 t)$$

$$-\frac{1}{2} b(t) = m(t) \sin(2\pi f_0 t)$$

$$\Psi_a(f) = \frac{1}{4} \left\{ \Psi_m(f+f_0) + \Psi_m(f-f_0) \right\} = \Psi_b(f)$$



Se faccio riferimento al Rumore:

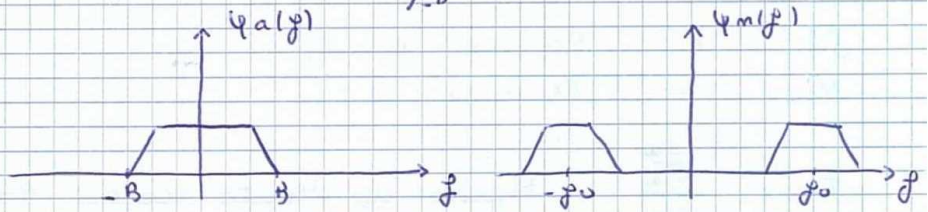


$$\Psi_a(f) = \begin{cases} N_0 & -B \leq f \leq B \\ \varnothing & \text{altrove} \end{cases}$$

$$a(t) = 2m(t) \cos(2\pi f_0 t)$$

$$E[a(t)] = E[2m(t) \cos(2\pi f_0 t)] = E[m(t)] 2 \cos(2\pi f_0 t) = 0$$

$$\sigma_a^2 = E[a^2] - \cancel{0^2} = E[a^2] = \int_{-B}^B \psi_a(f) df =$$



$$= \int_{-B}^B \psi_m(f+f_0) + \psi_m(f-f_0) df = \int_{-B}^B \psi_m(f+f_0) df + \int_{-B}^B \psi_m(f-f_0) df$$

$$\rightarrow \lambda = f+f_0, \quad \lambda = f-f_0$$

$$= \int_{f_0-B}^{f_0+B} \psi_m(\lambda) d\lambda + \int_{-B-f_0}^{+B-f_0} \psi_m(\lambda) d\lambda$$

$$\sigma_a^2 = \int_{f_0-B}^{f_0+B} \psi_m(\lambda) d\lambda + \int_{-f_0-B}^{-f_0+B} \psi_m(\lambda) d\lambda$$

$$\sigma_m^2 = E[m^2] = \int_{-\infty}^{+\infty} \psi_m(f) df = \int_{-f_0-B}^{-f_0+B} \psi_m(f) df + \int_{f_0-B}^{f_0+B} \psi_m(f) df$$

a, b Gaussian a media 0 e Varianza σ_m^2 .

$$R_{a,b}(\tau) = E[a(t)b(t+\tau)] = E[2m(t) \cos(2\pi f_0 t) \cdot$$

$$2m(t+\tau) \sin(2\pi f_0 (t+\tau))]$$

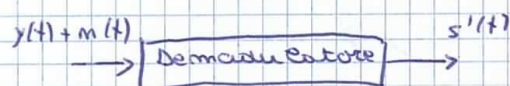
$$= -\frac{4}{2} E[m(t)m(t+\tau) (\sin(2\pi f_0 \tau) + \sin(4\pi f_0 t + 2\pi f_0 \tau))]$$

$$= -2 E[m(t)m(t+\tau) \sin(2\pi f_0 \tau)] - 2 E[m(t)m(t+\tau) \sin(4\pi f_0 t + 2\pi f_0 \tau)]$$

$$= -2 \sin(2\pi f_0 \tau) E[m(t)m(t+\tau)] - 2 E[m(t)m(t+\tau)] \cdot E[\sin(4\pi f_0 t + 2\pi f_0 \tau)]$$

$$R_{a,b}(\tau) = -2 \sin(2\pi f_0 \tau) R_m(\tau)$$

$$R_{a,b}(0) = 0$$



S_i : Potenza segnale Ingresso

N_i : // Rumore //

S_u : // Segnale Usata

N_u : // Rumore //

$$\frac{S_i}{N_i}, \frac{S_u}{N_u}$$

$$F_m = \frac{\frac{S_u}{N_u}}{\frac{S_i}{N_i}} : \text{Figura di Merito o Loss di Modulazione}$$

- 1) S_i, S_u si calcolano in assenza di Rumore.
- 2) N_u si calcola in assenza di segnale modulante ($s(t) = \phi$)
- 3) N_i si calcola nella Banda del segnale modulante

AN)

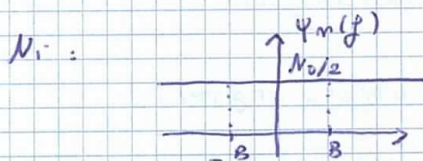
$$y(t) = V_0 [1 + k s(t)] \cos(2\pi f_0 t)$$

$$m(t) = a(t) \cos(2\pi f_0 t) + b(t) \sin(2\pi f_0 t)$$

$$x(t) = y(t) + m(t)$$

$$S_i : x(t) = y(t), \quad \overline{s(t)} = \phi$$

$$P_y = S_i = \frac{V_0^2}{2} [1 + k^2 \overline{s^2(t)}]$$

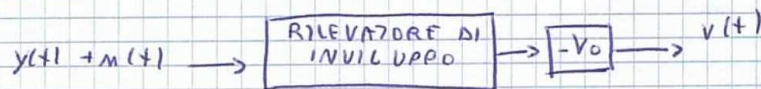


$$N_i : \int_{-B}^B \Psi_m(f) df = \frac{N_0}{2} \cdot 2B = N_0 B$$

$$\rightarrow \frac{S_i}{N_i} = \frac{V_0^2 [1 + k^2 \overline{s^2(t)}]}{2 N_0 B}$$

$$S_M: \quad x(t) = y(t) + n(t) \\ = \left\{ V_0 [1 + Ks(t)a(t)] \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t) \right\} \\ = R(t) \cos[2\pi f_0 t + \theta(t)]$$

$$R(t) = \sqrt{\left\{ V_0 [1 + Ks(t)] + a(t) \right\}^2 + b^2(t)}$$



$$R(t) = \sqrt{\left\{ V_0 [1 + Ks(t)] \right\}^2} = V_0 [1 + Ks(t)] \quad \text{in quanto } m(t) = \phi.$$

$$S_M = V_0^2 K^2 \overline{s^2(t)}$$

↓
Poi toglia V_0

$$v(t) = V_0 K s(t).$$

$$N_M; \quad s(t) = \phi$$

$$R(t) = \sqrt{\left\{ V_0 + a(t) \right\}^2 + b^2(t)} = V_0 \sqrt{\left(1 + \frac{a(t)}{V_0} \right)^2 + \left(\frac{b(t)}{V_0} \right)^2}$$

$$a(t), b(t) \ll V_0.$$

$$R(t) \approx V_0 \left(1 + \frac{a(t)}{V_0} \right) = V_0 + a(t)$$

↓
Poi toglia V_0

$$v(t) = a(t)$$

↑ non compare $b(t)$ perché $s(t)$ non ha componenti in quadratura.

$$N_M = \int_{-B}^B N_0 df = 2N_0 B$$

$$\frac{S_M}{N_M} = \frac{V_0^2 K^2 \overline{s^2(t)}}{2N_0 B}$$

$$\frac{V_0^2 K^2 \overline{s^2(t)}}{2N_0 B}$$

$$F_M = \frac{\frac{V_0^2 K^2 \overline{s^2(t)}}{2N_0 B}}{\frac{V_0^2 [1 + K^2 \overline{s^2(t)}]}{2N_0 B}} = \frac{K^2 \overline{s^2(t)}}{1 + K^2 \overline{s^2(t)}} < 1.$$

Rapporto $\frac{S_M}{N_M}$ peggiore di $\frac{S_i}{N_i}$

$$S(t) = V_m \cos(2\pi f_m t)$$

$$\overline{S^2(t)} = \frac{V_m^2}{2}$$

$$F_m = \frac{\frac{K^2 V_m^2}{2}}{1 + \frac{K^2 V_m^2}{2}} = \frac{K^2 V_m^2}{2 + K^2 V_m^2}, \quad m = K V_m$$

$$= \frac{m^2}{2 + m^2}, \quad m \leq 1$$

nel caso peggiore: $m = 1 \rightarrow F_m = 1/3$

30/04/2015

AK)

1. Si, Su un'assenza di Rumore ($n(t) = \emptyset$)
2. Nu, si considera $S(t) = \emptyset$
3. Ni, si calcola l'area sotto del segnale modulante $(-B, B)$

$$F_m = \frac{\frac{S_u}{N_u}}{\frac{S_i}{N_i}} = \frac{K^2 \overline{S^2(t)}}{1 + K^2 \overline{S^2(t)}} < 1 \quad \text{AK peggiora rapporto segnale/Rumore}$$

↓
dovuto alla Portante

DSB)

$$y(t) = V_0 s(t) \cos(2\pi f_0 t)$$

$$S_i = \frac{V_0^2}{2} \cdot \overline{S^2(t)}$$

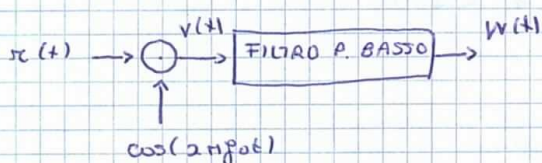
$$\frac{S_i}{N_i} = \frac{V_0^2 \overline{S^2(t)}}{2 N_0 B}$$

$$N_i = \int_{-B}^B \psi_m(f) df = N_0 B$$

$$\text{con } m(t) = a(t) \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t)$$

$$x(t) = [V_0 s(t) + a(t)] \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t)$$

$$y(t) = m(t)$$



$$Y(t) = [V_0 s(t) + a(t)] \cos^2(2\pi f_0 t) - b(t) \sin(2\pi f_0 t) \cos(2\pi f_0 t)$$

$$= [V_0 s(t) + a(t)] \cdot \frac{1}{2} [V_0 s(t) + a(t)] \cos(4\pi f_0 t) - b(t) \cdot \frac{1}{2} \sin(\emptyset) - b(t) \cdot \frac{1}{2} \sin(4\pi f_0 t)$$

$$W(t) = \frac{V_0 s(t) + a(t)}{2}$$

$$S_u: a(t) = \phi \rightarrow W(t) = \frac{V_0 s(t)}{2}$$

$$S_u = \frac{V_0^2}{4} \overline{s^2(t)}$$

$$N_u: s(t) = \phi \rightarrow W(t) = \frac{a(t)}{2}$$

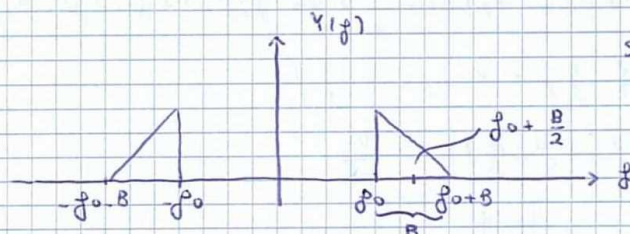
$$N_u = \int_{-B}^B \frac{1}{u} \psi_a(f) df = \frac{1}{4} \cdot 2 N_0 B = \frac{1}{2} N_0 B$$

$$\frac{S_u}{N_u} = \frac{\frac{V_0^2}{4} \overline{s^2(t)}}{\frac{1}{2} N_0 B} = \frac{V_0^2 \overline{s^2(t)}}{2 N_0 B}$$

$$F_m = \frac{S_u}{\frac{N_u}{S_i}} = 1 \quad \text{se rapporto segnale / rumore non peggiora}$$

SSB)

$$y(t) = V_0 s(t) \cos(2\pi f_0 t) - V_0 \hat{s}(t) \sin(2\pi f_0 t)$$



Spettro centrato su:

$$f = f_0 + \frac{B}{2}$$

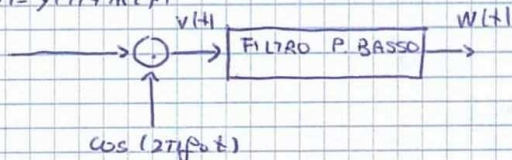
$$m(t) = a(t) \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t)$$

↳ Questo rumore è centrato in \$f = f_0\$: NON VA BENE!

$$m(t) = a(t) \cos\left[2\pi\left(f_0 + \frac{B}{2}\right)t\right] - b(t) \sin\left[2\pi\left(f_0 + \frac{B}{2}\right)t\right] : \text{OK!}$$

$$x(t) = V_0 s(t) \cos(2\pi f_0 t) - V_0 \hat{s}(t) \sin(2\pi f_0 t) + m(t)$$

$$x(t) = y(t) + m(t)$$



$$S_i = \frac{V_0^2}{2} \overline{s^2(t)} \cdot 2 = V_0^2 \overline{s^2(t)}$$

$$N_i = N_0 B$$

$$\rightarrow \frac{S_i}{N_i} = \frac{V_0^2 \overline{s^2(t)}}{N_0 B}$$

$$v(t) = V_0 s(t) \cos^2(2\pi f_0 t) - V_0 \hat{s}(t) \sin(2\pi f_0 t) \cos(2\pi f_0 t) \quad \leftarrow \text{con } m(t) = \phi$$

$$= \frac{V_0 s(t)}{2} + \frac{V_0 s(t)}{2} \cos(4\pi f_0 t) - \frac{V_0 \hat{s}(t)}{2} \sin(2\pi f_0 t) - \frac{V_0 \hat{s}(t)}{2} \sin(4\pi f_0 t)$$

$\swarrow$ FILTRO $\swarrow$ FILTRO

$$S_M = \frac{V_0^2}{4} s^2(t)$$

con $s(t), \hat{s}(t) = \phi$:

$$\pi(t) = m(t)$$

$$v(t) = a(t) \cos\left[2\pi\left(f_0 + \frac{B}{2}\right)t\right] \cos(2\pi f_0 t) - b(t) \sin\left[2\pi\left(f_0 + \frac{B}{2}\right)t\right] \cos(2\pi f_0 t)$$

$$= \frac{a(t)}{2} \cos\left[2\pi \frac{B}{2} t\right] + \frac{a(t)}{2} \cos\left[4\pi f_0 t + \pi B t\right] +$$

$$- \frac{b(t)}{2} \sin\left[2\pi \frac{B}{2} t\right] - \frac{b(t)}{2} \sin\left[4\pi f_0 t + \pi B t\right]$$

$\underbrace{\hspace{10em}}_{m(t)}$ $\swarrow$ FILTRO

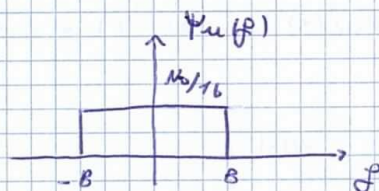
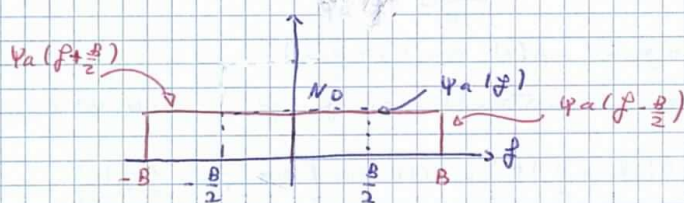
$$w(t) = \frac{a(t)}{2} \cos\left[2\pi \frac{B}{2} t\right] - \frac{b(t)}{2} \sin\left[2\pi \frac{B}{2} t\right]$$

$$x(t) \rightarrow X(f) \quad (-B, B)$$

$$z(t) = x(t) \cos(2\pi f_0 t) \rightarrow \psi_z(f) = \frac{1}{4} \left\{ \psi_x\left(f + f_0\right) + \psi_x\left(f - f_0\right) \right\}$$

$$\psi_u(f) = \frac{1}{4} \cdot \left\{ \frac{1}{4} \left[\psi_a\left(f + \frac{B}{2}\right) + \psi_a\left(f - \frac{B}{2}\right) \right] \right\}$$

$$= \frac{1}{16} \left\{ \psi_a\left(f + \frac{B}{2}\right) + \psi_a\left(f - \frac{B}{2}\right) \right\} = \frac{N_0}{16}$$



$$N_M = 2 \int_{-B}^B \frac{1}{16} N_0 df = 4 \frac{N_0 B}{16} = \frac{N_0 B}{4}$$

$$\frac{S_M}{N_M} = \frac{\frac{V_0^2}{4} s^2(t)}{\frac{N_0 B}{4}} = \frac{V_0^2 s^2(t)}{N_0 B} \rightarrow F_m = 1!$$

meo caso migliore, meee AR : $F_m = 1!$

Inoltre in questi esercizi abbiamo considerato solo la summa $m(t)$ del canale e non quello che introduce il demodulatore.

