FONDAMENTI DI TELECOMUNICAZIONI A

[Fotocopie di Appunti]

A CURA DI ALESSANDRO PAGHI

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LINK AL CORSO ANNO 2014/2015:
http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=54889&aa=2014

FREQUENTAZIONE: Consigliata.

Codifica di Sorgente



Quantità di Bit in realtà trasmessi!

Codifica di Canale

8 bit da trasmettere

8 bit								1 bit
0	·	·	·	·	·	·	0	0
0	1	·	·	·	·	0	1	

aggiungo 1 bit: 0 se il # di 1 è pari
1 se il # di 1 è dispari

nella sequenza c'è sempre un numero pari di 1.

Trasmetto: 0 1 0 0 0 0 0 0 1

Ricevo: 0 1 0 1 0 0 0 0 1 # 1 dispari!

C'è un errore!
Il ricevitore se ne accorge e chiede la ritrasmissione dei 9 bit.

Se:

0 1 0 0 0 0 0 0 1

⇓

0 1 1 0 0 0 1 0 1 Il ricevitore non si accorge dell'errore!

(9, 8): 9 bit trasmessi, 8 bit di informazione.

Prendiamo una matrice:

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$

(7, 4): 4 simboli informativi
parole lunghe 7

a_1, a_2, a_3, a_4 $\quad a_i = \{0, 1\}$

$a_1, a_2, a_3, a_4, c_1, c_2, c_3$

4 bit: a_1, a_2, a_3, a_4
7 bit: $a_1, a_2, a_3, a_4, c_1, c_2, c_3$

$\underline{c} = (0\ 1\ 0\ 0\ c_1 c_2 c_3)$
$a_1\ a_2 a_3 a_4$

Moltiplico $\underline{c}$ per la prima riga

$a_1 + a_2 + a_3 + \emptyset + c_1 + \emptyset \cdot c_2 + \emptyset \cdot c_3$

$\rightarrow \emptyset + 1 + \emptyset + \emptyset + c_1 + \emptyset + \emptyset$ $\rightarrow c_1 = \emptyset$ se c'è # pari di 1

$\rightarrow c_1 = 1$ se c'è # dispari di 1.

In questo caso $c_1 = 1$!

Adesso faccio per la seconda riga:

$a_1 + a_2 + \emptyset \cdot a_3 + a_4 + \emptyset \cdot c_1 + c_2 + \emptyset \cdot c_3$

$\emptyset + 1 + \emptyset + \emptyset + \emptyset + c_2 + \emptyset$ $\rightarrow c_2 = 1$!

adesso per la terza riga

$a_0 + \emptyset + a_3 + a_4 + \emptyset + \emptyset + c_3$

$\emptyset + \emptyset + \emptyset + \emptyset + \emptyset + \emptyset + c_3$ $\rightarrow c_3 = \emptyset$!

$\underline{c} = (0\ 1\ 0\ 0\ 1\ 1\ 0)$
$a_0\ a_1\ a_2\ a_3\ c_1\ c_2\ c_3$

Trasmetto quindi $\underline{c}$ (ci sono 4 bit informativi)

adesso ricevo: $\underline{c} = (0\ 1\ 1\ 0\ 1\ 1\ 0)$

Moltiplico $\underline{c}$ per la prima riga di H

$\emptyset + 1 + 1 + \emptyset + 1 + \emptyset + \emptyset$ => # dispari di 1 = 1

2° Riga

$\emptyset + 1 + \emptyset + \emptyset + \emptyset + 1 + \emptyset$ => # pari di 1 = $\emptyset$

3° Riga

$\emptyset + \emptyset + 1 + \emptyset + \emptyset + \emptyset + \emptyset$ => # dispari di 1 = 1

$1\ 0\ 1$ in quanto la terza colonna è $\begin{matrix}1\\0\\1\end{matrix}$

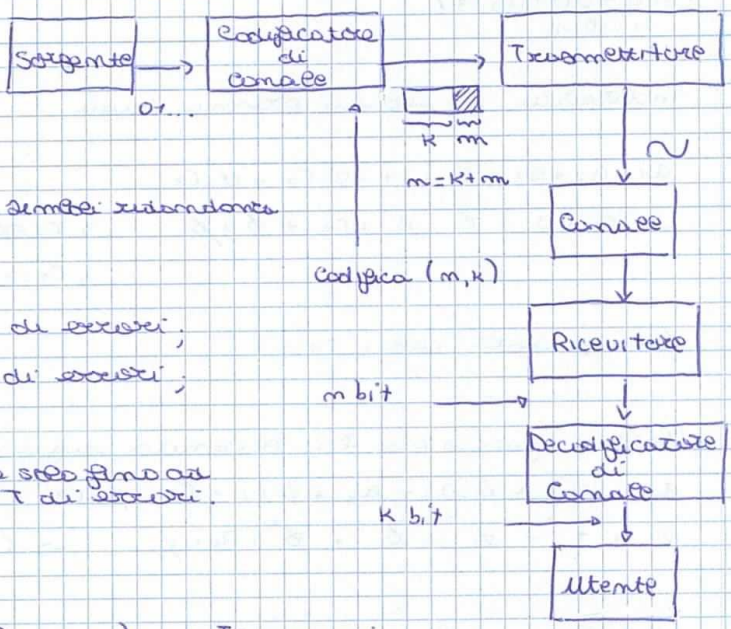
↑

Errore in terza posizione!

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23/03/2015

- Codifica di Canale



8 bit + 1

4 bit inf. + 3 simboli ridondanti

- Codici a rivelazione di errori;
- Codici a correzione di errori;

↑ Riescono a correggere solo fino ad un certo numero T di errori.

ARQ (Automatic Repeat Request) : Tecnica di ritrasmissione di pacchetti

Teorema della Codifica di Canale

C bit/s

P_e = Probabilità di avere un bit errato

$P_e \to \emptyset$ se prefissata opportunamente la codifica purché trasmetta ad una velocità (bit/s) $\leq C$.

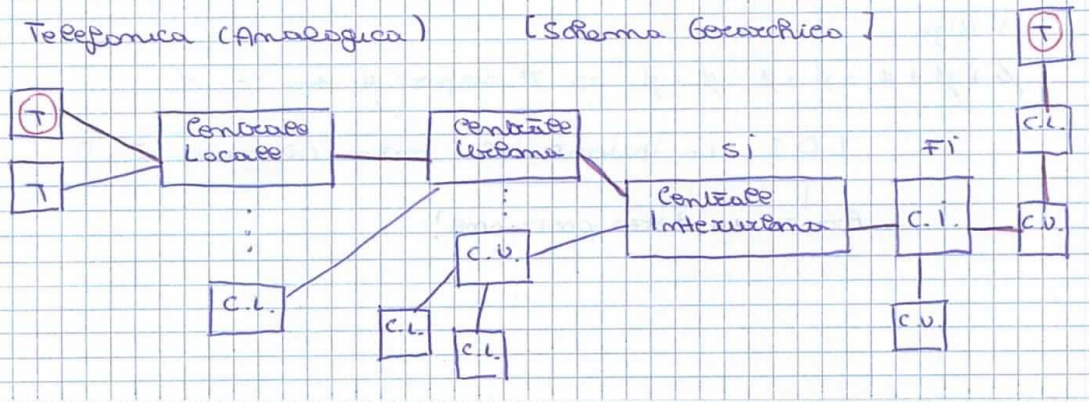
Comunicazione Punto - Punto

Codifica di Sorgente (o compressione dei dati)

Telefonia :



Rete Telefonica (Analogica) [Schema Gerarchico]



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nella telefonia è fondamentale che il circuito non sia aperto!

Fase 1: Costruzione del circuito (10 ÷ 20 s)
(costruisce il percorso T-T attraverso solo dai due T).

Fase 2: Conversazione (min)

Fase 3: Abbattimento del circuito (10 ÷ 20 s)

↑ Commutazione di Circuito (Circuit Switching)



Modulatore
Demodulatore

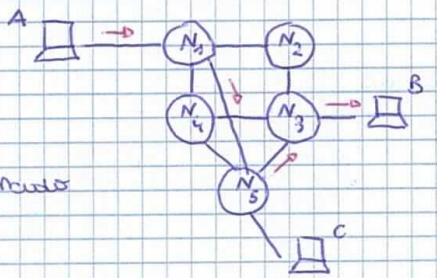
Si crea un circuito, ma non è un mezzo ottimale!



Tempo morto!

Arpanet → Internet

Commutazione di messaggio:



N: Nodo

Esempio di percorso.

I nodi sono i Router
Il processo si chiama Routing.

A → B [01 ... 10] messaggio

Prima dell'invio A aggiunge un Header



→ viene spedito questo messaggio

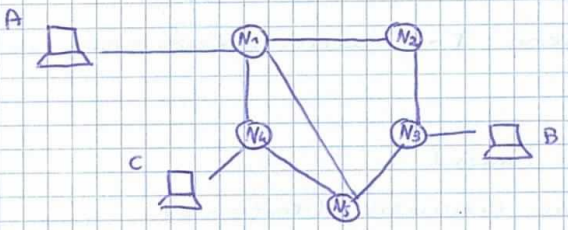
Ind. dest Ind. mitt.

Trasmissione dati: UMTS, GPRS
Trasmissione Voce: GSM
} Protocolli

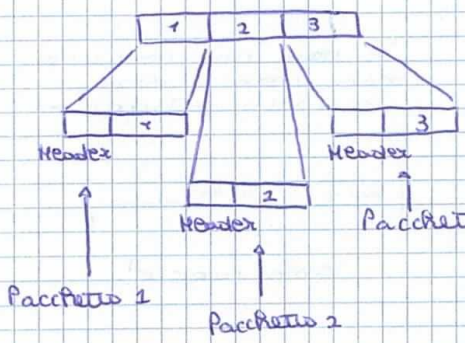
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Commutazione di Pacchetto (Packet Switching)

A -> B  messaggio



Il messaggio viene diviso in Pacchetti



Header formato da:
- Ind. dest;
- Ind. mitt;
- # pacchetto;

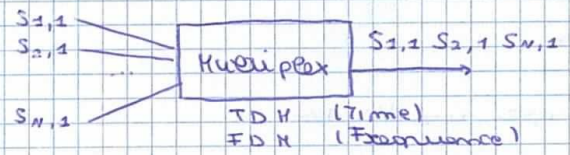
I Pacchetti possono seguire percorsi diversi ma il messaggio viene ricostruito dal ricevitore.