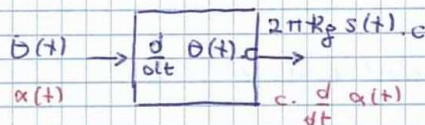
FH)

$$s(t) \rightarrow y(t) = V_0 \cos \left[2\pi f_0 t + \underbrace{2\pi K_f \int_0^t s(t) dt}_{\theta(t)} \right] =$$

$$= V_0 \cos [2\pi f_0 t + \theta(t)].$$

$$m(t) = a(t) \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t)$$

Demodulatore Ideale:



$$\frac{d}{dt} y(t) = \sin \left[2\pi f_0 t + 2\pi K_f \int_0^t s(t) dt \right] \cdot V_0 [2\pi f_0 + 2\pi K_f s(t)]$$

$$= V_0 2\pi f_0 \left[1 + \underbrace{\frac{K_f s(t)}{f_0}}_{\downarrow \text{generalmente } < 1} \right] \sin[\dots]$$

Questo sarebbe una modulazione mista AM, FH
ma lo posso demodulare con un rivelatore di
involuppo.

$$x(t) = y(t) + m(t) = V_0 \cos(\theta(t)) \cos(2\pi f_0 t) - V_0 \sin(\theta(t)) \sin(2\pi f_0 t) + a(t) \cos(2\pi f_0 t) - b(t) \sin(2\pi f_0 t).$$

$$= [V_0 \cos(\theta(t)) + a(t)] \cos(2\pi f_0 t) - [V_0 \sin(\theta(t)) + b(t)] \sin(2\pi f_0 t).$$

$$= \underbrace{R(t)}_{\uparrow \text{Inviluppo}} \cos[2\pi f_0 t + \underbrace{\alpha(t)}_{\uparrow \text{fase}}]$$

$$R(t) = \sqrt{[V_0 \cos(\theta(t)) + a(t)]^2 + [V_0 \sin(\theta(t)) + b(t)]^2}$$

$$\alpha(t) = \arctg \frac{V_0 \sin(\theta(t)) + b(t)}{V_0 \cos(\theta(t)) + a(t)}$$

$$y(t) = V_0 \cos[2\pi f_0 t + \theta(t)] ; \theta(t) = 2\pi k_f \int_{-\infty}^t s(t) dt$$

$$S_i = \frac{V_0^2}{2}$$

$$\frac{S_i}{N_i} = \frac{V_0^2}{2N_0 B}$$

$$N_i = N_0 B$$

$$S_u : m(t) = \varnothing$$

$$R(t) = \sqrt{V_0^2 \cos^2 \theta + V_0^2 \sin^2 \theta} = V_0$$

$$\alpha(t) = \arctg \frac{V_0 \sin \theta}{V_0 \cos \theta} = \theta$$

Ovviamente l'insieme di Fase di $y(t)$!

$$S_u = c^2 4\pi^2 k_f^2 \overline{s^2(t)} \leftarrow \text{Grazie al Derivative}$$

$$N_u : s(t) = \varnothing$$

$$R(t) = \sqrt{[V_0 + a(t)]^2 + b^2(t)}$$

$$\rightarrow \text{cosi } \theta(t) = \varnothing$$

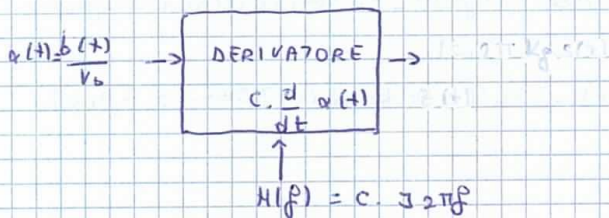
$$\alpha(t) = \arctg \frac{b(t)}{V_0 + a(t)}$$

Supponiamo alti rapporti Segnali - Rumore (20 dB).

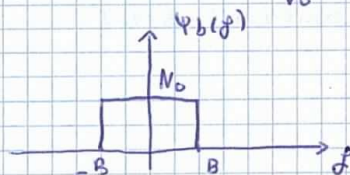
$$a(t), b(t) \ll V_0$$

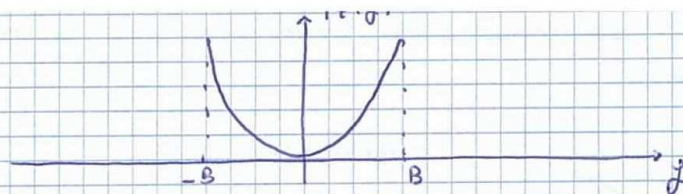
$$R(t) = \sqrt{V_0^2 \left\{ \left[1 + \frac{a(t)}{V_0} \right]^2 + \left[\frac{b(t)}{V_0} \right]^2 \right\}} \approx V_0$$

$$\alpha(t) \approx \arctg \frac{b(t)}{V_0} \approx \frac{b(t)}{V_0}$$



$$\Psi_z(f) = |H(f)|^2 \cdot \Psi_\alpha(f) = \frac{1}{V_0^2} c^2 4\pi^2 f^2 \cdot \Psi_b(f)$$





la frequenza viene a f forma una $\varphi_2(f)$ molto piccola.

Così è dovuto da f^2 in $\varphi_2(f)$, il quale è dovuto da $j2\pi f$ che è dovuto dalla derivata.

La derivata smorza la frequenza elevata!

$$N_u = \int_{-B}^B \varphi_2(f) df = \frac{c^2 4 \pi^2}{V_o^2} N_o \int_{-B}^B f^2 df = \frac{4 \pi^2 c^2 N_o}{V_o^2} \frac{2}{3} B^3$$

$$\frac{f_u}{N_u} = \frac{\cancel{f} 4 \pi^2 K_f^2 \overline{s^2(f)}}{2 B \cancel{f} N_o B^3} \quad 3 V_o^2 = \frac{3}{2} \frac{V_o^2 K_f^2 \overline{s^2(f)}}{N_o B^3}$$

$$F_m = \frac{\frac{3}{2} \frac{V_o^2 K_f^2 \overline{s^2(f)}}{N_o B^3}}{\frac{V_o^2}{2 N_o B}} = \frac{3}{2} \frac{K_f^2 \overline{s^2(f)}}{B^2}$$

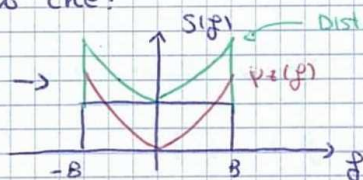
$$S(t) = V_m \cos(2\pi f_m t)$$

$$F_m = \frac{3 K_f^2 V_m^2}{2 B^2} = \frac{3}{2} \left(\frac{K_f V_m}{f_m} \right)^2 = \frac{3}{2} m^2$$

Il rapporto Segnale / Rumore è migliorato.

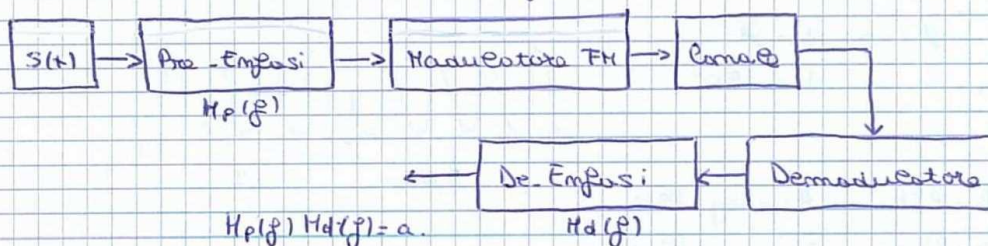
Supponiamo che:

Così il Rapporto $\frac{S}{N}$ è lo stesso a tutte le f .

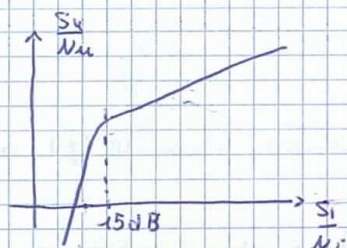


Distorsione $S(f)$ in frequenza

(per il problema dei disturbi sono state fatte f_{max}).



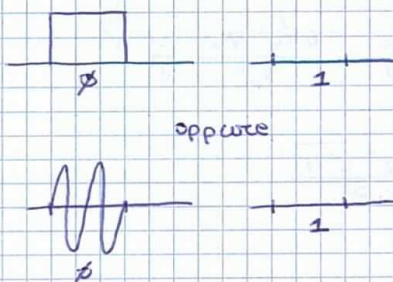
Tutto questo che abbiamo visto vale solo per reti rapporti Segnale/Rumore.



$$a(t), b(x) \ll V_0$$

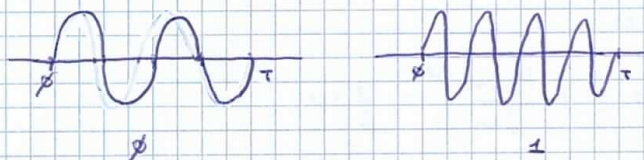
MODULAZIONI DIGITALI

Modulazioni di Ampiezza (OOK : On-Off Keying)



Modulazioni di Frequenza (FSK : Frequency Shift Keying).

Usa frequenze diverse per 0 e 1.

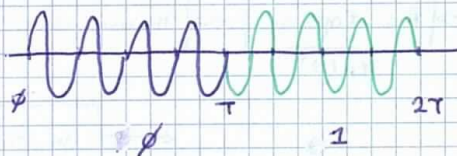


- B-FSK Binary FSK 2 frequenze
- M-FSK Multi FSK M num. Freq. : $M = 2^m$

Modulazioni di Fase (PSK : Phase Shift Keying)

Ho una Portante : $V_0 \cos(2\pi f_0 t + \phi)$

$$\begin{aligned} 0 &\rightarrow \phi = 0^\circ \\ 1 &\rightarrow \phi = 180^\circ \end{aligned}$$



- B-PSK Binary PSK
- Q-PSK Quaternary PSK
- 8-PSK ...

B-PSK : Trasmette un bit

Q-PSK : Trasmette due bit

M : numero di simboli da trasmettere

Bimodico : $M=2$

in generale $M=2^m$.

Simboli: $\{a_1, a_2, \dots, a_M\} = A$: alfabeto di Sorgente

$\{p(a_1), p(a_2), \dots, p(a_M)\}$: Probabilità o Priori della Sorgente.

Simboli equiprobabili: $p(a_i) = \frac{1}{M}$

$$\sum_{i=1}^M p(a_i) = 1.$$

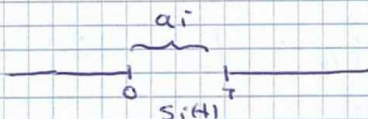
$a_1 \rightarrow s_1(t)$

$a_2 \rightarrow s_2(t)$ da ogni simbolo associare un segnale diverso.

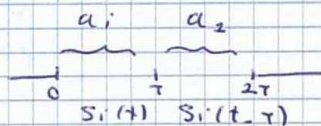
$\vdots$

$a_M \rightarrow s_M(t)$

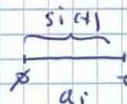
Se voglio trasmettere a_i , trasmetto $s_i(t)$.



$s_i(t) \neq \emptyset$ in $(\emptyset, T)$



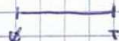
intervallo $\emptyset \div T$



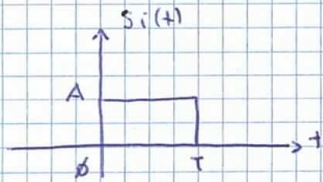
$$x(t) = s_i(t) + n(t)$$



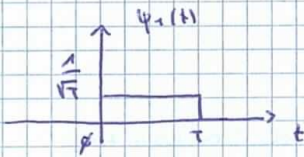
$$E_i \rightarrow s_i(t)$$



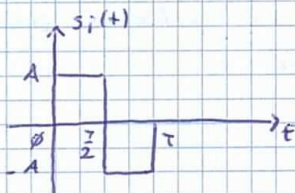
$$E_i = \int_{-\infty}^{+\infty} |s_i(t)|^2 dt = \int_{\emptyset}^T s_i^2(t) dt.$$



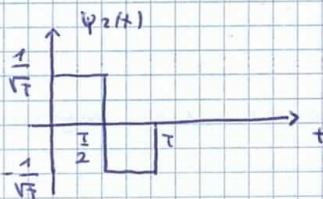
$$E_i = \int_0^T s_i^2(t) dt = \int_0^T A^2 dt = A^2 T.$$



$\psi_1(t)$ ha energia = 1 se $A = \frac{1}{\sqrt{T}}$
 $E_i = 1$.



$$E_i = A^2 T$$



$$E_i = 1.$$

$s_i(t) = A \cos(2\pi f_0 t)$, segnale modulato.

$$E_i = A^2 \int_0^T \cos^2(2\pi f_0 t) dt = \frac{A^2}{2} \left\{ \int_0^T dt + \int_0^T \cos(4\pi f_0 t) dt \right\}$$

$$= \frac{A^2}{2} T + \frac{A^2}{2} \cdot \frac{1}{4\pi f_0} \sin(4\pi f_0 T)$$

$$= A^2 \cdot \frac{T}{2}$$

$\sin(4\pi m) = 0$

$$T = mT'$$

$$T' = \frac{1}{f_0}$$

$$\rightarrow T \cdot f_0 = m$$

$\uparrow$ num. intero

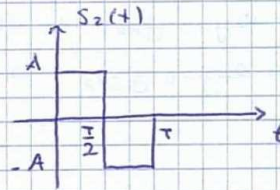
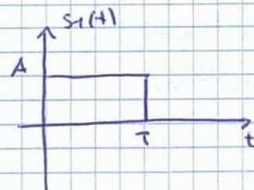
$$s_i(t) = \sqrt{\frac{2E_i}{T}} \cos(2\pi f_0 t).$$

$$\psi_i(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t).$$

$S_1(t), S_2(t) \neq \emptyset$ in $(\emptyset, T)$

Sono ortogonali se:

$$\int_0^T S_1(t) S_2(t) dt = 0$$

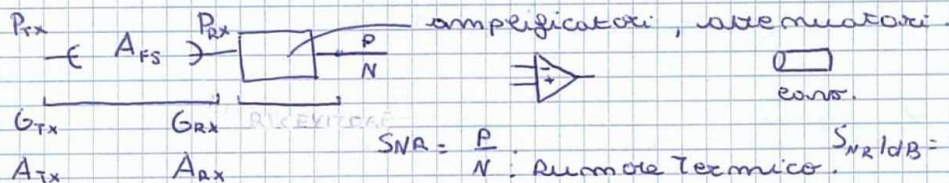


$S_1(t)$ e $S_2(t)$ sono ortogonali!

Esercizi Esame

04/05/2015

Equazioni delle Distanze



$$SNA = \frac{P}{N} \quad \text{Rumore Termico} \quad S_{NR} \text{ dB} = P_{rx} \text{ dB} - N_{dB}$$

$$N = k \cdot T_{S157} \cdot B \rightarrow N_{dB} = k T_{S157} \cdot B \text{ dB}$$

Banda segnale modulante.

$$X \rightarrow X_{dB} = 10 \log_{10} X$$

1,33, 10

G: Guadagno

A: Attenuazione

P: Potenza.

$$X \cdot Y \rightarrow X_{dB} + Y_{dB}$$

$$X/Y \rightarrow X_{dB} - Y_{dB}$$

$$P_{Rx} = \frac{P_{Tx} \cdot G_{Tx} \cdot G_{Rx}}{A}$$

$$P_{Rx} \text{ dB} = P_{Tx} \text{ dB} + G_{Tx} \text{ dB} + G_{Rx} \text{ dB} - A \text{ dB}$$

Diametro antenna

Guadagno di antenna: η, f, d, π, c .

f: Frequenza della Portante.

$$G = \eta \cdot \frac{\pi^2 f^2 d^2}{c^2}$$

Distanza antenne

$$A_{fs} = f, D, \pi, c$$

$$A_{fs} = \left(\frac{4\pi f D}{c} \right)^2$$

$$G_{dB} = 20,4 + 10 \log_{10} \eta + 20 \log_{10} f_{GHz} + 20 \log_{10} d$$

↑
incide $\pi, c = 10^9$ dalla f in GHz.

$$\begin{aligned} A_{gs1dB} &= 32,4 + 20 \log_{10} f_{kHz} + 20 \log_{10} D_{km} \\ &= 32,4 + 20 \log_{10} f_{GHz} + 20 \log_{10} D_{km} \end{aligned}$$

11/03/04 compito

$$\begin{array}{c} \text{---} \text{E} \text{---} \\ \text{P}_{tx} = 10 \text{ W} \quad D = 40 \text{ km} \end{array}$$

$$P_{rx} = ?$$

$$f_0 = 2,3 \text{ GHz} : f_{portante}$$

$$B = 10 \text{ kHz} : f_{modulante}$$

Due antenne paraboliche simmetriche :

$$d = 1 \text{ m}$$

$$\eta = 55 \% = 0,55$$

$$A_{tx} = A_{rx} = 3 \text{ dB}$$

$$P_{tx1dB} = 10 \log_{10} 10 = 10 \text{ dBW}$$

$$P_{rx1dB} = P_{tx1dB} + G_{tx1dB} + G_{rx1dB} - A_{gs1dB} - A_{tx1dB} - A_{rx1dB}$$

$$G = 0,55 \cdot \left(\frac{\pi \cdot 2,3 \cdot 10^8 \cdot 1}{3 \cdot 10^8} \right)^2 = 319$$

$$G_{dB} = 10 \log_{10} 319 = 25 \text{ dB}$$

$$G_{dB} = 20,4 - 2,6 + 7,2 = 25 \text{ dB}$$

$$A_{gs} = \left(\frac{4\pi \cdot 2,3 \cdot 10^8 \cdot 4 \cdot 10^4}{3 \cdot 10^8} \right)^2 = 1,48 \cdot 10^{13}$$

$$A_{gs1dB} = 131,7 \text{ dB}$$

$$A_{gs1dB} = 32,4 + 67,2 + 32 = 131,6 \text{ dB}$$

$$P_{rx1dB} = -77,7 \text{ dB}$$

Amplificatore:

$$P_i \rightarrow P_u \quad P_u = G \cdot P_i$$

$$G > 1$$

$$F: \text{figura di rumore} = \frac{SNR_i}{SNR_u}$$

Attenuatore

$$P_i \rightarrow P_u \quad P_u = G \cdot P_i$$

$$G < 1$$

$$A = \frac{1}{G}$$

$$T_{sist} = T_0 + T_{eq} \quad , \quad T_{eq} = T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2} + \frac{T_{eq4}}{G_1 G_2 G_3} + \dots$$

$$\downarrow$$

$$290 \text{ K}$$

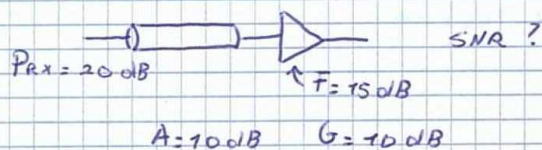
$$T_{eq} = \sum_{i=1}^N \frac{T_{eqi}}{\prod_{j=1}^{i-1} G_j}$$

$$\text{Amplificatore} : T_{eq} = T_0 (F - 1)$$

$$\text{Attenuatore} : T_{eq} = T_0 (A - 1)$$

31/05/04 Compito

Rivelatore:



$$B = 10 \text{ MHz}$$

$$T_i = T_0 = 290 \text{ K}$$

$$SNR = \frac{P}{N}$$

$$N = k \cdot T_{sist} \cdot B$$

$$T_{sist} = T_0 + T_{eq} = T_0 + T_{eq1} + \frac{T_{eq2}}{G_1}$$

$$= T_0 + T_0 (A - 1) + T_0 (F - 1) \cdot \frac{1}{G_1}$$

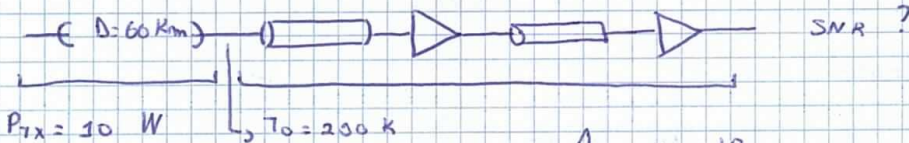
$$= T_0 + T_0 (A - 1) + T_0 (F - 1) \cdot A$$

$$= T_0 \left(1 + \frac{A-1}{A} + F \cdot \frac{A-1}{A} \right) = T_0 F A = 290 \cdot 31,62 \cdot 10 = 91698 \text{ K}$$

$$N = 1,38 \cdot 10^{-23} \cdot 9,17 \cdot 10^4 \cdot 10^7 = 12,65 \cdot 10^{-12}$$

$$SNR = P_{Tx} \text{ dB} - N \text{ dB} = 20 + 108 = 128 \text{ dB}$$

29/9/08 Compito



$$P_{Tx} = 10 \text{ W}$$

$$B = 20 \text{ KHz}$$

$$f_0 = 5 \text{ GHz}$$

$$G_{Tx} = G_{Rx} = 16 \text{ dB}$$

$$A_1 = 10 \text{ dB}$$

$$G_1 = F_1 = 10 \text{ dB}$$

$$A_2 = 13 \text{ dB}$$

$$G_2 = F_2 = 13 \text{ dB}$$

$$SNR = P_{Rx} \text{ dB} - N \text{ dB}$$

$$= P_{Tx} \text{ dB} + G_{Tx} \text{ dB} + G_{Rx} \text{ dB} - A_{fs} \text{ dB} - K T_{sys} B \text{ dB}$$

$$P_{Tx} \text{ dB} = 10 \text{ dB}$$

$$A_{fs} \text{ dB} = 32,3 + 20 \log_{10} 5000 + 20 \log_{10} 60 = 142 \text{ dB}$$

$$T_{sys} = T_0 + T_{eq} = T_0 + T_{eq1} + T_{eq2} \cdot \frac{1}{G_2} + T_{eq3} \cdot \frac{1}{G_1 G_2} + T_{eq4} \cdot \frac{1}{G_1 G_2 G_3}$$

$$= T_0 + T_0 (A_1 - 1) + T_0 (F_1 - 1) \cdot A_1 + T_0 (A_2 - 1) \cdot A_1 \cdot \frac{1}{G_2} + T_0 (F_2 - 1) \cdot \frac{A_1 A_2}{G_2}$$

$$= T_0 \left(1 + \frac{A_1}{G_1} + F_1 A_1 \cdot \frac{A_1}{G_2} + \frac{A_1 A_2}{G_2} - \frac{A_1}{G_2} + \frac{A_1 A_2 F_2}{G_2} - \frac{A_1 A_2}{G_2} \right)$$

$$= 290 (100 - 1 + 400) = 144710$$

$$SNR = \dots$$

05/05/2015

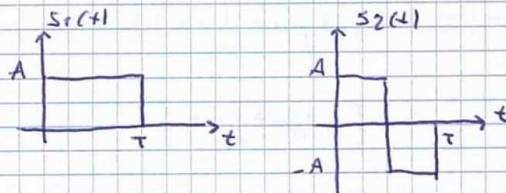
$$A = \{a_1, a_2, \dots, a_n\}, M = 2^m$$

$p(a_i)$: probabilità a priori del simbolo a_i .

$$a_i \rightarrow S_i(t) \neq \emptyset \text{ in } (0, T).$$

$$E_i = \int_0^T |S_i(t)|^2 dt$$

$S_i(t), S_j(t)$ sono ortogonali se: $\int_0^T S_i(t) S_j(t) dt = 0$.



$$S_1(t) = \sqrt{\frac{2E}{T}} \sin(2\pi f_0 t)$$

$$S_2(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t)$$

$$\int_0^T S_1(t) \cdot S_2(t) dt = \frac{2E}{T} \int_0^T \sin(2\pi f_0 t) \cos(2\pi f_0 t) dt$$

$$= \frac{2E}{T} \cdot \frac{1}{2} \left\{ \int_0^T \sin(4\pi f_0 t) dt + \int_0^T \sin(4\pi f_0 t) dt \right\}$$

$$= \frac{E}{T} \cdot \frac{1}{4\pi f_0} \left[-\cos(4\pi f_0 T) + 1 \right] = 0.$$

$$\rightarrow T = KT' = K \cdot \frac{1}{f_0} \rightarrow K = f_0 T$$

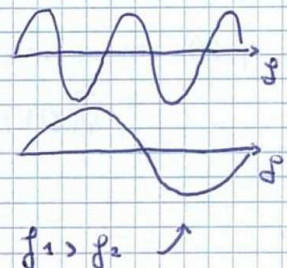
T : Tempo di Simbolo

$$\frac{1 \text{ sec}}{T} = R': \text{ n° simboli trasmessi in 1 sec.}$$

$$R = m \text{ bit/s} = m R'$$

$$\begin{cases} S_1(t) = \sqrt{\frac{2E}{T}} \cos[2\pi(f_0 + \Delta f)t] & , f_1 = f_0 + \Delta f \\ S_2(t) = \sqrt{\frac{2E}{T}} \cos[2\pi(f_0 - \Delta f)t] & , f_2 = f_0 - \Delta f \end{cases}$$

↑ FSK Bimario



Numero intero di cicli della sinusoidale in T .

$$T = KT' = K \cdot \frac{1}{f_0 + \Delta f}$$

$$T = mT'' = K \cdot \frac{1}{f_0 - \Delta f}$$

$$\rightarrow \begin{cases} (f_0 + \Delta f)T = K \\ (f_0 - \Delta f)T = m \end{cases}$$

$$\int_0^T s_1(t) s_2(t) dt = \frac{2E}{T} \int_0^T \cos[2\pi(f_0 + \Delta f)t] \cos[2\pi(f_0 - \Delta f)t] dt$$

$$= \frac{2E}{T} \cdot \frac{1}{2} \left\{ \int_0^T \cos[4\pi\Delta f t] dt + \int_0^T \cos[4\pi f_0 t] dt \right\}$$

$$= \frac{E}{T} \left\{ \frac{1}{4\pi\Delta f} \cdot \cancel{\text{sem}[4\pi\Delta f t]} + \frac{1}{4\pi f_0} \cdot \cancel{\text{sem}[4\pi f_0 t]} \right\} = 0$$

$$2f_0 T = K + m$$

$$2\Delta f T = K - m$$

Posso considerare come FSK Bimodale:

$$\begin{cases} s_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t) \\ s_2(t) = \sqrt{\frac{2E}{T}} \sin(2\pi f_0 t) \end{cases}$$

I segnali ortogonali vengono usati per semplificare i segnali stessi!

SERIE DI FOURIER GENERALIZZATA

$\psi_1(t), \psi_2(t), \dots$ funzioni ortogonali:

$$\int_0^T \psi_i(t) \psi_j(t) dt = 0 \quad \forall i, j, i \neq j.$$

$\psi_i(t)$ sono normalizzate ($E_i = 1$).

$$\int_0^T |\psi_i(t)|^2 dt = 1.$$

$\psi_i(t)$ sono quindi otto - membri

Basi ammissibili:

$$\begin{cases} \psi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi(f_0 + \Delta f)t) \\ \psi_2(t) = \sqrt{\frac{2}{T}} \cos(2\pi(f_0 - \Delta f)t) \end{cases}$$

$$S_i(t) = \sum_{k=1}^{\infty} S_{i,k} \psi_k(t) \rightarrow S_{i,k} = \int_0^T S_i(t) \psi_k(t) dt$$

Da $S_i(t)$ posso calcolare $S_{i,k}$ e viceversa.