IP: Internet Protocol

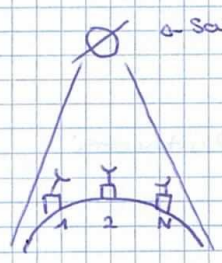
TCP: Transmission Control Protocol

TCP/IP

Multiplex



Sistemi di Accesso Multiplo



a saccente

Più utenti trasmettono senza relazionarsi fra loro.

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TDMA : Time Division multiple Access

FDMA : Frequency Division multiple Access

$s(t) \rightarrow S(f)$

$$E_s = \int_{-\infty}^{+\infty} |s(t)|^2 dt = \int_{-\infty}^{+\infty} |S(f)|^2 df$$

$$s(t) \rightarrow s_m = s\left(\frac{m}{f_c}\right)$$

$$s(t) = \sum_{m=-\infty}^{+\infty} s\left(\frac{m}{f_c}\right) \operatorname{sinc}\left[f_c\left(t - \frac{m}{f_c}\right)\right]$$

$$E_s = \int_{-\infty}^{+\infty} s^2(t)\, dt = \int_{-\infty}^{+\infty} \underbrace{s(t)\cdot s(t)}_{\sum_{m=-\infty}^{+\infty} s\left(\frac{m}{f_c}\right) \operatorname{sinc}\left[f_c\left(t - \frac{m}{f_c}\right)\right]} \cdot \sum_{k=-\infty}^{+\infty} s\left(\frac{k}{f_c}\right) \operatorname{sinc}\left[f_c\left(t - \frac{k}{f_c}\right)\right] dt$$

$$= \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} s\left(\frac{m}{f_c}\right) s\left(\frac{k}{f_c}\right) \int_{-\infty}^{+\infty} \operatorname{sinc}\left[f_c\left(t - \frac{m}{f_c}\right)\right] \operatorname{sinc}\left[f_c\left(t - \frac{k}{f_c}\right)\right] dt$$

$\tau = t - \frac{m}{f_c}$

$$= \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} s\left(\frac{m}{f_c}\right) s\left(\frac{k}{f_c}\right) \int_{-\infty}^{+\infty} \operatorname{sinc}(f_c \tau) \operatorname{sinc}\left[f_c\left(\tau - \frac{k-m}{f_c}\right)\right] d\tau$$

$$s(t) * g(t) = \int_{-\infty}^{+\infty} s(\tau)\, g(t-\tau)\, d\tau$$

$$s(t) * s(t) = \int_{-\infty}^{+\infty} s(\tau)\, s(t-\tau)\, d\tau = c(t) \text{ funzione del tempo.}$$

$$\int_{-\infty}^{+\infty} \operatorname{sinc}\left[f_c \tau\right] \operatorname{sinc}\left[f_c\left(\tau - \frac{k-m}{f_c}\right)\right] d\tau$$

$\frac{\sin \pi x}{\pi x}$ è una funzione pari

$$= \int_{-\infty}^{+\infty} \operatorname{sinc}\left[f_c \tau\right] \operatorname{sinc}\left[f_c\left(\frac{k-m}{f_c} - \tau\right)\right] d\tau$$

È la convoluzione dell'istante $t = \frac{k-m}{f_c}$

$S(-f)$

$\Rightarrow \int_{-\infty}^{+\infty} S(f)\, S(f)\, e^{j2\pi f t}\, df$ → la convoluzione è l'antitrasformata del prodotto delle trasformate.

con $t = \frac{k-m}{f_c}$

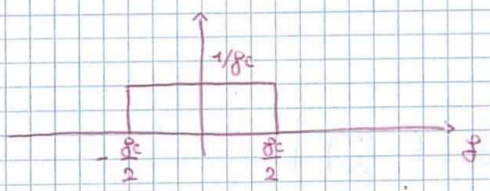
XKÈ INTEGRALE ? $\int_{-\infty}^{+\infty} s(\tau)\, s(t-\tau)\, d\tau = S(f) \cdot S(f)$.

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$\text{rect}\left(\frac{t}{\tau}\right) \rightarrow \tau \cdot \text{sinc}(f\tau)$

$\tau \,\text{sinc}(t\tau) \rightarrow \text{rect}\left(\frac{f}{\tau}\right)$

$\text{sinc}(f_c t) \rightarrow \frac{1}{f_c}\,\text{rect}\left(\frac{f}{f_c}\right)$ con t tempo



$$= \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} \frac{1}{f_c^2} \cdot e^{j2\pi f t}\, df \qquad \text{con } t = \frac{k-m}{f_c}$$

$$= \frac{1}{f_c^2} \cdot \frac{1}{j2\pi t}\, e^{j2\pi f t}\Big|_{-\frac{f_c}{2}}^{\frac{f_c}{2}} = \frac{1}{f_c^2} \cdot \frac{1}{j2\pi t} \cdot \left[e^{j2\pi \frac{f_c}{2} t} - e^{-j2\pi \frac{f_c}{2} t} \right]$$

$$= \frac{1}{f_c^2} \cdot \frac{1}{\pi t} \cdot \text{sen}\left[\pi f_c t\right] = \frac{1}{f_c^2} \cdot \frac{1}{\pi\left(\frac{k-m}{f_c}\right)} \cdot \text{sen}\left[\pi f_c \cdot \frac{k-m}{f_c}\right]$$

$$= \frac{1}{f_c}\,\text{sinc}\left[f_c\left(\frac{k-m}{f_c}\right)\right]$$

In conclusione

$$= \sum_{m=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} s\left(\frac{m}{f_c}\right) s\left(\frac{k}{f_c}\right) \frac{1}{f_c} \cdot \text{sinc}\left[\cancel{f_c} \cdot \frac{(k-m)}{\cancel{f_c}}\right]$$

$$\frac{\text{sen}[(k-m)\pi]}{(k-m)\pi} \rightarrow \begin{cases} 0 & k \neq m \\ 1 & k = m \end{cases}$$

$$= \frac{1}{f_c} \sum_{m=-\infty}^{+\infty} s^2\left(\frac{m}{f_c}\right) = E_s$$

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3(1) Segnale deterministico

Probabilità

Dati A, B, C, ...

n prove : A m_a volte esce A su n.

Frequenza dell'evento A: $V_A = \frac{m_a}{n}$

Se $n = \infty$: $\lim_{n\to\infty} V_A = \lim_{n\to\infty} \frac{m_a}{n} = P(A)$: Probabilità dell'evento A.

$0 \le m_a \le n$, $0 \le P(A) \le 1$.

$\underbrace{A_1, A_2, \dots, A_N}_{\text{Risultati}}$ $\quad n_1 \to A_1$, $n_2 \to A_2$, ...

$\sum_{i=1}^{N} n_i = n$

$\sum_{i=1}^{N} \frac{n_i}{n} = 1 \Rightarrow \sum_{i=1}^{n} P(A_i) = 1$

A_1, A_2 si dicono equiprobabili se $P(A_1) = P(A_2)$.

$A_1, A_2, \dots, A_m$ se sono tutti equiprobabili : $P(A_i) = \frac{1}{N}$.

1 dado 6 faccie : $p(1) = \frac{1}{6}$.

Probabilità composta di A_1, A_2 : $P(A_1 \circ A_2)$ =

= Probabilità che si verifichi A_1 o A_2 = $\frac{m_1}{n} + \frac{m_2}{n} = P(A_1) + P(A_2)$

$P(A_1, A_2)$ = Probabilità congiunta di A_1 e A_2

= probabilità che si verifichi A_1 e A_2

A_1 e A_2 statisticamente indipendenti

3 dadi rossi, 2 bianchi, 5 verdi

Pr(1 dado rosso, numero 2) = $\frac{3}{10} \cdot \frac{1}{6} = \frac{3}{60} = \frac{1}{20}$.

$P(A_1, A_2) = P(A_1) \cdot P(A_2)$ se A_1, A_2 stat. indip.

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Tutti gli apparati ad una temperatura > 0 K producono un rumore costante in frequenza!

Prob. condizionata (A_1 e A_2)

$P(A_1/A_2)$ = Prob. che una volta verificatosi A_2 si verifichi A_1

$$P(A_1/A_2) = \frac{m_{12}}{m_2}$$

$$P(A_1/A_2) = \frac{m_{12}}{m} = \frac{m_{12}}{m_2} \cdot \frac{m_2}{m}$$

$$P(A_1, A_2) = P(A_1/A_2) \cdot P(A_2) \qquad P(A_2) = \frac{m_2}{m}$$

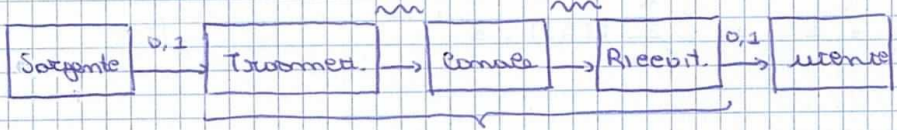
$$P(A_1, A_2) = \frac{m_{12}}{m} = \frac{m_{12}}{m_1} \cdot \frac{m_1}{m}$$

$\uparrow$ $P(A_2/A_1)$

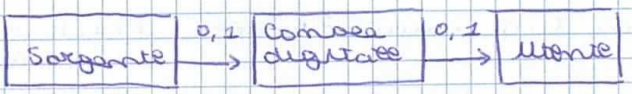
$$P(A_1, A_2) = P(A_2/A_1)\, P(A_1)$$

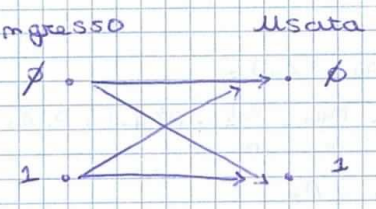
$$\Rightarrow P(A_1, A_2) = P(A_1/A_2)\, P(A_2) = P(A_2/A_1)\, P(A_1)$$