$S_i(t)$ continuo nell'intervallo $0 \leq t \leq T$.

$$\underline{S_i} = (S_{i,1}, S_{i,2}, \dots) = \underline{S_i}$$

Prendiamo $S_i(t)$, $S_j(t)$:

$$S_i(t) = \sum_{k=1}^{\infty} S_{i,k} \psi_k(t) \rightarrow \underline{S_i} = (S_{i,1}, S_{i,2}, \dots)$$

$$S_j(t) = \sum_{k=1}^{\infty} S_{j,k} \psi_k(t) \rightarrow \underline{S_j} = (S_{j,1}, S_{j,2}, \dots)$$

$$\int_0^T S_i(t) S_j(t) dt = \int_0^T \sum_{k=1}^{\infty} S_{i,k} \psi_k(t) \cdot \sum_{l=1}^{\infty} S_{j,l} \psi_l(t) dt$$

$$= \sum_{k=1}^{\infty} \sum_{l=1}^{\infty} S_{i,k} S_{j,l} \underbrace{\int_0^T \psi_k(t) \psi_l(t) dt}_{\substack{\text{per } k \neq l \\ \int_0^T \psi_k^2(t) dt = 1 \\ \text{per } k = l}}$$

$$\int_0^T \psi_k^2(t) dt = 1 \quad \text{per } k = l$$

$$= \sum_{k=1}^{\infty} S_{i,k} S_{j,k} = \langle \underline{S_i}, \underline{S_j} \rangle \quad \text{Prodotto Scalare}$$

$$A = \{a_1, a_2, \dots, a_N\}$$

M segnali $S_1(t), S_2(t), \dots, S_N(t)$.

Posso ridurre il n° di funzioni sapendo che ho solo m segnali da rappresentare.



Procedimento di Gram - Schmidt

$$S_1(t) \rightarrow \psi_1(t) = \frac{S_1(t)}{|S_1(t)|}, \quad \int_0^T \psi_1^2(t) dt = \int_0^T \frac{S_1^2(t)}{|S_1(t)|^2} dt$$

$$= \frac{1}{|S_1(t)|^2} \int_0^T S_1^2(t) dt = \frac{|S_1(t)|^2}{|S_1(t)|^2} = 1$$

$$S_2(t) \rightarrow V_2(t) = S_2(t) - \angle S_2(t), \psi_1(t) \rangle \psi_1(t)$$

$$\int_0^T V_2(t) \psi_1(t) dt = \int_0^T S_2(t) \psi_1(t) dt - \int_0^T \angle S_2(t), \psi_1(t) \rangle \psi_1^2(t) dt$$

$$= \angle S_2(t), \psi_1(t) \rangle - \angle S_2(t), \psi_1(t) \rangle \underbrace{\int_0^T \psi_1^2(t) dt}_1 = 0.$$

$V_2(t)$ ortogonale a $\psi_1(t)$

Ho tolto la componente parallela di $S_2(t)$ a $S_1(t)$.

$$\psi_2(t) = \frac{V_2(t)}{|V_2(t)|}$$

$$S_3(t) \rightarrow V_3(t) = S_3(t) - \angle S_3(t), \psi_2(t) \rangle \psi_2(t) - \angle S_3(t), \psi_1(t) \rangle \psi_1(t)$$

$$\int_0^T V_3(t) \psi_1(t) dt = \int_0^T S_3(t) \psi_1(t) dt - \angle S_3(t), \psi_2(t) \rangle \int_0^T \psi_2(t) \psi_1(t) dt$$

$$- \angle S_3(t), \psi_1(t) \rangle \underbrace{\int_0^T \psi_1^2(t) dt}_1 - \angle S_3(t), \psi_1(t) \rangle$$

$$= 0$$

$$\int_0^T V_3(t) \psi_2(t) dt = \int_0^T S_3(t) \psi_2(t) dt - \angle S_3(t), \psi_2(t) \rangle \int_0^T \psi_2^2(t) dt$$

$$- \angle S_3(t), \psi_1(t) \rangle \int_0^T \psi_1(t) \psi_2(t) dt$$

$$= 0$$

$V_3(t)$ ortogonale a $\psi_1(t)$ e $\psi_2(t)$

$$\psi_3(t) = \frac{V_3(t)}{|V_3(t)|}$$

$$S_4(t) \rightarrow V_4(t) = S_4(t) - \angle S_4(t), \psi_3(t) \rangle \psi_3(t) - \angle S_4(t), \psi_2(t) \rangle \psi_2(t)$$

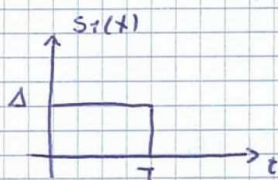
$$- \angle S_4(t), \psi_1(t) \rangle \psi_1(t)$$

$$\psi_4(t) = \frac{V_4(t)}{|V_4(t)|}$$

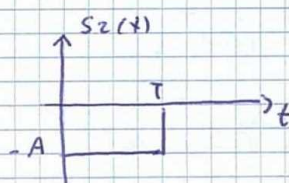
Se venisse $V_2(t) = \emptyset$, posso a trovare M funzioni ortonormali.

$$\begin{array}{c} S_1(t), \dots, S_M(t) \\ \downarrow \\ \psi_1(t), \dots, \psi_N(t) \end{array} \quad N \leq M$$

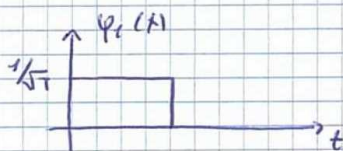
Max M funzioni ortonormali!



$$\begin{aligned} \psi_1(t) &= \frac{S_1(t)}{|S_1(t)|}, \quad E_1 = \int_0^T S_1^2(t) dt \\ &= A^2 \int_0^T dt = A^2 T \end{aligned}$$



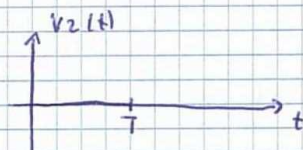
$$\psi_1(t) = \frac{S_1(t)}{A\sqrt{T}} = \frac{1}{\sqrt{T}}$$



$$V_2(t) = S_2(t) - \langle S_2(t), \psi_1(t) \rangle \psi_1(t)$$

$$\Rightarrow \langle S_2(t), \psi_1(t) \rangle =$$

$$= - \int_0^T A \frac{1}{\sqrt{T}} dt = - \frac{A}{\sqrt{T}} \cdot T = -A\sqrt{T}$$



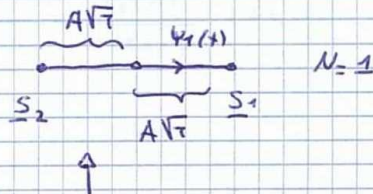
$$-A - (-A\sqrt{T}) \cdot \frac{1}{\sqrt{T}} = \emptyset$$

$$\begin{aligned} \rightarrow S_1(t) &= A\sqrt{T} \psi_1(t) \\ S_2(t) &= -A\sqrt{T} \psi_1(t) \end{aligned}$$

Solo con $\psi_1(t)$ posso a descrivere $S_1(t)$ e $S_2(t)$!

$$M=2, \quad N=1$$

$$\begin{aligned} S_1(t) &\rightarrow S_1 = (A\sqrt{T}) \\ S_2(t) &\rightarrow S_2 = (-A\sqrt{T}) \end{aligned}$$



Segnali Antiparalleli

$$\begin{cases} S_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t) \\ S_2(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t + \pi) \end{cases} \quad \text{BPSK} \quad M=2$$

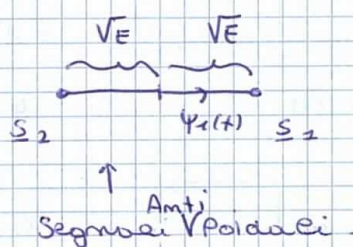
$$\varphi_1(t) = \frac{S_1(t)}{|S_1(t)|} = \frac{S_1(t)}{\sqrt{E}} = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t)$$

$$\begin{aligned} S_2(t) &= \sqrt{\frac{2E}{T}} \left\{ \underbrace{\cos \pi}_{-1} \cos(2\pi f_0 t) - \underbrace{\sin \pi}_{0} \sin(2\pi f_0 t) \right\} \\ &= -\sqrt{\frac{2E}{T}} \cos(2\pi f_0 t) \end{aligned}$$

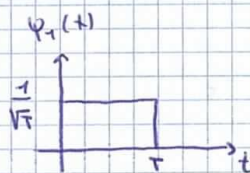
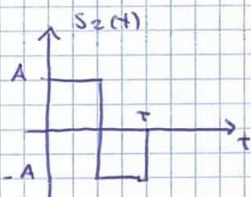
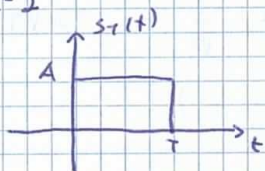
$N=1$?

$$S_1(t) \rightarrow \underline{s}_1 = (\sqrt{E})$$

$$S_2(t) \rightarrow \underline{s}_2 = (-\sqrt{E})$$



$N=2$

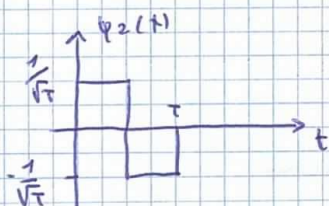


$$S_1(t) = A\sqrt{T} \varphi_1(t)$$

$$V_2(t) = S_2(t) - \angle S_2(t), \varphi_1(t) \rangle \varphi_1(t)$$

$$\angle S_2(t), \varphi_1(t) \rangle = \int_0^T S_2(t) \varphi_1(t) dt = \int_0^{T/2} \frac{A}{\sqrt{T}} dt + \int_{T/2}^T -\frac{A}{\sqrt{T}} dt = 0$$

$$V_2(t) = S_2(t) \quad , \quad \varphi_2(t) = \frac{S_2(t)}{|S_2(t)|} = \frac{S_2(t)}{A\sqrt{T}}$$

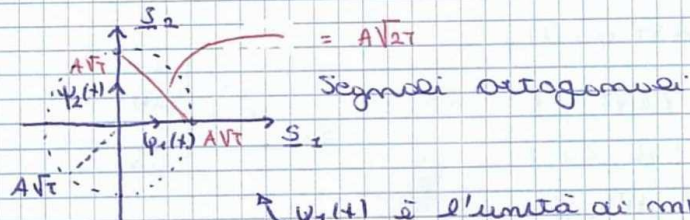


$N=2$

Occorrono due funzioni
ortonormali.

$$S_2(t) = A\sqrt{T} \varphi_2(t)$$

$$\begin{cases} S_1(t) \rightarrow \underline{S}_1 = (A\sqrt{T}, \emptyset) \\ S_2(t) \rightarrow \underline{S}_2 = (\emptyset, A\sqrt{T}) \end{cases}$$



$\psi_1(t)$ è l'unità di misura!

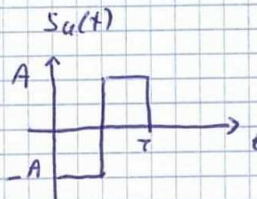
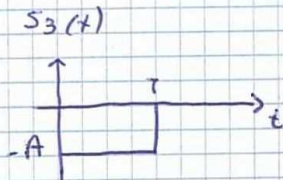
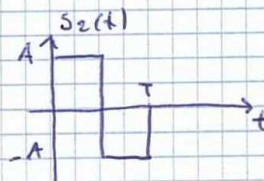
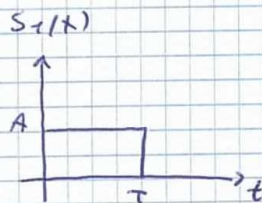
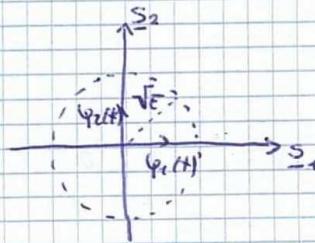
$S_1(t)$ è $A\sqrt{T}$ volte $\psi_1(t)$.

$\rightarrow S_1(t)$ è sull'asse di $\psi_1(t)$.

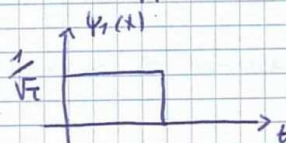
$$\begin{cases} S_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t) \\ S_2(t) = \sqrt{\frac{2E}{T}} \sin(2\pi f_0 t) \end{cases} \quad M=2$$

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t) \rightarrow S_1(t) = \sqrt{E} \psi_1(t) \quad \underline{S}_1 = (\sqrt{E}, \emptyset)$$

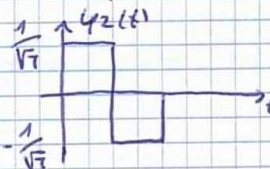
$$\psi_2(t) = \sqrt{\frac{2}{T}} \sin(2\pi f_0 t) \rightarrow S_2(t) = \sqrt{E} \psi_2(t) \quad \underline{S}_2 = (\emptyset, \sqrt{E})$$



$$\psi_1(t) = \frac{S_1(t)}{A\sqrt{T}}$$



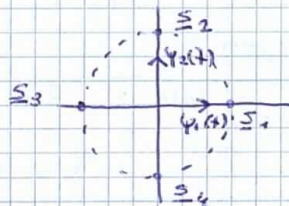
$$\psi_2(t) = \frac{S_2(t)}{A\sqrt{T}}$$



$$\begin{cases} s_1(t) = A\sqrt{T} \psi_1(t) \\ s_2(t) = A\sqrt{T} \psi_2(t) \\ s_3(t) = -A\sqrt{T} \psi_1(t) \\ s_4(t) = -A\sqrt{T} \psi_2(t) \end{cases}$$

$$K=4; \quad N=2$$

$$\begin{aligned} \underline{s}_1 &= (A\sqrt{T}, \phi) \\ \underline{s}_2 &= (\phi, A\sqrt{T}) \\ \underline{s}_3 &= (-A\sqrt{T}, \phi) \\ \underline{s}_4 &= (\phi, -A\sqrt{T}) \end{aligned}$$



Segnali Bi-ortogonali

$$\begin{cases} s_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t) \\ s_2(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t + \frac{\pi}{2}) \\ s_3(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t + \pi) \\ s_4(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t + \frac{3\pi}{2}) \end{cases}$$

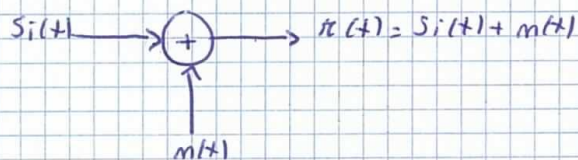
4-PSK

Q-PSK (Quadrature - PSK)

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t)$$

$$\psi_2(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t + \frac{\pi}{2}) = -\sqrt{\frac{2}{T}} \sin(2\pi f_0 t)$$

$$\begin{cases} \underline{s}_1 = (\sqrt{E}, \phi) \\ \underline{s}_2 = (\phi, \sqrt{E}) \\ \underline{s}_3 = (-\sqrt{E}, \phi) \\ \underline{s}_4 = (\phi, -\sqrt{E}) \end{cases}$$



$$s_i(t) \rightarrow \underline{s}_i \quad N \leq K$$

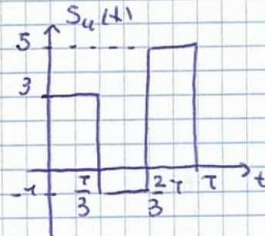
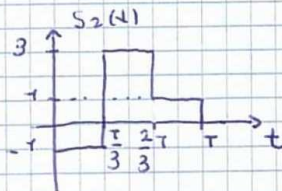
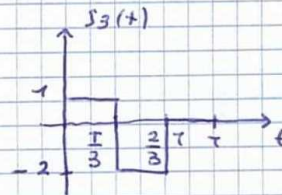
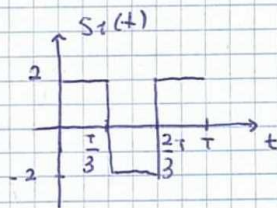
$$m(t) \rightarrow \underline{m} = (m_1, m_2, \dots, m_N, m_{N+1}, \dots)$$

non c'è legame fra le componenti di $\underline{m}$: non sono correlate!

Rumore Bianco, Additivo, Gaussiano: dotato di N componenti

07/05/2015

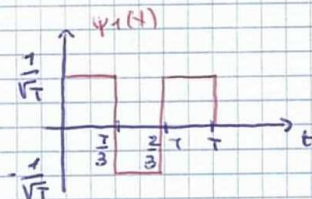
Esercizio:



$$\psi_1(t) = \frac{S_1(t)}{|S_1(t)|}$$

$$\int_0^T S_1^2(t) dt = \int_0^{T/3} 4 dt + \int_{T/3}^{2T/3} 4 dt + \int_{2T/3}^T 4 dt = 4T$$

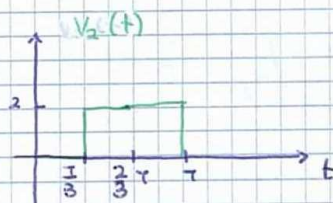
$$\psi_1(t) = \frac{S_1(t)}{2\sqrt{T}}$$



$$V_2(t) = S_2(t) - \langle S_2(t), \psi_1(t) \rangle \psi_1(t)$$

$$\langle S_2(t), \psi_1(t) \rangle = \int_0^T S_2(t) \psi_1(t) dt = \int_0^{T/3} -\frac{1}{\sqrt{T}} dt + \int_{T/3}^{2T/3} 3 \left(-\frac{1}{\sqrt{T}}\right) dt + \int_{2T/3}^T \frac{1}{\sqrt{T}} dt = -\frac{1}{\sqrt{T}} \cdot \frac{T}{3} - \frac{3}{\sqrt{T}} \cdot \frac{T}{3} + \frac{1}{\sqrt{T}} \cdot \frac{T}{3} = -\sqrt{T}$$

$$V_2(t) = S_2(t) + \sqrt{T} \psi_1(t)$$



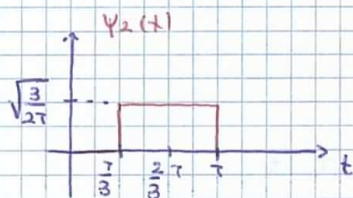
$V_2(t)$ ortogonale a $\psi_1(t)$?

Sì, ha fatto come sempre!

$$\psi_2(t) = \frac{V_2(t)}{|V_2(t)|}$$

$$\int_0^T V_2^2(t) dt = \int_{T/3}^T 4 dt = 4 \cdot \frac{2}{3}T = \frac{8}{3}T$$

$$\psi_2(t) = V_2(t) \cdot \sqrt{\frac{3}{8T}}$$

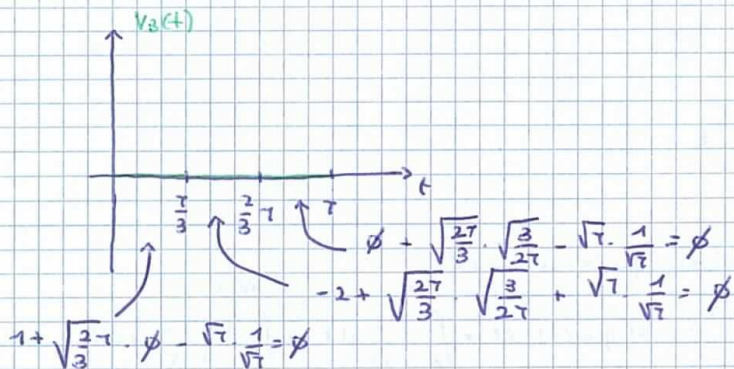


$$V_3(t) = S_3(t) - \angle S_3(t), \psi_2(t) \rightarrow \psi_2(t) - \angle S_3(t), \psi_1(t) \rightarrow \psi_1(t)$$

$$\angle S_3(t), \psi_1(t) \rightarrow \int_0^{T/3} \frac{1}{\sqrt{T}} dt + \int_{T/3}^{2T/3} 2 \cdot \frac{1}{\sqrt{T}} dt = \frac{1}{\sqrt{T}} \cdot \frac{T}{3} + \frac{2}{\sqrt{T}} \cdot \frac{T}{3} = \frac{3}{\sqrt{T}} \cdot \frac{T}{3} = \sqrt{T}$$

$$\angle S_3(t), \psi_2(t) \rightarrow \int_{T/3}^{2T/3} -2 \cdot \sqrt{\frac{3}{2T}} dt = -2 \cdot \sqrt{\frac{3}{2T}} \cdot \frac{T}{3} = -\sqrt{\frac{2T}{3}}$$

$$V_3(t) = S_3(t) + \sqrt{\frac{2T}{3}} \psi_2(t) - \sqrt{T} \psi_1(t)$$



$\psi_1(t)$ e $\psi_2(t)$ Bottomo anche per descrivere $S_3(t)$!

$$V_4(t) = S_4(t) - \angle S_4(t), \psi_2(t) \rightarrow \psi_2(t) - \angle S_4(t), \psi_1(t) \rightarrow \psi_1(t)$$

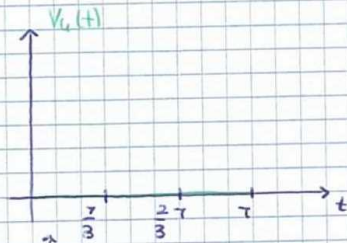
$$\angle S_4(t), \psi_1(t) \rightarrow \int_0^{T/3} 3 \cdot \frac{1}{\sqrt{T}} dt + \int_{T/3}^{2T/3} 1 \cdot \frac{1}{\sqrt{T}} dt + \int_{2T/3}^T 3 \cdot \frac{1}{\sqrt{T}} dt =$$

$$3 \cdot \frac{1}{\sqrt{T}} \cdot \frac{T}{3} + \frac{1}{\sqrt{T}} \cdot \frac{T}{3} + \frac{3}{\sqrt{T}} \cdot \frac{T}{3} = \frac{5}{\sqrt{T}} \cdot \frac{T}{3} = 3\sqrt{T}$$

$$\langle S_4(t), \psi_2(t) \rangle = \int_{\frac{1}{3}}^{\frac{2}{3}\tau} -1 \cdot \sqrt{\frac{3}{2\tau}} dt + \int_{\frac{2}{3}\tau}^{\tau} 5 \cdot \sqrt{\frac{3}{2\tau}} dt =$$

$$= -\sqrt{\frac{3}{2\tau}} \cdot \frac{\tau}{3} + 5 \cdot \sqrt{\frac{3}{2\tau}} \cdot \frac{\tau}{3} = 4 \cdot \sqrt{\frac{3}{2\tau}} \cdot \frac{\tau}{3} = \sqrt{\frac{8\tau}{3}}$$

$$V_4(t) = S_4(t) - \sqrt{\frac{8\tau}{3}} \cdot \psi_2(t) + 3\sqrt{\tau} \cdot \psi_1(t)$$



$$-1 \cdot \sqrt{\frac{8\tau}{3}} \cdot \sqrt{\frac{3}{2\tau}} + 3\sqrt{\tau} \cdot \frac{1}{\sqrt{\tau}} = 0$$

$\psi_1(t)$ e $\psi_2(t)$ lavorano anche per decomporre $S_4(t)$.

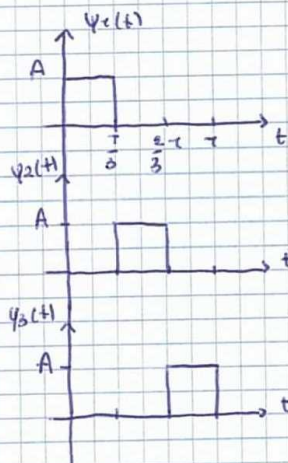
$$S_1(t) = |S_1(t)| \psi_1(t) \Rightarrow S_1 = (2\sqrt{\tau}, 0)$$

$$S_2(t) = (-\sqrt{\tau}, \sqrt{\frac{8\tau}{3}})$$

$$S_3(t) = (\sqrt{\tau}, -\sqrt{\frac{8\tau}{3}})$$

$$S_4(t) = (3\sqrt{\tau}, \sqrt{\frac{8\tau}{3}})$$

Altro Metodo:



$$\int_0^{\tau/3} \psi_1^2(t) dt = 1 = \int_0^{\tau/3} A^2 dt = 1$$

$$\rightarrow A = \sqrt{\frac{3}{\tau}}$$

$$\langle S_1(t), \psi_1(t) \rangle = \int_0^{1/3} \sqrt{\frac{3}{t}} \cdot 2 \, dt = 2 \cdot \sqrt{\frac{3}{t}} \cdot \frac{t}{3} = 2 \sqrt{\frac{t}{3}}$$

$$\underline{S}_1 = \left(2\sqrt{\frac{t}{3}}, -2\sqrt{\frac{t}{3}}, 2\sqrt{\frac{t}{3}} \right);$$

$$\langle S_2(t), \psi_1(t) \rangle = \int_0^{1/3} -1 \cdot \sqrt{\frac{3}{t}} \, dt = -\sqrt{\frac{t}{3}}$$

$$\underline{S}_2 = \left(-\sqrt{\frac{t}{3}}, 3\sqrt{\frac{t}{3}}, \sqrt{\frac{t}{3}} \right)$$

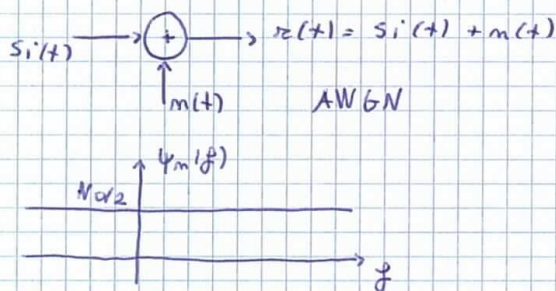
$$\underline{S}_3 = \left(\sqrt{\frac{t}{3}}, -2\sqrt{\frac{t}{3}}, \phi \right)$$

$$\underline{S}_4 = \left(3\sqrt{\frac{t}{3}}, -\sqrt{\frac{t}{3}}, 5\sqrt{\frac{t}{3}} \right)$$

$$S_i(t) \rightarrow \underline{S}_i = (S_{i,1}, S_{i,2}, \dots, S_{i,N})$$

Il simbolo, regolare

$$\psi_1(t), \psi_2(t), \dots, \psi_N(t), \quad N \leq M$$



$n(t) \rightarrow$ infinite funzioni ortogonali

$$\phi_1(t), \phi_2(t), \dots, \phi_m(t), \dots$$

$$\int_0^T \phi_i(t) \phi_j(t) \, dt = \delta_{ij}$$

$$\rightarrow \phi_1(t) = \psi_1(t), \dots, \phi_N(t) = \psi_N(t), \phi_{N+1}(t), \dots$$

$$m_i = \int_0^T m(t) \phi_i(t) \, dt$$

$$\text{Per } 1 \leq i \leq N \rightarrow \phi_i(t) = \psi_i(t)$$

$$x(t) \rightarrow \underline{x} = (x_1, x_2, \dots, x_N, x_{N+1}, \dots)$$

$$x_i = \int_0^T x(t) \phi_i(t) \, dt \quad 1 \leq i \leq N \rightarrow \phi_i(t) = \psi_i(t)$$

$$\pi(t) = s_j(t) + m(t)$$

$$\pi_i = \int_{\phi}^T [s_j(t) + m(t)] \cdot \phi_i(t) dt = \int_{\phi}^T s_j(t) \phi_i(t) dt + \int_{\phi}^T m(t) \phi_i(t) dt$$

$$1 \leq i \leq N \rightarrow \pi_i = s_{j,i} + m_i$$

$$\underline{s}_j = (s_{j,1}, s_{j,2}, \dots, s_{j,N})$$

$$\underline{m} = (m_1, m_2, \dots, m_N, m_{N+1}, \dots)$$

$$\underline{\pi} = (\pi_1, \pi_2, \dots, \pi_N, \pi_{N+1}, \dots)$$

$$\underline{\pi} = \underline{s}_j + \underline{m} \rightarrow \begin{cases} \pi_i = s_{j,i} + m_i & \text{se } 1 \leq i \leq N \\ \pi_i = m_i & \text{se } i > N \end{cases}$$

$$\phi_i(t), \quad 1 \leq i \leq N, \text{ coincidono con } \psi_i(t)$$

N funzioni ortonormali coincidono con $\psi_1(t), \dots, \psi_N(t)$.