↳ Teorema di Bayes



Canale Digitale (numerico)





schema del canale digitale

$P(0/0)$: Prob. trasmesso 0 ricevo 0 = q_0

$P(1/0) = p_0$

$P(0/1) = p_1$

$P(1/1) = q_1$

q: ricezioni corrette

p: ricezioni errate

$P(0/0) + P(1/0) = 1$

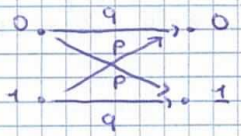
$P(0/1) + P(1/1) = 1$

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24/03/2015







$P(0/0) = q_0$ $\quad$ $P(1/0) = p_0$ $\quad$ $P(0/1) = p_1$ $\quad$ $P(1/1) = q_1$

Spesso: $q_0 = q_1$ $\quad$ $p_0 = p_1$

Quindi: $q = q_0 = q_1$, $p = p_0 = p_1$

Canale Simmetrico Binario (CSB)

$$\begin{cases} q_0 + p_0 = 1 \\ q_1 + p_1 = 1 \end{cases} \Rightarrow q + p = 1.$$

$p < \frac{1}{2}$ Sempre, Spesso $p \ll \frac{1}{2}$. $(p = 10^{-2})$

Nelle ipotesi in cui $p > \frac{1}{2}$, inverto la ricezione $\emptyset \to 1$, $1 \to \emptyset$

Consideriamo 2 Bit

$\emptyset\emptyset$ → Probabilità di ricevere $\emptyset\emptyset$ = P_c = Pr. corretta ricezione

$$P_c = P(\emptyset, \emptyset) = P_c(\emptyset) \cdot P_c(\emptyset) = q \cdot q = q^2$$

↑ Fenomeni indipend.

$\emptyset\emptyset \to \emptyset 1$, $P(\emptyset, 1) = q \cdot p$

$\emptyset\emptyset \to 1\emptyset$, $P(1, \emptyset) = p \cdot q$

Probabilità di un errore: $P_r(1 \text{ errore}) = P(\emptyset, 1) + P(1, \emptyset) = 2pq$

$\emptyset\emptyset \to 11$, $P(1, 1) = p \cdot p = p^2$

P_e: Prob. di Sbagliare (prob. che la parola ricevuta sia sbagliata) $= P(\emptyset, 1) + P(1, \emptyset) + P(1, 1) = 2pq + p^2$.

$P_e + P_c = 1$!

$2pq + p^2 + q^2 = 1 \Rightarrow (p+q)^2 = 1$!

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$\not{0}\not{0}\not{0}$ viene trasmesso.

Ricevo:

000	$P_c = P(0,0,0) = q^3$
001	$P(0,0,1) = pq^2$
010	$P(0,1,0) = pq^2$
011	$P(0,1,1) = p^2q$
100	$P(1,0,0) = pq^2$
101	$P(1,0,1) = p^2q$
110	$P(1,1,0) = p^2q$
111	$P(1,1,1) = p^3$

$P(1 \text{ errore}) = 3pq^2$

$P(2 \text{ errori}) = 3p^2q$

$P(3 \text{ errori}) = p^3$

Quindi:

$P_c = q^3$

$P_e = 3pq^2 + 3p^2q + p^3$

$P_c + P_e = 1$

$q^3 + 3pq^2 + 3p^2q + p^3 = (p+q)^3 = 1.$

Consideriamo la variabile casuale X: $-\infty < X < +\infty$

X variabile aleatoria o casuale

$P_X(x) = Pr.$ che $X \leq x$: Funzione di distribuzione cumulativa di X.

$P_X(-\infty) = \not{0}$, $P_X(+\infty) = 1$ $\qquad 0 \leq P_X(x) \leq 1$

$P_X(a \leq X \leq b) = Pr\{X \leq b\} - Pr\{X \leq a\} = P_X(b) - P_X(a)$.

$P_X(x \leq X \leq x+dx) = P_X(x+dx) - P_X(x)$

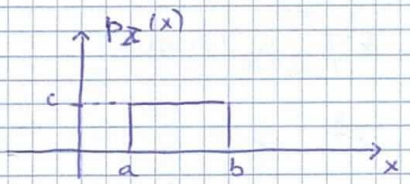
$p_X(x) = \frac{dP_X(x)}{dx}$, Funzione distribuzione densità di X.

$P_X(x) = \int_{-\infty}^{x} p_X(x)\,dx$

$P_X(+\infty) = 1 \Rightarrow \int_{-\infty}^{+\infty} p_X(x)\,dx = 1$

$Pr\{a \leq X \leq b\} = P_X(b) - P_X(a) = \int_a^b p_X(x)\,dx$

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$$\int_{-\infty}^{+\infty} p_X(x)\,dx = \int_a^b c\,dx = 1$$

$$\Rightarrow cx\Big|_a^b = 1$$

$$\Rightarrow c(b-a) = 1$$

$$\Rightarrow c = \frac{1}{b-a}$$ altrimenti $p_X(x)$ non è una densità di prob.!

Tutti i valori di x:

$a < x < b$

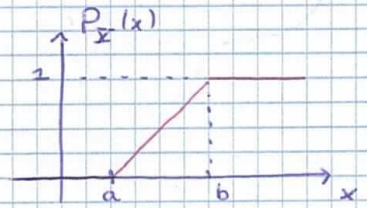
sono equiprobabili.

$$P_X(x) = \int_{-\infty}^{x} p_X(x)\,dx$$

$P_X(x) = 0$ per $x \leq a$

$$P_X(x) = \int_a^x p_X(x)\,dx = \frac{1}{b-a}\int_a^x dx = \frac{1}{b-a}\,x\Big|_a^x = \frac{x-a}{b-a}.$$

$x > b$, $P_X(x) = \int_a^x p_X(x)\,dx = \int_a^b p_X(x)\,dx = 1.$



X valore medio

→ media statistica $\bar{X} = E(x) = \int_{-\infty}^{+\infty} x \cdot p_X(x)\,dx$

momento di ordine m di X: $E(x^m)$, m num. intero.

$$E(x^m) = \int_{-\infty}^{+\infty} x^m \cdot p_X(x)\,dx$$

$E(X^2)$ = Valore quadratico medio $= \overline{X^2}$

momento centrato di ordine m: $E[(x - \bar{x})]^m$

$$= \int_{-\infty}^{+\infty} [(x-\bar{x})^m]\, p_X(x)\,dx$$

$$m = 2,\quad E[(x-\bar{x})^2] = \int_{-\infty}^{+\infty} (x-\bar{x})^2 p_X(x)\,dx$$

$$= E[x^2 - 2\bar{x}x + \bar{x}^2] = \int_{-\infty}^{+\infty} x^2 p_X(x)\,dx - 2\int_{-\infty}^{+\infty} \bar{x}x\, p_X(x)\,dx$$

$$+ \int_{-\infty}^{+\infty} \bar{x}^2 p_X(x)\,dx.$$

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$$= \int_{-\infty}^{+\infty} x^2 p_{\bar{X}}(x)\,dx - 2\bar{x}\int_{-\infty}^{+\infty} x\, p_{\bar{X}}(x)\,dx + \bar{x}^2 \int_{-\infty}^{+\infty} p_{\bar{X}}(x)\,dx$$

$$= \overline{x^2} - 2\bar{x}\cdot\bar{x} + \bar{x}^2 \cdot 1$$

$$\Rightarrow E[x^2 - 2\bar{x}x + \bar{x}^2] = E[(x-\bar{x})^2] = E(x^2) - \bar{x}^2$$

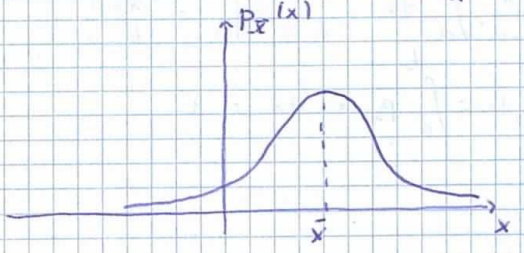
$\bar{x} = 0 \rightarrow \sigma_x^2 = E(x^2)$

$p_{\bar{X}}(x)$ Distribuzione Gaussiana

$\bar{x} = E(x)$

$\sigma_x^2 = E(x^2) - \bar{x}^2$

$$p_{\bar{X}}(x) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \cdot e^{-\frac{(x-\bar{x})^2}{2\sigma_x^2}}$$



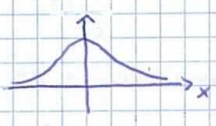
La larghezza dipende dalla varianza.

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{+\infty} e^{-\frac{y^2}{2}}\,dy$$

$$\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-y^2}\,dy \quad : \text{Error Function}$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_x^{+\infty} e^{-y^2/2}\,dy = \underbrace{\frac{1}{\sqrt{2\pi}} \int_0^{+\infty} e^{-y^2/2}\,dy}_{1/2} - \frac{1}{\sqrt{2\pi}} \int_0^x e^{-y^2/2}\,dy$$

$\frac{1}{\sqrt{2\pi}} e^{-y^2/2} \rightarrow$



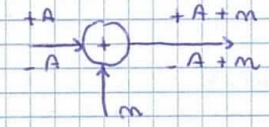
$$Q(x) = \frac{1}{2} - \frac{1}{\sqrt{\pi}} \int_0^{\frac{x}{\sqrt{2}}} e^{-t^2}\,dt \cdot \frac{2}{2} = \frac{1}{2}\left[1 - \operatorname{erf}\left(\frac{x}{\sqrt{2}}\right)\right]$$

$t = \frac{y}{\sqrt{2}}$

Supponiamo di trasmettere ϕ, 1.

$\phi \rightarrow +A$

$1 \rightarrow -A$



n: noise (Rumore)

$$p_n(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} \cdot e^{-n^2/2\sigma_n^2}$$

ϕ si manda trasmesso, +A segnale, segnale ricevuto $+A+n = r$

$$P(e/\phi) = P_{e/\phi} = Pr\{r = A+n \leq \phi / \phi\} \rightarrow A+n \leq \phi$$

$$n \leq -A$$

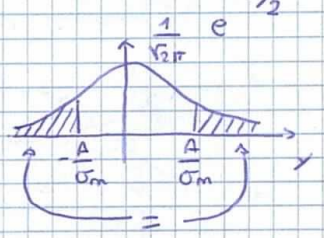
se $r > \phi$ si decide ϕ

se $r < \phi$ si decide 1

$$P_{e/\phi} = Pr\{n \leq -A\} = \int_{-\infty}^{-A} p_n(n)\,dn = \int_{-\infty}^{-A} \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-n^2/\sigma_n^2}\,dn$$

$$y = \frac{n}{\sigma_n}$$

$$P_{e/\phi} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{-A/\sigma_n} e^{-y^2/2}\,dy \rightarrow$$



$$P_{e/\phi} = \frac{1}{\sqrt{2\pi}} \int_{\frac{A}{\sigma_n}}^{+\infty} e^{-y^2/2}\,dy = Q\left(\frac{A}{\sigma_n}\right)$$

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_{x}^{+\infty} e^{-y^2/2}\,dy$$

$$\sigma_n^2 = E(n^2) - \bar{n}^2, \text{ ma } \bar{n}^2 = \phi \rightarrow \sigma_n^2 = E(n^2)$$

$-A+n = r$, segnale se $r \geq \phi$!

$$\rightarrow P_{e/1} = Pr\{r \geq \phi\} = Pr\{n \geq A\}$$

$$p_n(n) = \frac{1}{\sqrt{2\pi\sigma_n^2}} e^{-n^2/2\sigma_n^2}$$

$$P_{e/1} = \int_{A}^{+\infty} p_n(n)\,dn = \frac{1}{\sqrt{2\pi\sigma_n^2}} \int_{A}^{+\infty} e^{-n^2/2\sigma_n^2}\,dn \quad \begin{matrix} t = n/\sigma_n \\ \downarrow \end{matrix} \quad \frac{1}{\sqrt{2\pi}} \int_{\frac{A}{\sigma_n}}^{+\infty} e^{-t^2/2}\,dt = Q\left(\frac{A}{\sigma_n}\right)$$

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$\emptyset$ → $+A$

1 → $-A$

$\overline{m} = \emptyset$

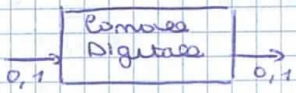
$$P_e = P_{e/\emptyset} \cdot p(\emptyset) + \underbrace{P_{e/1} \cdot p(1)}_{P_r(e,1)}$$

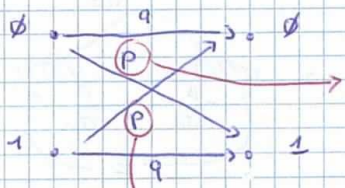
$p(0) = p(1) = $ simmetrici equiprobabili $= \frac{1}{2}$

$$P_e = \frac{1}{2} \cdot Q\left(\frac{A}{\sigma_m}\right) + \frac{1}{2} Q\left(\frac{A}{\sigma_m}\right) = Q\left(\frac{A}{\sigma_m}\right)$$

$$P_e = P_{e/\emptyset} \cdot p(\emptyset) + P_{e/\emptyset}\, p(1) = P_{e/\emptyset} [p(\emptyset) + p(1)] = P_{e/\emptyset}$$







$$q = 1 - Q\left(\frac{A}{\sigma_m}\right)$$

PROPRIETÀ TRASFORMATE — Esercizi Esame — 26/03/2013

- $s(t) \leftrightarrow S(f)$
  $s^*(t) \leftrightarrow S^*(-f)$ (Teorema di Parseval)

- $s(t) \leftrightarrow S(f)$ (Trasformata del rect è il sinc)
  $s(-t) \leftrightarrow S(-f)$

- $s(t) : \mathbb{R} \to \mathbb{R}$, $s(t) \leftrightarrow S(f)$
  $S(-f) = S^*(f)$

- $s(t) : \mathbb{R} \to \mathbb{R}$, $s(t) = s(-t)$ (Rect e Sinc)
  $S(f) \in \mathbb{R}$, $S(f) = S(-f)$

- $s(t) : \mathbb{R} \to \mathbb{R}$, $s(t) = -s(-t)$
  $S(f) \in \mathbb{C}$, $S(f) = -S(-f)$

- $s(t) : \mathbb{R} \to \mathbb{R}$, $s(t) \leftrightarrow S(f)$
  $s_p(t) \leftrightarrow Re\{S(f)\}$
  $s_d(t) \leftrightarrow Im\{S(f)\}$

- Proprietà di Linearità ← Deriva dalla linearità dell'integrale.
  $s(t) = a\,s_1(t) + b\,s_2(t)$
  $S(f) = a\,S_1(f) + b\,S_2(f)$

- Fattore di Scalatura
  $s(at) \leftrightarrow \frac{1}{|a|} S\left(\frac{f}{a}\right)$

- Principio di Dualità
  $s(t) \leftrightarrow S(f)$
  $S(t) \leftrightarrow s(-f)$

- Traslazione Temporale
  $s(t - t_0) \leftrightarrow S(f) \cdot e^{-j 2\pi t_0 f}$

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- Traslazione in Frequenza
  $s(t)\, e^{-j2\pi f_0 t} \leftrightarrow S(f+f_0)$

- $\int_{-\infty}^{+\infty} s(t)\, dt = S(\emptyset)$ ← Per il calcolo dell'energia al posto di T. di Parseval.
  $\int_{-\infty}^{+\infty} S(f)\, df = s(\emptyset)$

- Derivazione nel tempo
  $s(t) \leftrightarrow S(f)$
  $\frac{d}{dt} s(t) \leftrightarrow j2\pi f \cdot S(f)$

- Derivazione in frequenza
  $s(t) \leftrightarrow S(f)$
  $-j2\pi t\, s(t) \leftrightarrow \frac{d}{df} S(f)$ ← utilizzata spesso nel verso ←

- $s(t) \leftrightarrow S(f)$
  $\int_{-\infty}^{t} s(\tau)\, d\tau \leftrightarrow \frac{1}{j2\pi f} \cdot S(f)$

- Prodotto di convoluzione
  $s(t) \leftrightarrow S(f)$, $z(t) \leftrightarrow Z(f)$
  $s(t) * z(t) = S(f) \cdot Z(f)$
  $s(t) \cdot z(t) = S(f) * Z(f)$

Esercizi

$s(t) = rect(t)$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty} rect(t)\, e^{-j2\pi f t}\, dt$$

$$= \int_{-\frac{1}{2}}^{\frac{1}{2}} 1 \cdot e^{-j2\pi f t}\, dt = \frac{1}{-j2\pi f} \cdot \left[e^{-j2\pi f t} \right]_{-\frac{1}{2}}^{\frac{1}{2}}$$

$$= +\frac{1}{j2\pi f} \cdot \left[-e^{-j\pi f} + e^{j\pi f} \right] = \frac{sen(\pi f)}{\pi f} = sinc(f)$$

$s(t) = rect(4t+1) = rect\left(4\left(t+\frac{1}{4}\right)\right)$

$S(f) = \frac{1}{4} sinc\left(\frac{f}{4}\right) e^{j\frac{f}{2}\pi}$

↑

$$\begin{cases} rect(4t) \longleftrightarrow \frac{1}{4} sinc\left(\frac{f}{4}\right) \\ rect\left(4\left(t+\frac{1}{4}\right)\right) \longleftrightarrow \frac{1}{4} sinc\left(\frac{f}{4}\right) e^{j2\pi f \cdot \frac{1}{4}} \end{cases}$$

Per casa:

$s(t) = tri(2t+3)$

$s(t) = sinc^2(t-1)\, e^{j4\pi t}$

$sinc^2(t-1) \longleftrightarrow tri(f)\, e^{-j2\pi f}$

$sinc^2(t-1)\, e^{j4\pi t} \longleftrightarrow tri(f-2)\, e^{-j2\pi(f-2)}$

$= tri(f-2)\, e^{-j2\pi f + 4j\pi}$

$= tri(f-2)\, e^{-j2\pi f} \cdot e^{4j\pi}$

$e^{j4\pi} = \underbrace{\cos(4\pi)}_{1} + j\underbrace{sen(4\pi)}_{0}$

$= 1$

$$s(t) = sinc(2t-4) \cdot \cos(400\pi t) = sinc(2t-4)\left[\frac{e^{j400\pi t} + e^{-j400\pi t}}{2}\right]$$

$sinc(2(t-2)) = \frac{1}{2} rect\left(\frac{f}{2}\right) e^{-j4\pi f}$

$$S(f) = \frac{1}{2}\left[\frac{1}{2} rect\left(\frac{f-200}{2}\right) e^{-j4\pi(f-200)} + \frac{1}{2} rect\left(\frac{f+200}{2}\right) e^{-j4\pi(f+200)} \right]$$

$e^{j800\pi} = 1$; $e^{-j800\pi} = 1$

$$= \frac{1}{4}\left[rect\left(\frac{f-200}{2}\right) e^{-j4\pi f} + rect\left(\frac{f+200}{2}\right) e^{-j4\pi f} \right]$$

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$s(t) = \text{sinc}(2t-4)\,\text{sen}(400\pi t)$

$$S(f) = \frac{1}{4j}\cdot\left[\text{rect}\left(\frac{f-200}{2}\right)e^{-j4\pi f} - \text{rect}\left(\frac{f+200}{2}\right)e^{-j4\pi f}\right]$$

Per casa:

$s(t) = \text{rect}\left(\frac{t+1}{3}\right)\text{sen}(2\pi f_0 t)$ Es. 07/01/02