$\underline{s}_j$ = vettore trasmesso

$$\underline{s}_j = (s_{j,1}, s_{j,2}, \dots, s_{j,N})$$

$\underline{\pi}$ = vettore ricevuto = (∞ componenti)

$\underline{m}$ = vettore rumore = (∞ componenti)

$$m_i = \int_{\phi}^T m(t) \phi_i(t) dt$$

AWGN con media ϕ ($\overline{m(t)} = \phi$) e varianza $\sigma_m^2 = \frac{N_0}{2}$.

m_i = Gaussiano

$$E(m_i) = E \left\{ \int_{\phi}^T m(t) \phi_i(t) dt \right\} = \int_{\phi}^T E(m(t)) \phi_i(t) dt = \phi$$

$$\begin{aligned} \sigma_i^2 &= E \left\{ m_i^2 - \overline{m_i}^2 \right\} = E(m_i^2) = E \left\{ \int_{\phi}^T m(t) \phi_i(t) dt \int_{\phi}^T m(s) \phi_i(s) ds \right\} \\ &= \int_{\phi}^T \int_{\phi}^T E[m(t) m(s)] \phi_i(t) \phi_i(s) dt ds \end{aligned}$$

$$\rightarrow s = t + \tau \rightarrow E[m(t) m(t + \tau)] = R_m(\tau)$$

$$= \int_{\phi}^T \int_{\phi}^T R_m(\tau) \phi_i(t) \phi_i(s) dt ds$$

$$\psi_m(\tau) = \frac{N_0}{2} \rightarrow R_m(\tau) = \frac{N_0}{2} \delta(\tau)$$

$$= \frac{N_0}{2} \int_{\phi}^T \int_{\phi}^T \delta(\tau) \phi_i(t) \phi_i(s) dt ds, \quad s = t + \tau$$

$$= \frac{N_0}{2} \underbrace{\int_{\phi}^T \phi_i(t) \phi_i(t) dt}_1 = \frac{N_0}{2}$$

$$E(m_i, m_k) = E \left(\int_{\phi}^T m(t) \phi_i(t) dt - \int_{\phi}^T m(s) \phi_k(s) ds \right)$$

$$= \int_{\phi}^T \int_{\phi}^T \underbrace{E[m(t) m(s)]}_{R_m(\tau)} \phi_i(t) \phi_k(s) dt ds, \quad s = t + \tau$$

$$= \frac{N_0}{2} \int_{\phi}^T \int_{\phi}^T \delta(\tau) \phi_i(t) \phi_k(s) dt ds, \quad \delta(\tau) \neq \phi \text{ use } \tau = \phi!$$

$$= \frac{N_0}{2} \int_{\phi}^T \phi_i(t) \phi_k(t) dt = \phi$$

$1 \leq i \leq N \rightarrow \pi_1, \pi_2, \dots, \pi_N$ costituiscono una statistica sufficiente per il ricevitore

$$\underline{\pi} = (\pi_1, \pi_2, \dots, \pi_N)$$

$$\underline{m} = (m_1, m_2, \dots, m_N)$$

$$\Rightarrow \underline{\pi} = \underline{s}_j + \underline{m}$$

Ricevitore Attivo: Minimizza la Probabilità di errore

$$\underline{\pi} \rightarrow P(e/\underline{\pi}) \quad \text{Probabilità errore!}$$

$\underline{s}_j$ segnale trasmesso

$\downarrow$
 a_j

$$P(e/\underline{\pi}) = 1 - P(c/\underline{\pi}) = 1 - p(a_j/\underline{\pi})$$

$$P_e = \int_{\underline{\pi}} P(e/\underline{\pi}) p_{\underline{\pi}}(\underline{\pi}) d\underline{\pi} = \int_{\underline{\pi}} [1 - p(a_j/\underline{\pi})] p_{\underline{\pi}}(\underline{\pi}) d\underline{\pi}$$

$$= \underbrace{\int_{\underline{\pi}} p_{\underline{\pi}}(\underline{\pi}) d\underline{\pi}}_1 - \int_{\underline{\pi}} p(a_j/\underline{\pi}) p_{\underline{\pi}}(\underline{\pi}) d\underline{\pi}$$

$$= 1 - \int_{\underline{\pi}} p(a_j/\underline{\pi}) p_{\underline{\pi}}(\underline{\pi}) d\underline{\pi} \leftarrow \text{minimizzare}$$

massimizzare $\int_{\mathbf{r}} p(\mathbf{a}_j/\mathbf{r}) p(\mathbf{r}) d\mathbf{r}$, $p(\mathbf{r}) > 0$

massimizzare $p(\mathbf{a}_j/\mathbf{r})$

Spesso è più facile calcolare $p(\mathbf{r}/\mathbf{a}_j)$

$$p(\mathbf{r}/\mathbf{a}_j) p(\mathbf{a}_j) = p(\mathbf{a}_j/\mathbf{r}) p(\mathbf{r})$$

$$\max p(\mathbf{a}_j/\mathbf{r}) = \max \frac{p(\mathbf{r}/\mathbf{a}_j) \cdot p(\mathbf{a}_j)}{p(\mathbf{r})}$$

posso eliminare $\frac{1}{p(\mathbf{r})}$ perché tutti i simboli ce l'hanno

Ricevitore ottimo: $\max p(\mathbf{r}/\mathbf{a}_j) p(\mathbf{a}_j)$

$$\max p(\mathbf{r} = \mathbf{s}_j + \mathbf{m} / \mathbf{a}_j) p(\mathbf{a}_j)$$

$$\max p_{\mathbf{m}}(\mathbf{m} = \mathbf{r} - \mathbf{s}_j / \mathbf{a}_j) p(\mathbf{a}_j)$$

$$p_{\mathbf{m}}(\mathbf{m} = \mathbf{r} - \mathbf{s}_j / \mathbf{a}_j) = p_{\mathbf{r}}\{m_1 = r_1 - s_{j1}, m_2 = r_2 - s_{j2}, \dots, m_N = r_N - s_{jN} / \mathbf{a}_j\}$$

$$\mathbf{m} = (m_1, m_2, \dots, m_N)$$

$$= \prod_{i=1}^N p_{m_i}(m_i = r_i - s_{ji} / \mathbf{a}_j)$$

in quanto le m_i non sono correlate!

$$p_{m_i}(m_i) = \frac{1}{\sqrt{2\pi N_0/2}} e^{-\frac{m_i^2/2 \cdot N_0/2}{2}} = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{m_i^2}{N_0}}$$

$$= \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r_i - s_{ji})^2}{N_0}}$$

$$= \prod_{i=1}^N \frac{1}{\sqrt{\pi N_0}} e^{-\frac{(r_i - s_{ji})^2}{N_0}}$$

$$= \left(\frac{1}{\sqrt{\pi N_0}} \right)^N e^{-\sum_{i=1}^N (r_i - s_{ji})^2 / N_0}$$

$$\max \left\{ \left(\frac{1}{\sqrt{\pi N_0}} \right)^N e^{-\sum_{i=1}^N (r_i - s_{ji})^2 / N_0} \cdot p(\mathbf{a}_j) \right\}$$

costante per un ricevitore ottimo!

$$= \max \left[-\sum_{i=1}^N (r_i - s_{ji})^2 \cdot \frac{1}{N_0} + \ln p(\mathbf{a}_j) \right]$$

$$= \min \left[\sum_{i=1}^N (r_i - s_{ji})^2 - N_0 \ln p(\mathbf{a}_j) \right]$$

11/09/08

$$y(t) = [A + B s(t)] \cdot \cos(2\pi f_0 t) + C \hat{s}(t) \cdot \sin(2\pi f_0 t)$$

$$P_c = 1 \text{ W} \quad (\text{Portante})$$

Al variare di A, B, C è possibile ottenere una modulazione AM, DSB, SSB.

$f_0 \gg$ Banda Segnale.

AM:

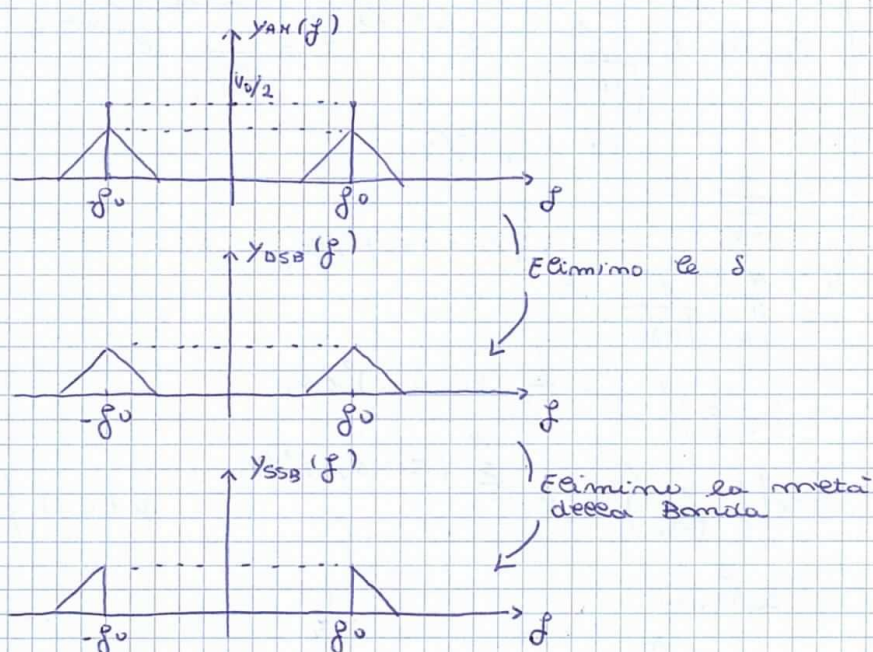
$$y_{AM}(t) = V_0 (1 + K s(t)) \cos(2\pi f_0 t), \quad C = \emptyset, A = V_0, B = V_0 \cdot K$$

$$P_c = P_{cos} = \frac{V_0^2}{2} = 1 \text{ W} \rightarrow V_0 = \sqrt{2}$$

↑
Se conosco la Potenza della Portante conosco anche la sua ampiezza.

DSB:

$$y_{DSB}(t) = V_0 s(t) \cos(2\pi f_0 t), \quad A = \emptyset, B = V_0, C = \emptyset$$



SSB:

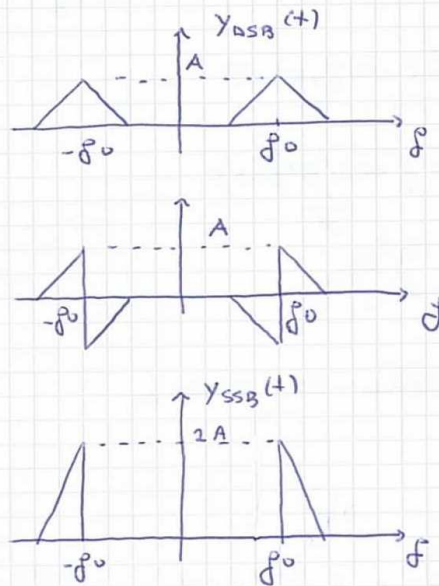
$$y_{SSB}(t) = V_0 s(t) \cos(2\pi f_0 t) \pm V_0 \hat{s}(t) \sin(2\pi f_0 t)$$

$$A = \phi$$

$$B = V_0$$

$$C = V_0$$

Cosa accade in frequenza!?



$$s(t) = 4 \cos(2\pi f_m t) \quad , \quad f_m = 10 \text{ MHz}$$

$$N_0 = 10^{-11}$$

$$K = 0,5$$

SNR per tutte e tre le modulazioni!?

AM:

$$SNR_i = \frac{V_0^2 (1 + K^2 P_m)}{2 N_0 B} = \frac{P_y}{P_N} = \frac{P_y}{N_0 B} \rightarrow P_y = \frac{V_0^2 (1 + K^2 P_m)}{2}$$

$$SNR_i = \frac{2 (1 + 1/4 \cdot 8)}{2 \cdot 10^{-11} \cdot 10^7} = 3 \cdot 10^4$$

$m = K \max |s(t)| \leq 1$. $\rightarrow$ In questo caso $m = K \cdot V_m = 2 > 1$
la modulazione non funziona bene!

$$SNR_M = \frac{V_0^4 K^2 P_H}{2 N_0 B} \quad , \quad F_H = \frac{SNR_M}{SNR_i}$$

$$= 2 \cdot 10^4$$

$\rightarrow$ il rapporto segnale rumore è diminuito.

DSB:

$$SNR_i = \frac{V_0^2 P_H}{2 N_0 B} : SNR_u \text{ in questo } F_H = 1$$

$$SNR_i = \frac{P_Y}{P_N}$$

P_H : Potenza
Modulante!

$$P_Y = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} V_0^2 s^2(t) \cos^2(2\pi f_0 t) dt$$

$$\downarrow$$

$$\frac{1}{2} + \frac{1}{2} \cos(4\pi f_0 t)$$

$$= \lim_{T \rightarrow \infty} \frac{1}{T} \frac{V_0^2}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} s^2(t) dt = \frac{V_0^2 P_H}{2}$$

$$SNR_i = \frac{2 \cdot 8}{2 \cdot N_0 B} = 8 \cdot 10^4 = SNR_u \gg SNR_{u, AH}$$

SSB:

$$SNR_i = \frac{V_0^2 P_H}{N_0 B} = \frac{2 \cdot 8}{N_0 B} = 16 \cdot 10^4 = 2 SNR_{i, DSB} = SNR_u$$

17/07/08

$$m(t) = \cos(2\pi f_m t) \quad f_m = 10 \text{ KHz}$$

$$s(t) = X_{in}^2(t) + \frac{1}{2} X_{in}(t) - \frac{1}{2}$$

$$m(t) \rightarrow \boxed{\text{NON LIN.}} \rightarrow s(t)$$

$$s(t) \rightarrow \boxed{\text{AH}} \rightarrow y_{AH}(t)$$

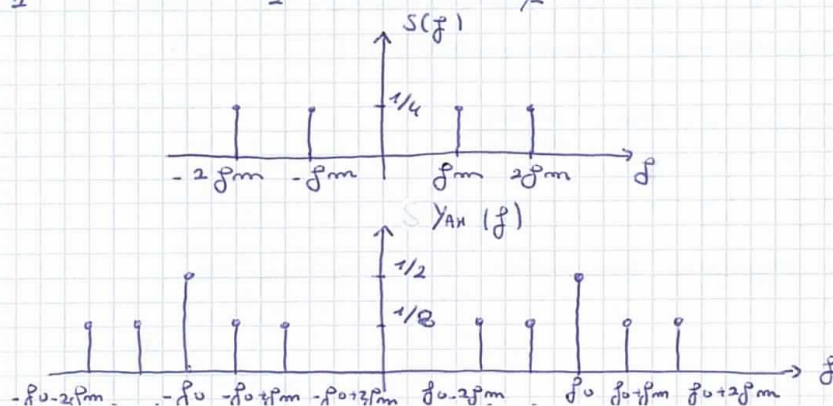
$$c(t) = V_0 \cos(2\pi f_0 t), \quad V_0 = 1 \text{ V}$$

$$f_0 = 1 \text{ MHz}$$

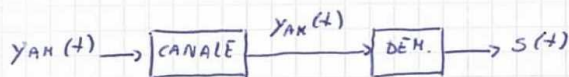
$$K = 1$$

$$s(t) = \cos^2(2\pi f_m t) + \frac{1}{2} \cos(2\pi f_m t) - \frac{1}{2}$$

$$= \frac{1}{2} + \frac{1}{2} \cos(4\pi f_m t) + \frac{1}{2} \cos(2\pi f_m t) - \frac{1}{2}$$



$$N_0 = 10^{-8}$$



↑
nel canale non c'è attenuazione!