Es. 16/04/04

$s(t) = \cos(2\pi f_0 t)\cdot\frac{d}{dt}\text{sinc}^2(3t)$

$\text{sinc}^2(3t) = \frac{1}{3}\,\text{tri}\left(\frac{f}{3}\right)$

$\frac{d}{dt}\text{sinc}^2(3t) = j2\pi f\cdot\frac{1}{3}\,\text{tri}\left(\frac{f}{3}\right)$

$$S(f) = \frac{1}{2}\left[\frac{2}{3}j\pi(f-f_0)\,\text{tri}\left(\frac{f-f_0}{3}\right) + \frac{2}{3}j\pi(f+f_0)\,\text{tri}\left(\frac{f-f_0}{3}\right)\right]$$

Es: $\int_{-\infty}^{+\infty}|s(t)|^2\,dt = \int_{-\infty}^{+\infty}|S(f)|^2\,df$ ← Parseval

$\int_{-\infty}^{+\infty}x(t)\,y^*(t)\,dt = \int_{-\infty}^{+\infty}X(f)\,Y^*(f)\,df$ ← Parseval generalizzato

Quando $y(t) \in \mathbb{R}$ $\Rightarrow$ $y(t) \to y^*(t)$

$y^*(t) \to Y(f)$

$Y(f) \to Y^*(f)$

Es. 24/04/02

$$\int_{-\infty}^{+\infty} e^{-j4\pi t}\,\text{sinc}^2(4t)\,dt$$

Attenzione: non uso Parseval!

$\to (\text{sinc}(4t)e^{-j2\pi t})^2 \neq |\text{sinc}(4t)e^{-j2\pi t}|^2 = \text{sinc}^2(4t)$!

Provo Parseval generalizzato:

$$= \int_{-\infty}^{+\infty} \underbrace{e^{-j4\pi t}\,\text{sinc}(4t)}_{x(t)}\cdot\underbrace{\text{sinc}(4t)}_{y^*(t)}\,dt$$

Oppure uso la proprietà $\int_{-\infty}^{+\infty}s(t)\,dt = S(0)$.

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$\rightarrow s(t) = e^{-j4\pi t}\, \mathrm{sinc}^2(4t)$

$S(f) = \frac{1}{4}\, tri\left(\frac{f+2}{4}\right)\Big|_{\phi} = \frac{1}{8}$

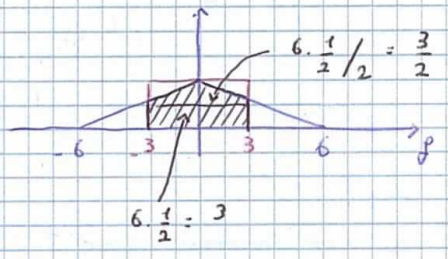
Es. 16/04/04

$\int_{-\infty}^{+\infty} \mathrm{sinc}^3(6t)\, dt = \int_{-\infty}^{+\infty} \underbrace{\mathrm{sinc}^2(6t)}_{x(t)} \cdot \underbrace{\mathrm{sinc}(6t)\, dt}_{y^*(t)}$

$= \int_{-\infty}^{+\infty} \frac{1}{6}\, tri\left(\frac{f}{6}\right) \cdot \frac{1}{6}\, rect\left(\frac{f}{6}\right) df$

$= \frac{1}{36} \int_{-3}^{3} tri\left(\frac{f}{6}\right) df$

$= \frac{1}{36} \cdot \frac{9}{2} = \frac{1}{8}$



Calcolare l'energia di

N.B: $|S(f)|^2 = S(f) \cdot S^*(f)$

$s(t) = \mathrm{sinc}(t-1)$

$Es = \int_{-\infty}^{+\infty} |s(t)|^2\, dt = \int_{-\infty}^{+\infty} \mathrm{sinc}^2(t-1)\, dt = \int_{-\infty}^{+\infty} |S(f)|^2\, df$

$\Rightarrow S(f) = rect(f)\, e^{-j2\pi f}$

$= \int_{-\infty}^{+\infty} rect(f)\, e^{-j2\pi f}\, rect(f)\, e^{j2\pi f}\, df$

$= \int_{-\infty}^{+\infty} rect^2(f)\, df = 1$

Oppure: $\mathrm{sinc}^2(t-1) \leftrightarrow S(f) = tri(f)\, e^{-j2\pi f}\Big|_{f=\phi} = 1$

Per casa:

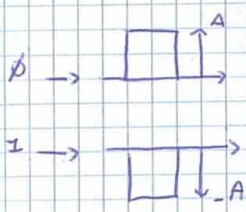
$s(t) = \mathrm{sinc}\left(\frac{t}{2} - 1\right) \cdot e^{j4\pi t}$ Es?

$\int_{-\infty}^{+\infty} s(t)\, dt = S(\phi)$

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30/03/2015





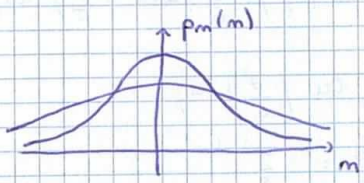
$\emptyset \to +A \to r = A + n$ $\qquad P_{e/\emptyset} = Q\left(\frac{A}{\sigma_n}\right)$

dipende da Pot. segnale

dipende da Pot. rumore

$E(n) = \emptyset$ gaussiana

↑ l'errore è gaussiano!

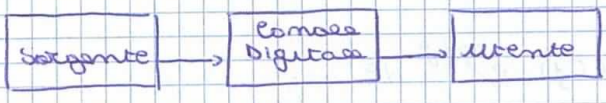


Quindi nel nostro caso: $\sigma_m^2 = E(n^2)$

$1 \to -A \to r = -A + n$ $\qquad P_{e/1} = P_{e/\emptyset}$

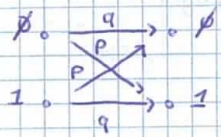
$P_e = P_{e/\emptyset} \cdot p(\emptyset) + P_{e/1} \cdot p(1) = P_{e/\emptyset} = P_{e/1} = Q\left(\frac{A}{\sigma_n}\right)$

Con $p(\emptyset)$ e $p(1)$ probabilità a priori: prob. con cui vengono generati dalla sorgente $\emptyset$ e 1.



$\begin{cases} P_{e/\emptyset} \\ P_{e/1} \end{cases}$

Traduzione grafica dello schema:

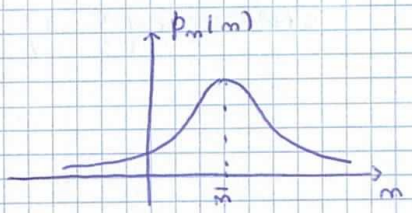


$\Rightarrow$ con $p = Q\left(\frac{A}{\sigma_n}\right)$ quando $P_{e/\emptyset} = P_{e/1} = p$.

Prendiamo una gaussiana:

$\bar{m}$

σ_m^2 $\qquad p_m(m) = \frac{1}{\sqrt{2\pi\sigma_m^2}} \cdot e^{-(m-\bar{m})^2/2\sigma_m^2}$



$P_m(m) = \int_{-\infty}^{m} p_m(m)\,dm = \frac{1}{\sqrt{2\pi\sigma_m^2}} \cdot \int_{-\infty}^{m} e^{-(m-\bar{m})^2/2\sigma_m^2}$

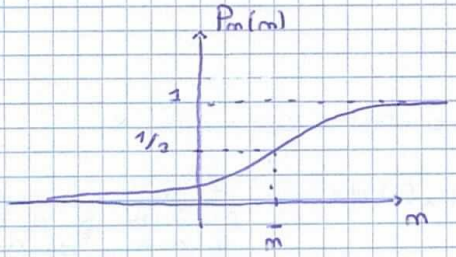
$\to t = \frac{m-\bar{m}}{\sigma_m}$

$= \frac{1}{\sqrt{2\pi}} \cdot \int_{-\infty}^{\frac{m-\bar{m}}{\sigma_m}} e^{-t^2/2}\,dt$

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$$\rightarrow Q(x) = \frac{1}{\sqrt{2\pi}} \cdot \int_{y}^{+\infty} e^{-y^2/2}\, dy$$

$$= \underbrace{\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-t^2/2}\, dt}_{1} - \frac{1}{\sqrt{2\pi}} \int_{\frac{m-\bar{m}}{\sigma_m}}^{+\infty} e^{-t^2/2}\, dt = 1 - Q\left[\frac{m-\bar{m}}{\sigma_m}\right]$$



Esercizio: calcolo $E(m)$

$$p_m(m) = \frac{1}{\sqrt{2\pi\sigma_m^2}}\, e^{-(m-\bar{m})^2/2\sigma_m^2}$$

$$E(m) = \int_{-\infty}^{+\infty} m\, p_m(m)\, dm = \int_{-\infty}^{+\infty} (m - \bar{m} + \bar{m})\, p_m(m)\, dm$$

$$= \int_{-\infty}^{+\infty} (m-\bar{m}) \cdot \frac{1}{\sqrt{2\pi\sigma_m^2}} \cdot e^{-(m-\bar{m})^2/2\sigma_m^2}\, dm + \int_{-\infty}^{+\infty} \bar{m}\, \frac{1}{\sqrt{2\pi}}\, e^{-(m-\bar{m})^2/2\sigma_m^2}\, dm$$

$$= \int_{-\infty}^{+\infty} t \cdot \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}\, dt + \bar{m} \int_{-\infty}^{+\infty} \frac{1}{\sqrt{2\pi}}\, e^{-t^2/2}\, dt = \bar{m} \cdot 1$$

con $t = \frac{m-\bar{m}}{\sigma_m}$

Prodotto funz. pari · funz. dispari = 0

Prendiamo una gaussiana con v. medio $= \bar{x}$ e σ_m^2.