$$SNR_M = \frac{V_0^2 K^2 P_H}{2 N_0 B} = \frac{1 \cdot 1 \cdot 1/4}{2 \cdot 10^{-8} \cdot 2 \cdot 10^4} = \frac{1}{16} \cdot 10^4$$

P_H : Somma di due
coseni

$$\rightarrow P_H = \frac{1}{8} + \frac{1}{8} = \frac{1}{4}$$

Oppure sommo le
ampiezze dei 4 δ al
quadrato:

$$P_H = \frac{1}{16} + \frac{1}{16} + \frac{1}{16} + \frac{1}{16} = \frac{1}{4}$$

$$B = 2 f_m = 20 \text{ KHz}$$

29/09/08

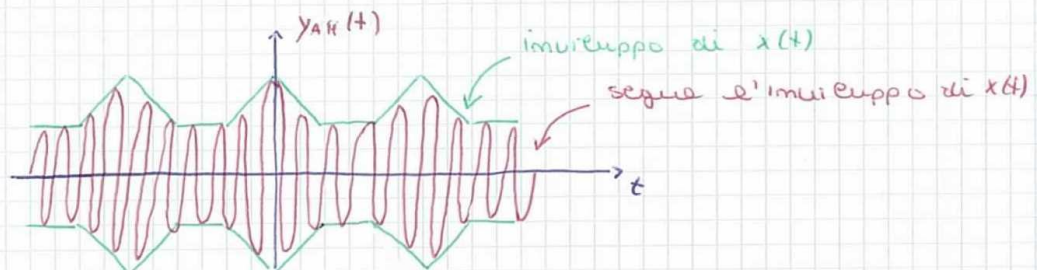
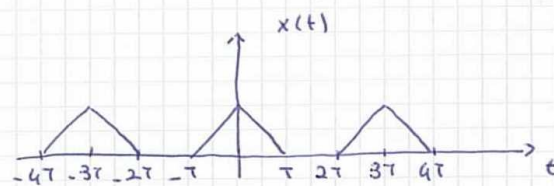
$$x(t) = \sum_{m=-\infty}^{+\infty} g(t - m3\tau)$$

$$g(t) = \text{tri}\left(\frac{t}{\tau}\right)$$

$$V_0 = 2V, \quad f_0 \gg \frac{1}{3\tau}, \quad \tau = 1 \text{ ms}, \quad K = \frac{1}{2}$$

Disegnare $y_{AH}(t)$.

$$y_{AH}(t) = V_0 (1 + K s(t)) \cos(2\pi f_0 t)$$



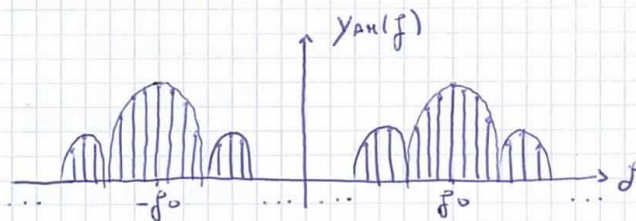
$$y_{AH}(t) = V_0 \left(1 + K \sum_{m=-\infty}^{+\infty} g(t - m3\tau) \right) \cos(2\pi f_0 t)$$

$$y_{AH}(f) = \frac{V_0}{2} \left(\delta(f - f_0) + \delta(f + f_0) \right) + \dots$$

$$G(f) = T \operatorname{sinc}^2(fT)$$

$$X(f) = \frac{1}{3T} \sum_{m=-\infty}^{+\infty} \operatorname{sinc}^2\left(m \frac{\pi}{3T}\right) \cdot \delta\left(f - \frac{m}{3T}\right)$$

$$Y_{AK}(f) = \frac{V_0}{2} \left(\delta(f - f_0) + \delta(f + f_0) \right) + \frac{V_0}{2} \cdot K \left[\frac{1}{3} \sum_{m=-\infty}^{+\infty} \operatorname{sinc}^2\left(\frac{m}{3}\right) \cdot \delta\left(f - f_0 - \frac{m}{3T}\right) + \frac{1}{3} \sum_{m=-\infty}^{+\infty} \operatorname{sinc}^2\left(\frac{m}{3}\right) \cdot \delta\left(f + f_0 - \frac{m}{3T}\right) \right]$$



Calcolare SNR_i , SNR_u prima e dopo il demodulatore.

Posso usare le formule semplificate? NO!

Il segnale $x(t)$ non è a media nulla!

12/05/2015

$$A = \{a_1, a_2, \dots, a_H\}$$

$$p(a_1), p(a_2), \dots, p(a_H)$$

$$s_1(t), s_2(t), \dots, s_H(t)$$

$$s_i(t) \rightarrow \underline{s}_i = (s_{i1}, \dots, s_{iN}), \quad N \leq H \quad \psi_1(t), \dots, \psi_N(t)$$

$$x(t) \rightarrow \underline{x} = (x_1, x_2, \dots, x_N, \dots)$$

$$m(t) \rightarrow \underline{m} = (m_1, m_2, \dots, m_N, \dots)$$

AWGN

$$\underline{x} = (x_1, \dots, x_N)$$

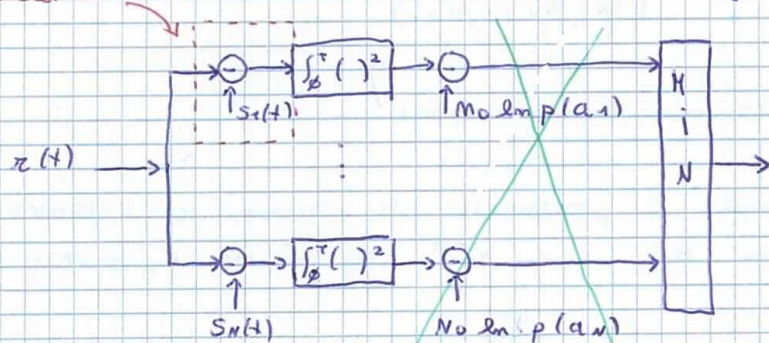
$$\underline{m} = (m_1, \dots, m_N)$$

 s_j : Segnale Transmesso

$$\begin{aligned} |\underline{x} - \underline{s}_j|^2 - N_0 \ln p(a_j) & \text{ min} \rightarrow \text{Ricevitore a max} \\ |\underline{x} - \underline{s}_i|^2 - N_0 \ln p(a_i), i \neq j & \text{ verosimiglianza!} \end{aligned}$$

$$|\underline{x} - \underline{s}_j|^2 = \int_0^T [x(t) - s_j(t)]^2 dt : \text{distanza euclidea fra } x(t) \text{ e } s_j(t)$$

Corrente



$$\text{Se } p(a_i) = \frac{1}{H}, \forall i$$

Ricevitore a distanza minima

Sincronismo di Simbolo: $x(t)$ e $s_j(t)$ devono avere la stessa scala di tempi $\rightarrow$ cominciano in t e finiscono in T .

$$(\underline{r}^2 - 2 \underline{r} \underline{s}_j + \underline{s}_j^2) - N_0 \ln p(a_j)$$

$$\text{Se } p(a_i) = \frac{1}{H}, \forall i \rightarrow (\underline{r}^2 - 2 \underline{r} \underline{s}_j + \underline{s}_j^2)$$

$$\min \left(\frac{1}{2} r^2 - 2 r s_j + s_j^2 \right) - N_0 \ln p(a_j)$$

non influenza
in quanto è un
termine costante

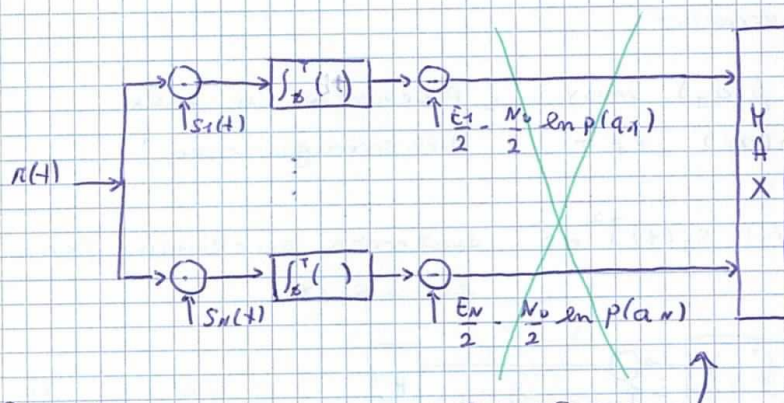
$$\rightarrow \int_0^T s_j^2 dt = E_j$$

$$\min \left\{ \left(-2 r s_j + E_j \right) - N_0 \ln p(a_j) \right\} =$$

$$= \max \left\{ 2 r s_j - E_j + N_0 \ln p(a_j) \right\} =$$

$$= \max \left\{ \underbrace{r s_j}_{\downarrow} - \frac{E_j}{2} + \frac{N_0}{2} \ln p(a_j) \right\}$$

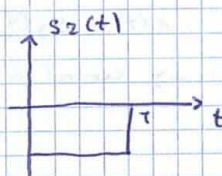
$$\int_0^T r(t) s_j(t) dt = \text{Correlazione tra } x(t) \text{ e } s_j(t).$$



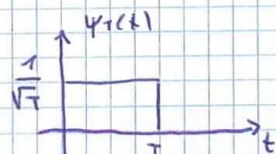
Ricevitore a massima correlazione

Spesso il secondo è preferito al primo (ricevitore) perché il primo fa un x^2 che è un'operazione non lineare!

$$\min \left\{ \|x - s_j\|^2 - N_0 \ln p(a_j) \right\}$$

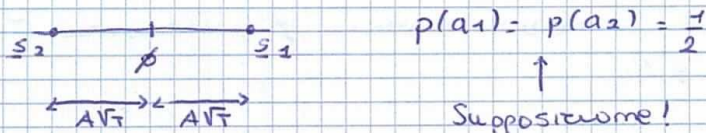


$$M=2, N=1$$



$$\underline{s}_1 = (A\sqrt{T})$$

$$\underline{s}_2 = (-A\sqrt{T})$$



$$p(a_1) = p(a_2) = \frac{1}{2}$$

Supposizione!

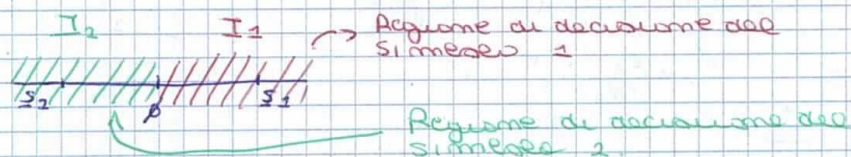
$$\left(\frac{\pi - s_1}{\sigma} \right)^2 - N_0 \ln p(a_1) \rightarrow p(a_1) = \frac{1}{H}, \forall i$$

$$\left(\frac{\pi - s_2}{\sigma} \right)^2 - N_0 \ln p(a_2)$$

$$\left\{ \begin{array}{l} \left(\frac{\pi - s_1}{\sigma} \right)^2 \\ \left(\frac{\pi - s_2}{\sigma} \right)^2 \end{array} \right\} \text{ Scego quello a distanza minima}$$

$$\pi = \pi_1$$

$$\left. \begin{array}{l} s_1 = (AVT) \\ s_2 = (-AVT) \end{array} \right\} \rightarrow \left. \begin{array}{l} (\pi_1 - AVT)^2 \\ (\pi_1 + AVT)^2 \end{array} \right\} \min$$



Se $\pi_1 \hat{x} > \phi$ scego s_1

Se $\pi_2 \hat{x} < \phi$ scego s_2

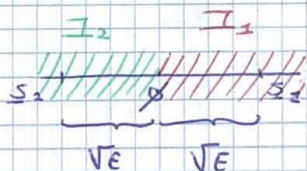
$$s_1(t) = \sqrt{\frac{2E}{T}} \cdot \cos(2\pi f_0 t)$$

$$s_2(t) = \sqrt{\frac{2E}{T}} \cdot \cos(2\pi f_0 t + \pi) = -s_1(t)$$

$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t)$$

$$s_1 = (\sqrt{E})$$

$$s_2 = (-\sqrt{E})$$



Riferendosi al precedente esempio:

$$s_1(AVT)$$

Scego quando $\pi_1 = s_{11} + m_1 \in I_2$

$$\rightarrow P_{e/s_1} = \Pr\{r_1 \notin I_1\}$$

ome casuale

Sapendo che:

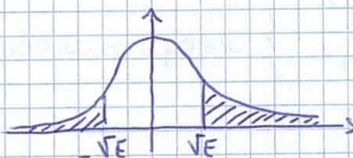
$m_i \rightarrow$ Gaussiano a media ϕ , varianza $\sigma_i^2 = \frac{N_0}{2}$
 $\forall 1 \leq i \leq N$.

Aggiungendo il secondo esempio:

Supponiamo di avere trasmissore $\underline{s}_1 = (\sqrt{E})$

$$P_{e/\underline{s}_1} = \Pr\{r_1 \neq \phi\} = \Pr\{\sqrt{E} + m_1 \neq \phi\} = \Pr\{m_1 \neq -\sqrt{E}\}$$

$$P_{e/\underline{s}_1} = \int_{-\infty}^{-\sqrt{E}} p_{m_1}(m_1) dm_1 + \int_{\sqrt{E}}^{+\infty} p_{m_1}(m_1) dm_1$$



$$= \int_{-\infty}^{-\sqrt{E}} p_{m_1}(m_1) dm_1 + \int_{\sqrt{E}}^{+\infty} p_{m_1}(m_1) dm_1 = *$$

$$p_{m_1}(m_1) = \frac{1}{\sqrt{\frac{N_0}{2} \pi}} e^{-\frac{m_1^2}{\frac{N_0}{2}}} = \frac{1}{\sqrt{\pi N_0}} e^{-\frac{m_1^2}{N_0}}$$

$$* = \int_{-\infty}^{-\sqrt{E}} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{m_1^2}{N_0}} dm_1 + \int_{\sqrt{E}}^{+\infty} \frac{1}{\sqrt{\pi N_0}} e^{-\frac{m_1^2}{N_0}} dm_1 = *$$

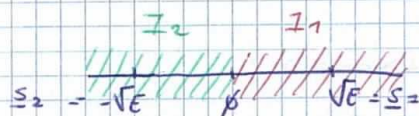
$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-x^2/2} dx$$

$$\rightarrow \frac{m_1}{N_0} = \frac{x}{\sqrt{2}} \rightarrow x = \frac{\sqrt{2}}{\sqrt{N_0}} m_1$$

$$dx = \frac{\sqrt{2}}{\sqrt{N_0}} dm_1$$

$$* = \frac{\sqrt{2}}{\sqrt{2 N_0 \pi}} \int_{-\sqrt{E} \frac{\sqrt{2}}{\sqrt{N_0}}}^{-\sqrt{E} \frac{\sqrt{2}}{\sqrt{N_0}}} e^{-x^2/2} dx + \frac{1}{\sqrt{2\pi}} \int_{\sqrt{E} \frac{\sqrt{2}}{\sqrt{N_0}}}^{+\infty} e^{-x^2/2} dx$$

$$P_{e/\underline{s}_1} = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$



Secondo trasmissore $\underline{s}_2$:

$$r_1 = -\sqrt{E} + m_1 \neq \phi$$

$$m_1 \neq \sqrt{E}$$

$$P_{e/\underline{s}_2} = \int_{\sqrt{E}}^{+\infty} p_{m_1}(m_1) dm_1$$

$$P_{e/\underline{s}_2} = P_{e/\underline{s}_1} = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

$$\rightarrow P_e = P_{e/s_1} p(a_1) + P_{e/s_2} p(a_2)$$

$$= Q\left(\sqrt{\frac{2E}{N_0}}\right) \cdot \frac{1}{2} + Q\left(\sqrt{\frac{2E}{N_0}}\right) \cdot \frac{1}{2} = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$

nel caso precedente: $\sqrt{E} = A\sqrt{T}$!