$$Pr\left\{\bar{x} - k\sigma_m \leq X \leq \bar{x} + k\sigma_m\right\} = Pr\left\{X \leq \bar{x} + k\sigma_m\right\} - Pr\left\{X \leq \bar{x} - k\sigma_m\right\}$$

$$= \int_{-\infty}^{\bar{x}+k\sigma_m} p_x(x)\, dx - \int_{-\infty}^{\bar{x}-k\sigma_m} p_x(x)\, dx$$

$$\int_{-\infty}^{\bar{x}+k\sigma_m} p_x(x)\, dx = \int_{-\infty}^{\bar{x}+k\sigma_m} \frac{1}{\sqrt{2\pi\sigma_m^2}}\, e^{-(x-\bar{x})^2/2\sigma_m^2}\, dx$$

$$\rightarrow t = \frac{x-\bar{x}}{\sigma_m} = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\frac{k\sigma_m}{\sigma_m}} e^{-t^2/2}\, dt = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{+\infty} e^{-t^2/2}\, dt - \frac{1}{\sqrt{2\pi}} \int_{k}^{+\infty} e^{-t^2/2}\, dt$$

$$= 1 - Q(k)$$

$$\int_{-\infty}^{\bar{x}-k\sigma_m} p_x(x)\, dx = 1 - Q(-k)$$

Il 99,7% dei valori cadono in questa area

$$\rightarrow Pr\left\{\bar{x} - k\sigma_m \leq X \leq \bar{x} + k\sigma_m\right\} = 1 - Q(k) - 1 + Q(-k) = Q(-k) - Q(k) = 0,997$$

con $k = 3$

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Se io ho n variabili aleatorie, indipendenti e con la stessa densità di probabilità

$n_1, n_2, \dots, n_N$

$x = \sum_{i=1}^{N} n_i$. Per $N \to \infty$, x è una gaussiana

con $\bar{x} = \sum_{i=1}^{N} \bar{n}_i$, $\sigma_x^2 = \sum_{i=1}^{N} \sigma_{n_i}^2$

↑ Teorema del limite centrale

Funzione caratteristica della variabile casuale x.

$$\phi_x(f) = E\left\{ e^{j2\pi f x} \right\} = \int_{-\infty}^{+\infty} e^{j2\pi f x} p_x(x)\, dx$$

La funzione caratt. è la trasf. di Fourier a meno di un segno ⊖ di $p_x(x)$.

Prendiamo:

X, Y , $p_X(x)$, $p_Y(y)$ $\to$ $Z = X + Y$, $p_Z(z)$?

$z = x + y$ $\to$ fissato x , $y = z - x$

$p_x(x)$ $\quad$ $p_y(y = z - x)$.

$$p_z(z/x) = p_x(x)\, p_y(z - x)$$

$$p_z(z) = \int_{-\infty}^{+\infty} p_x(x)\, p_y(z - x)\, dx = p_x(x) * p_y(y)$$

Se definisco $\phi_x(f)$ $\phi_y(f)$ $\to$ $\phi_z(f) = \phi_x(f) \cdot \phi_y(f)$

$p_z(z) \longleftrightarrow \phi_z(f)$

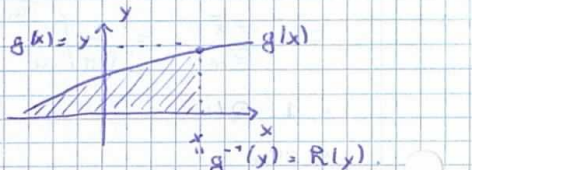
Prendiamo: x v.a. con $p_x(x)$

$y = g(x)$ continua invertibile

$x = g^{-1}(y) = R(y)$.

x → [sistema] → $y = g(x)$

$p_x(x)$ $\quad$ $p_y(y)$



1° caso)

$g(x)$ funzione monotona crescente

$$P_y(y) = P_r\{Y \le y\} = P_r\{X \le x = R(y)\}$$

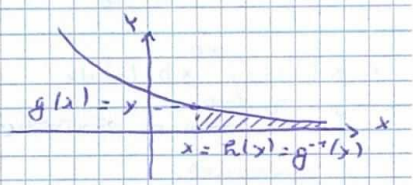
$$= \int_{-\infty}^{R(y)} p_x(x)\, dx$$

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$$p_Y(y) = \frac{dP_Y(y)}{dy} = p_X(x = R(y)) \cdot \frac{dR(y)}{dy}$$

2° caso)

$g(x)$ funzione monotona decrescente



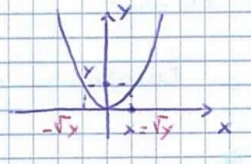
$$P\{Y \le y\} = P_r\{Y \le y\} = P_r\{x \ge R(y)\}$$

$$= \int_{R(y)}^{+\infty} p_X(x)\,dx$$

$$p_Y(y) = \frac{dP_Y(y)}{dy} = -p_X(x = R(y)) \cdot \frac{dR(y)}{dy}$$

Prendiamo una funzione:

$y = x^2$, x v.a. gaussiana con $\bar{x} = 0$, σ_x^2



$$P_Y(y) = P_r\{Y \le y\}$$
$$= P_r\{-\sqrt{y} \le X \le \sqrt{y}\}$$
$$= P_r\{X \le \sqrt{y}\} - P_r\{X \le -\sqrt{y}\}$$
$$= P_X(\sqrt{y}) - P_X(-\sqrt{y})$$

$$P_X(\sqrt{y}) = \int_{-\infty}^{\sqrt{y}} p_X(x)\,dx = \sqrt{\cdot} \int_{-\infty}^{\sqrt{y}} \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-t/2\sigma_x^2} \, d\left(\frac{1}{2\sqrt{y}}\right)$$

con $t = \sqrt{y}$, $dt = \frac{1}{2\sqrt{y}}\,dy$

$$= \frac{1}{2\sqrt{y}} \int_{-\infty}^{\sqrt{y}} e^{-t/2\sigma_x^2}\,dt = \frac{1}{2\sqrt{y}} \frac{1}{-2\sigma_x^2} e^{-\sqrt{y}/2\sigma_x^2}$$

$$p_Y(y) = p_X(R(y)) \frac{d}{dy} R(y) = \int_{-\infty}^{R(y)} p_X(x)\,dx$$

Quindi: $p_Y(y) = p_X(x = R(y)) \cdot \frac{d}{dy} R(y)$ per $x \ge 0$

con $R(y) = \sqrt{y}$

$$p_Y(y) = \frac{1}{\sqrt{2\pi\sigma_x^2}} \cdot e^{-y/2\sigma_x^2} \cdot \frac{1}{2\sqrt{y}} \quad p$$

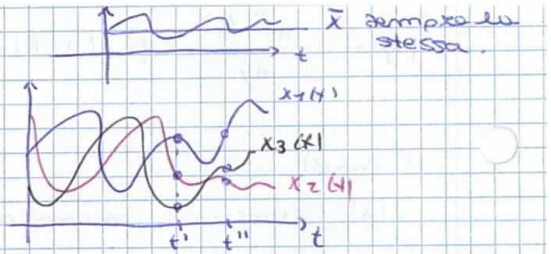
$$p_Y(y) = -p_X(x) \frac{d}{dy} R(y) \quad \text{per } x \le 0$$

$$= \frac{1}{\sqrt{2\pi\sigma_x^2}} e^{-y/2\sigma x^2} \cdot \frac{1}{2\sqrt{y}}$$

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media temporale

$x(t) \rightarrow \bar{x} = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\,dt$



$x(t) \neq E(x) = \int_{-\infty}^{+\infty} x\, p_x(x)\,dx$

media statistica.

in generale.

Generalmente $E(x(t')) \neq E(x(t''))$.

Quando la media temporale e la media statistica coincidono:

$x(t) = E(x)$ → Processo Ergodico.

Processo Stazionario in senso stretto

$\bar{x}$, $\overline{x^2}$, ..., $(x-\bar{x})^2$ non dipendono dagli istanti k effettuate!

Correlazione:

data $x(t)$ si def. correlazione mediante tempo:

$$R_x(\tau) = \int_{-\infty}^{+\infty} x(t)\, x(t+\tau)\,dt$$ ← Funzione di autocorrelazione.

Proprietà:

$$R_x(-\tau) = \int_{-\infty}^{+\infty} x(t)\, x(t-\tau)\,dt \quad (t_1 = t-\tau) = \int_{-\infty}^{+\infty} x(t_1+\tau)\, x(t_1)\,dt_1 = R_x(\tau).$$

↑ Funzione Pari: $R_x(-\tau) = R_x(\tau)$.

Se $R_x(\tau) = 0$ significa che $x(t+\tau)$ non è legato a $x(t)$.

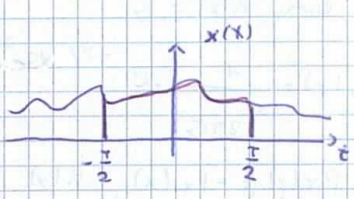
$$R_x(0) = \int_{-\infty}^{+\infty} x^2(t)\,dt = E_x$$

$$\int_{-\infty}^{+\infty} x^2(t)\,dt = \int_{-\infty}^{+\infty} |S(f)|^2\,df$$, $x(t)$ segnale deterministico.

Segnale casuale → $E_x = \infty$.

Prendiamo $x(t)$ segnale aleatorio

→ Costruisco $x_T(t) = x(t)\,\mathrm{rect}\left(\frac{t}{T}\right)$



$T \to \infty$, $x_T(t) \to \infty$

$$\bar{x} = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x_T(t)\,dt$$

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$$x_T(t) \to X_T(f) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x_T(t)\, e^{-\pi 2 n f t}\, dt$$

$$\lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |x_T(t)|^2 dt = \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{+\infty} |X_T(f)|^2 df$$

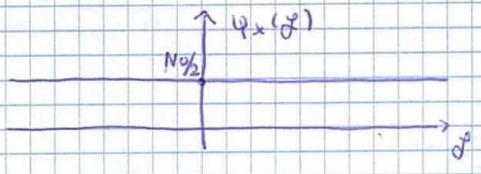
↑ Teorema di Parseval per segnali casuali.

$\lim_{T\to\infty} \frac{1}{T} |X_T(f)|^2 = \psi_x(f)$ = densità spettrale di potenza di $x(t)$.

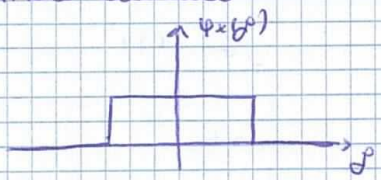
↑ Indica come la potenza è distribuita in frequenza.

$\psi_x(f) \neq \emptyset$, $\psi_x(f) = \psi_x(-f)$

Rumore con $\psi_x(f)$ costante : Rumore Bianco.



Rumore colorato :



L'antitrasformata di una costante è un δ.

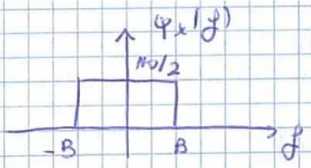
Se $R_x(\tau) = \emptyset$ posso passare istantaneamente da $f = +\infty$ a $f = -\infty$ → i valori non sono legati fra loro!

Rumore termico → Rumore Bianco e Gaussiano.

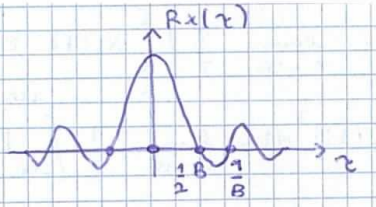
Rumore Gaussiano → Info sulla probabilità di uscita dei valori.

Rumore Bianco → Info in frequenza come sono distribuite le frequenze. => fra $x(t)$ e $x(t+\tau)$ non c'è legame.

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$\Rightarrow$



$\tau = \frac{1}{2}B \Rightarrow x(t+\tau)$ non dipende da $x(t)$.

Quando non ho τ cioè campionamento $\rightarrow R_x(\tau) = \not{0}$!

Il numero dell'istante $n+1$ non dipende dal numero dell'istante n.

Esercizi Esame 31/03/2015

$x(t) \rightarrow$ [LTI] $\rightarrow y(t) = ? = x(t) * R_i(t)$

$\delta(t) \rightarrow R_i(t) \rightarrow R_i(t)$

Un sistema LTI è caratterizzato da una certa risposta impulsiva $R_i(t)$.

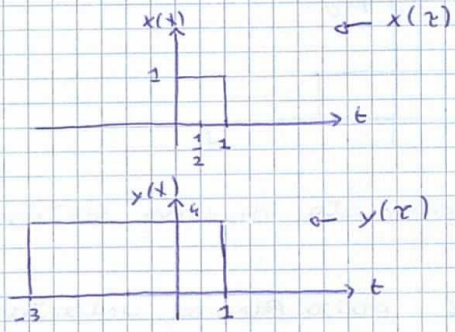
$Y(f) = X(f) \cdot H(f)$

$$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, R_i(t-\tau)\, d\tau$$

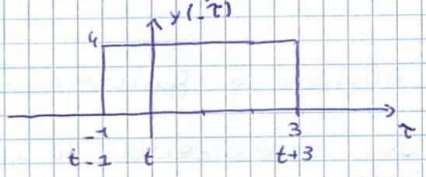
$x(t) = \text{rect}\left(t - \frac{1}{2}\right)$

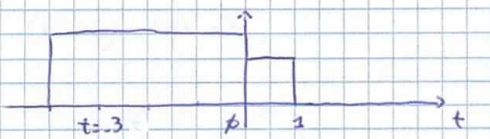
$y(t) = 4\,\text{rect}\left(\frac{t+1}{4}\right)$

$s(t) = x(t) * y(t)$

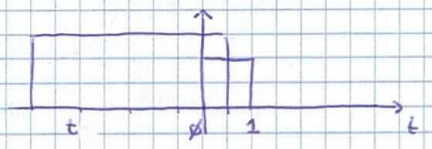


Io devo valutare $y(-\tau)$



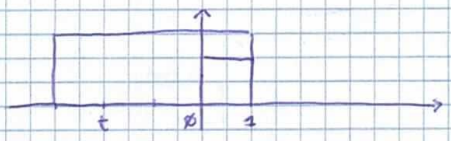


$t+3 < 0 \quad , t < -3 \Rightarrow s(t) = 0$



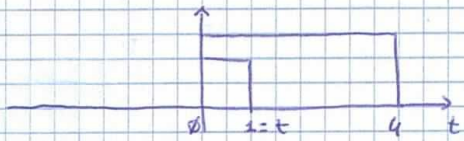
$$s(t) = \int_0^{t+3} 1 \cdot 4 \, d\tau = 4\tau \Big|_0^{t+3} = 4(t+3)$$

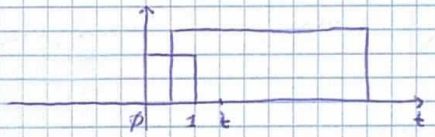
$-3 < t < -2$



$$s(t) = \int_0^1 1 \cdot 4 \, d\tau = 4$$

$-2 < t < 1$

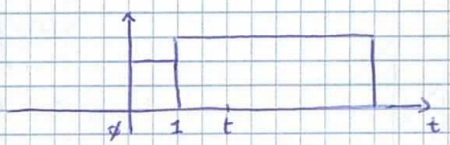




$$s(t) = \int_{t-1}^1 1 \cdot 4 \, d\tau = 4\tau \Big|_{t-1}^1$$

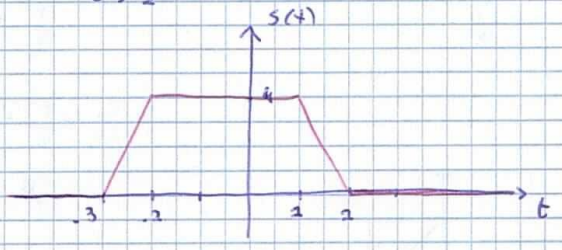
$= 4 - 4(t-1) = 8 - 4t$

$1 < t < 2$



$s(t) = 0 \quad , t > 2$

$$s(t) = \begin{cases} 0 & t \leq -3 \\ 4(t+3) & -3 < t \leq -2 \\ 4 & -2 < t \leq 1 \\ 4(-t+2) & 1 < t \leq 2 \\ 0 & t > 2 \end{cases}$$

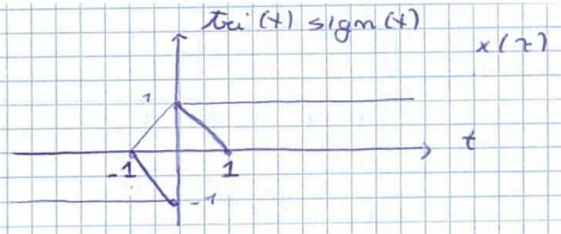


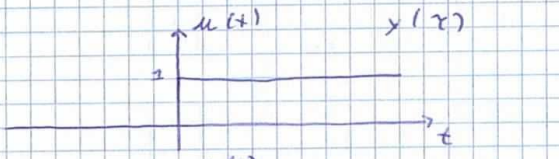
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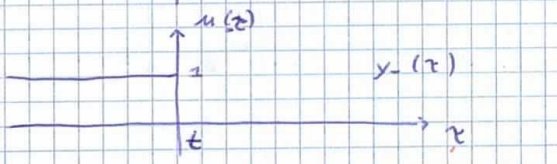
Convoluzione fra:

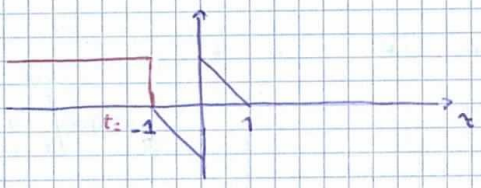
$x(t) = tri(t)\ sign(t)$

$y(t) = u(t)$

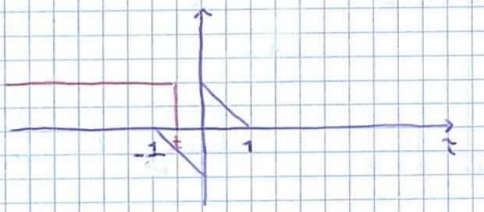








$s(t) = \emptyset \quad , \quad t \leq -1$

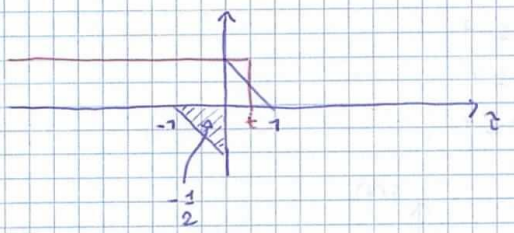


$$s(t) = \int_{-1}^{t} 1 \cdot (1+\tau)\, d\tau$$

$$= -\tau - \frac{\tau^2}{2}\Big|_{-1}^{t} = -t - \frac{t^2}{2} - \frac{1}{2}$$

$$= -\frac{t^2}{2} - t - \frac{1}{2}$$

$$-1 < t \leq \emptyset$$

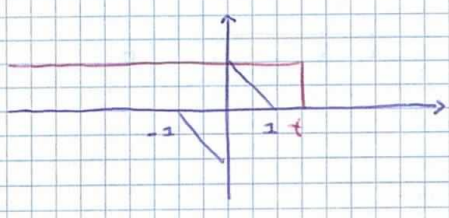


$$s(t) = \int_{\emptyset}^{t} 1 \cdot (-\tau + 1)\, d\tau - \frac{1}{2}$$

$$= -\frac{\tau^2}{2} + \tau\Big|_{\emptyset}^{t} - \frac{1}{2}$$

$$= -\frac{t^2}{2} + t - \frac{1}{2}$$

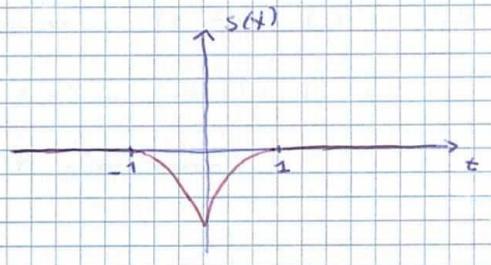
$$\emptyset < t \leq 1$$



$s(t) = \emptyset \quad t > 1$

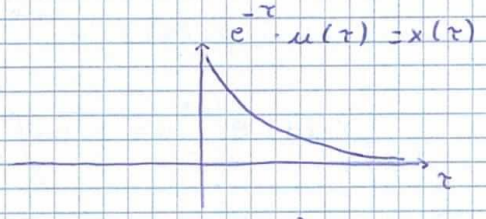
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$$s(t) = \begin{cases} 0 & t \le -1 \\ -\frac{t^2}{2} - t - \frac{1}{2} & -1 < t \le 0 \\ -\frac{t^2}{2} + t - \frac{1}{2} & 0 < t \le 1 \\ 0 & t > 1 \end{cases}$$