$s_1(t), s_2(t)$: Distanza Euclidea fra $s_1(t)$ e $s_2(t)$

$$D_{1/2} = \int_0^T [s_1(t) - s_2(t)]^2 dt = *$$

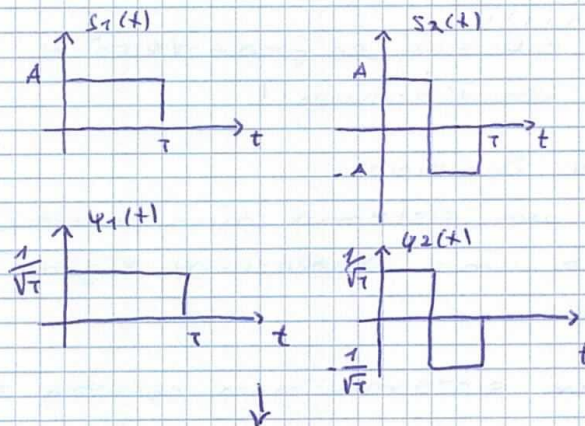
$$\begin{array}{ccc} \text{---} \bullet \text{---} \bullet \text{---} & \psi_1(t) & s_2 = -s_1 \\ s_2 & & s_2(t) = -s_1(t) \end{array}$$

$$\begin{aligned} * &= \int_0^T [s_1^2(t) + s_2^2(t) - 2s_1(t)s_2(t)] dt \\ &= \int_0^T s_1^2(t) dt + \int_0^T s_2^2(t) dt + 2 \int_0^T s_1^2(t) dt = 4E \end{aligned}$$

$\downarrow$ $\downarrow$ $\downarrow$
 E E $2E$

com $s_1(t), s_2(t)$ segnali antipodali.

Prendiamo due segnali ortogonali



$$s_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t)$$

$$s_2(t) = \sqrt{\frac{2E}{T}} \sin(2\pi f_0 t)$$

$$M=2, N=2$$

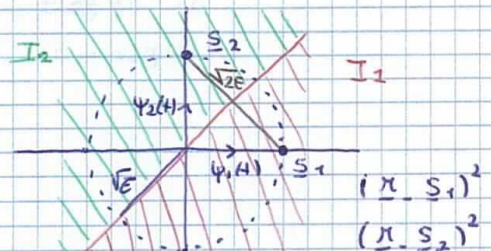
$$s_1 = (\sqrt{E}, 0)$$

$$s_2 = (0, \sqrt{E})$$

$$s_1 = (s_{11}, s_{12}) = (A\sqrt{T}, 0)$$

$$s_2 = (s_{21}, s_{22}) = (0, A\sqrt{T})$$

uguale se $A\sqrt{T} = \sqrt{E}$



$$D_{1,2} = \int_0^T [s_1(t) - s_2(t)]^2 dt = \int_0^T s_1^2(t) dt + \int_0^T s_2^2(t) dt - 2 \int_0^T s_1(t)s_2(t) dt = 2E$$

$\downarrow$ $\downarrow$ $\downarrow$
 E E $\emptyset$

com $s_1(t)$ e $s_2(t)$ segnali ortogonali

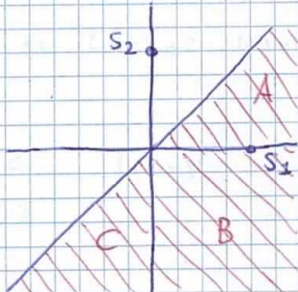
$$\underline{s}_1 = (\sqrt{E}, \emptyset) \quad \underline{x} = (x_1, x_2)$$

$$\underline{s}_2 = (\emptyset, \sqrt{E}) \quad \underline{m} = (m_1, m_2)$$

$$P_{e/s_1} = \begin{cases} x_1 = \sqrt{E} + m_1 \\ x_2 = m_2 \end{cases}$$

$$P_{e/s_1} = \Pr\{\underline{x} \in I_1\}$$

$$= \Pr\{\underline{x} \in A\} + \\ + \Pr\{\underline{x} \in B\} + \\ + \Pr\{\underline{x} \in C\}$$



$$A: \begin{cases} x_1 \geq \emptyset \\ x_2 \geq \emptyset \\ x_1 \leq x_2 \end{cases} \rightarrow \begin{cases} \sqrt{E} + m_1 \geq \emptyset \\ m_2 \geq \emptyset \\ \sqrt{E} + m_1 \leq m_2 \end{cases} \rightarrow \begin{cases} m_1 \geq -\sqrt{E} \\ m_2 \geq \emptyset \\ m_2 \leq \sqrt{E} + m_1 \end{cases} \rightarrow \begin{cases} m_1 \geq -\sqrt{E} \\ \emptyset \leq m_2 \leq \sqrt{E} + m_1 \end{cases}$$

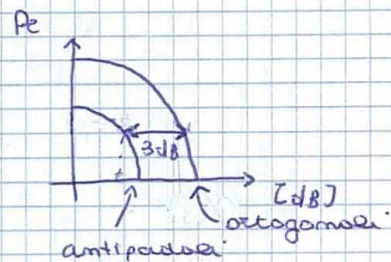
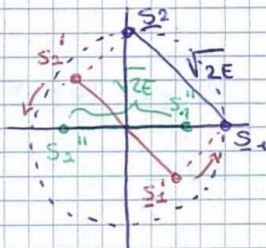
$$\Pr\{\underline{x} \in A\} = \int_{-\sqrt{E}}^{+\infty} p_{m_1}(m_1) \int_{\emptyset}^{\sqrt{E} + m_1} p_{m_2}(m_2) dm_2 dm_1 \\ = \int_{-\sqrt{E}}^{+\infty} p_{m_1}(m_1) \int_{\emptyset}^{\sqrt{E} + m_1} \frac{1}{\sqrt{\pi N_0}} e^{-m_2^2/N_0} dm_2 dm_1$$

→ Non riesco quindi a calcolare la probabilità di errore con la maniera tradizionale!

Teorema sulla Regione di Decisione

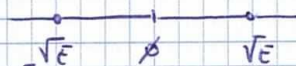
Trasmissione: La P_e di un sistema non cambia se si effettua una rotazione rigida tra i segnali.

Rotazione: La P_e di un sistema non cambia se si effettua una rotazione rigida tra i segnali.



$$\begin{cases} \underline{S}_1'' = \left(\sqrt{\frac{E}{2}} \right) \\ \underline{S}_2'' = \left(-\sqrt{\frac{E}{2}} \right) \end{cases}$$

$$P_e = Q\left(\sqrt{\frac{2E}{N_0}}\right)$$



$$P_e = Q\left(\sqrt{\frac{2 \cdot \frac{E}{2}}{N_0}}\right) = Q\left(\sqrt{\frac{E}{N_0}}\right) : \text{Probabilità di errore per due segnali ortogonali equiprobabili.}$$

Quattro segnali ortogonali:

$$S_1(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t)$$

$$S_2(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t + \frac{\pi}{2})$$

$$S_3(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t + \pi) = -S_1(t)$$

$$S_4(t) = \sqrt{\frac{2E}{T}} \cos(2\pi f_0 t + \frac{3\pi}{2}) = -S_2(t)$$

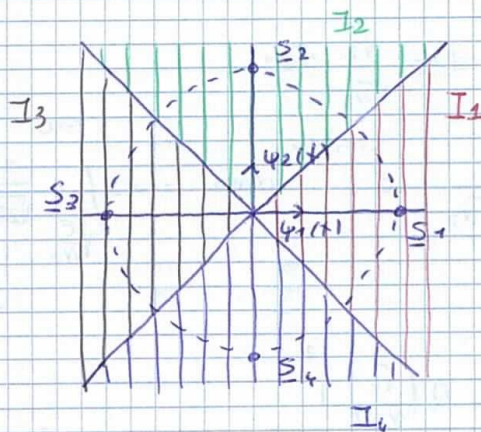
$$\psi_1(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t) \quad , \quad \psi_2(t) = \sqrt{\frac{2}{T}} \cos(2\pi f_0 t + \frac{\pi}{2})$$

$$\underline{S}_1 = (\sqrt{E}, 0)$$

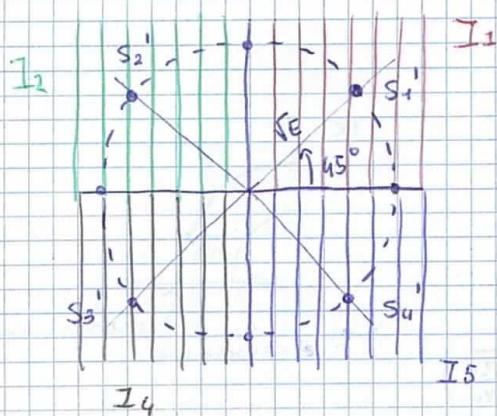
$$\underline{S}_2 = (0, \sqrt{E})$$

$$\underline{S}_3 = (-\sqrt{E}, 0)$$

$$\underline{S}_4 = (0, -\sqrt{E})$$



$$P_{c|S_1} = P_r\{t \in I_1\}$$



$$P_{C/S_1'} = \Pr \{ r \in I_1 = 1^{\text{a}} \text{ Quadrante} \}$$

$$P_{C/S_1}$$

$$\underline{S_1'} = \left(\sqrt{E} \cdot \frac{\sqrt{2}}{2}, \sqrt{E} \cdot \frac{\sqrt{2}}{2} \right) = \left(\sqrt{\frac{E}{2}}, \sqrt{\frac{E}{2}} \right)$$

$$\underline{r} = (r_1, r_2)$$

$$r_1 = \sqrt{\frac{E}{2}} + m_1, \quad r_2 = \sqrt{\frac{E}{2}} + m_2$$

$$P_{C/S_1'} = \begin{cases} r_1 \geq \varnothing \\ r_2 \geq \varnothing \end{cases}$$

$$P_{C/S_1'} = \Pr \{ r_1 \geq \varnothing, r_2 \geq \varnothing \} = \Pr \{ r_1 \geq \varnothing \} \cdot \Pr \{ r_2 \geq \varnothing \}$$

$$r_1 \geq \varnothing \rightarrow m_1 \geq -\sqrt{\frac{E}{2}}$$

$$r_2 \geq \varnothing \rightarrow m_2 \geq -\sqrt{\frac{E}{2}}$$

$$P_{C/S_1'} = \int_{-\sqrt{\frac{E}{2}}}^{+\infty} p_{m_1}(m_1) dm_1 \int_{-\sqrt{\frac{E}{2}}}^{+\infty} p_{m_2}(m_2) dm_2 = \left[\int_{-\sqrt{\frac{E}{2}}}^{+\infty} p_{m_1}(m_1) dm_1 \right]^2$$

$$\int_{-\sqrt{\frac{E}{2}}}^{+\infty} \frac{1}{\sqrt{\pi N_0}} e^{-m_1^2/N_0} dm_1 = \int_{-\infty}^{+\infty} \frac{1}{\sqrt{\pi N_0}} e^{-m_1^2/N_0} dm_1 - \int_{-\infty}^{-\sqrt{\frac{E}{2}}} \frac{1}{\sqrt{\pi N_0}} e^{-m_1^2/N_0} dm_1$$

$$= 1 - \int_{-\sqrt{\frac{E}{2}}}^{+\infty} \frac{1}{\sqrt{\pi N_0}} e^{-m_1^2/N_0} dm_1 = 1 - \int_{\sqrt{\frac{E}{2}}}^{+\infty} \frac{1}{\sqrt{\pi N_0}} e^{-m_1^2/N_0} dm_1$$

$$\frac{x}{\sqrt{2}} = \frac{m_1}{\sqrt{N_0}}, \quad dx = \frac{\sqrt{2}}{\sqrt{N_0}} dm_1$$

$$= 1 - \int_{\sqrt{\frac{E}{2}}}^{+\infty} \frac{\sqrt{2}}{\sqrt{2}} \frac{1}{\sqrt{\pi N_0}} e^{-m_1^2/N_0} dm_1 = 1 - \int_{\sqrt{\frac{E}{2}}}^{+\infty} \frac{1}{\sqrt{2\pi}} e^{-x^2/2} dx$$

$$= 1 - Q\left(\sqrt{\frac{E}{N_0}}\right)$$

$$\Rightarrow P_{C/S_1'} = \left[1 - Q\left(\sqrt{\frac{E}{N_0}}\right) \right]^2$$

$$P_{C/S_2'} = 1 - P_{C/S_1'} = 1 - \left[1 - Q\left(\sqrt{\frac{E}{N_0}}\right) \right]^2$$

Siccome il disegno è simmetrico:

$$P_e = P_{e/s_1'} p(a_1) + P_{e/s_2'} p(a_2) + P_{e/s_3'} p(a_3) + P_{e/s_4'} p(a_4) \\ = P_{e/s_1'}$$

in quanto $p(a_i) = \frac{1}{4}$.

$$P_{e/s_1'} = \dots = P_{e/s_4'}$$

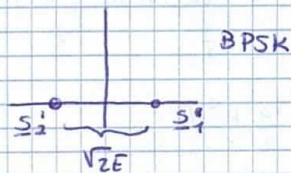
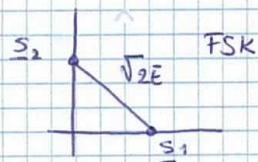
$$D_{1,2} = 2E$$

$$D_{1,3} = 4E$$

$$D_{1,4} = 2E$$

... e così posso fare per gli altri

Più frequentemente combaciamo s_1 con s_2 o
 s_1 con s_3 , più difficile combaciamo con s_4 .



Sistemi equivalenti dal punto di vista di P_e
ma sono diversi da un punto di vista della
realizzazione