Per casa: $s(t) = rect(t) * tri(t)$

$s(t) = e^{-t}u(t) * e^{-t}u(t)$



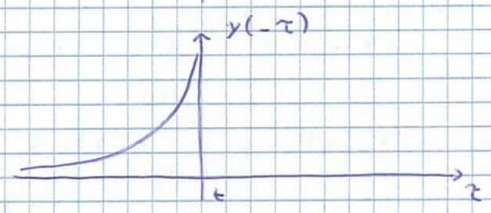
$s(t) = 0 \quad , t \le 0$

$s(t) \neq 0 \quad , t > 0$

$$s(t) = \int_0^t e^{-\tau} e^{-(t-\tau)} d\tau$$

$$= e^{-t} \int_0^t 1 \, d\tau$$

$$= t e^{-\tau}$$



$$s(t) = \begin{cases} 0 & t \le 0 \\ t e^{-t} & t > 0 \end{cases}$$

Sistemi LTI

Quando un sistema è LTI?

$T[x(t)] = y(t)$, T trasformazione.

$T[a_1 x_1(t) + a_2 x_2(t)] = a_1 y_1(t) + a_2 x_2(t)$.

$T[x(t-t_0)] = y(t-t_0)$.

Dobbiamo:

$y(t) = K x(t) u(t-\tau)$ è LTI?

$K \cdot (a_1 x_1(t) + a_2 x_2(t)) \cdot u(t-\tau)$

$= K a_1 x_1(t) \cdot u(t-\tau) + K a_2 x_2(t) \cdot u(t-\tau)$

$= a_1 y_1(t) + a_2 y_2(t)$ è lineare!

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$K\, x(t-t_0)\, u(t-\tau)$ quando $x(t) = x(t-t_0)$

$y(t-t_0) = K\, x(t-t_0)\, u(t-t_0-\tau)$ non è Tempo Invariante!

L non non Ti

$y(t) = \int_{-\infty}^{+\infty} x(\tau)\, e^{t+\tau}\, d\tau$ è Lti?

$\rightarrow \int_{-\infty}^{+\infty} (a_1 x_1(\tau) + a_2 x_2(\tau)) \cdot e^{t+\tau}\, d\tau = a_1 y_1(t) + a_2 y_2(t)$

$= a_1 \int_{-\infty}^{+\infty} x_1(\tau)\, e^{t+\tau}\, d\tau + a_2 \int_{-\infty}^{+\infty} x_2(\tau)\, e^{t+\tau}\, d\tau$ È lineare.

$\rightarrow \int_{-\infty}^{+\infty} x(\tau - t_0)\, e^{t+\tau}\, d\tau$ con $x(\tau - t_0)$ invece di $x(\tau)$.

$\stackrel{?}{=} y(t-t_0) = \int_{-\infty}^{+\infty} x(\tau)\, e^{t+\tau-t_0}\, d\tau$

$K = \tau - t_0$

$\Rightarrow \int_{-\infty}^{+\infty} x(K)\, e^{t+K+t_0}\, dK$ non è Ti

Sistemi Lti:

- $y(t) = K \cdot x(t)$, $R(t) = K\,\delta(t)$, $H(f) = K$
- $y(t) = x(t-t_0)$, $R(t) = \delta(t-t_0)$, $H(f) = e^{-j2\pi f t_0}$
- $y(t) = \int_{-\infty}^{t} x(\tau)\, d\tau$
- $y(t) = \frac{d}{dt} x(t)$
- Trasformata di Hilbert:

  $y(t) = x(t) * \frac{1}{\pi t}$, $R(t) = \frac{1}{\pi t}$, $H(f) = -j\,\mathrm{sign}(f)$

  ↳ sfasa di segnale di $\frac{\pi}{2}$.

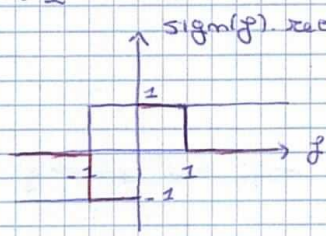
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$s(t) = \text{sinc}(2t)$ Fai la Trasformata di Hilbert

$\hat{s}(t) = \text{sinc}(2t) * \frac{1}{\pi t}$

$S(f) = \frac{1}{2}\,\text{rect}\left(\frac{f}{2}\right)$

$\hat{S}(f) = \frac{1}{2}\,\text{rect}\left(\frac{f}{2}\right) \cdot \left(-j\,\text{segn}(f)\right) = -\frac{j}{2}\,\text{sign}(f)\,\text{rect}\left(\frac{f}{2}\right)$



$= \text{rect}\left(f - \frac{1}{2}\right) - \text{rect}\left(f + \frac{1}{2}\right)$

$\hat{S}(f) = \frac{1}{2j}\left[\text{rect}\left(f - \frac{1}{2}\right) - \text{rect}\left(f + \frac{1}{2}\right)\right]$

$\Downarrow$

$\hat{s}(t) = \text{sinc}(t) \cdot \text{sen}\left(2\pi t \cdot \frac{1}{2}\right) = \text{sinc}(t) \cdot \text{sen}(\pi t)$

$s(t) = \text{sinc}(t) \cdot \cos(\pi t)$

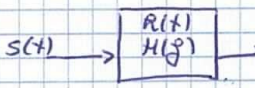
$S(f) = \frac{1}{2}\left[\text{rect}\left(f - \frac{1}{2}\right) + \text{rect}\left(f + \frac{1}{2}\right)\right]$

09/04/2015

$s(t) \quad S(f)$

$$E_s = \int_{-\infty}^{+\infty} |s(t)|^2\,dt = \int_{-\infty}^{+\infty} |S(f)|^2\,df$$

$|S(f)^2| = \Psi_s(f)$



$Y(f) = H(f) \cdot S(f)$

$\Psi_y(f) = |H(f)|^2 \cdot \Psi_s(f)$

Se $s(t) \to \int_{-\infty}^{+\infty} |s(t)|^2\,dt < \infty$ non è sempre, $S(f) = ?$

allora $s(t) \to s_T(t) = s(t)\,\text{rect}\left(\frac{t}{T}\right)$

con $P_s = \lim_{T\to\infty} \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} |s_T(t)|^2\,dt = \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{+\infty} |S_T(f)|^2\,df.$

$\Psi_s(f) = \lim_{T\to\infty} \frac{1}{T}|S_T(f)|^2$, densità spettrale di potenza.

PROPRIETÀ:

$\Psi_s(f) = \Psi_s(-f)$

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$\psi_s(f) = ?$

$R_s(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} s(t)\, s(t+\tau)\, dt \longrightarrow \psi_s(f)$

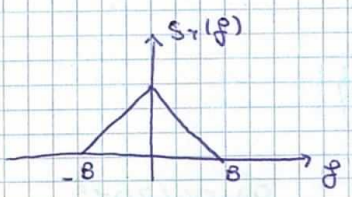
↑ Teorema di Wiener - Khintchine

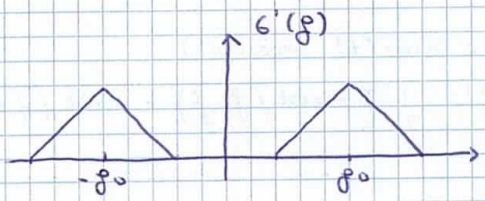
$g(t) = s(t) \cdot \cos(2\pi f_0 t)$, $s(t)$ non ha energia finita.

$s_T(t) = s(t)\, rect\left(\frac{t}{T}\right)$

$g'(t) = s(t)\, rect\left(\frac{t}{T}\right) \cdot \cos(2\pi f_0 t)$

$G'(f) = \frac{1}{2}\left[S_T(f+f_0) + S_T(f-f_0) \right]$



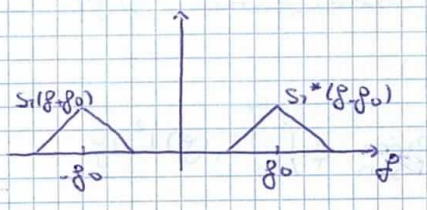


$\psi_{g'}(f) = |G'(f)|^2 = \frac{1}{4}\left\{ \left[S_T(f+f_0) + S_T(f-f_0) \right] \cdot \left[S_T(f+f_0)^* + S_T(f-f_0)^* \right] \right\}$

↑ $G'(f) \cdot G'(f)^*$

$= \frac{1}{4}\left\{ S_T(f+f_0)\, S_T^*(f+f_0) + S_T(f+f_0)\, S_T^*(f-f_0) + S_T(f-f_0)\, S_T^*(f+f_0) + S_T(f-f_0)\, S_T^*(f-f_0) \right\}$

$= \frac{1}{4}\left\{ |S_T(f+f_0)|^2 + |S_T(f-f_0)|^2 \right\}$



$= \frac{1}{4}\left\{ \psi_s(f+f_0) + \psi_s(f-f_0) \right\}$

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$$x(t) \to \bar{x} = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\,dt$$

$$g(x) \to \overline{g(x)} = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} g(x)\,dt$$

$$R_x(\tau) = \lim_{T\to\infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t)\,x(t+\tau)\,dt$$

$$\updownarrow$$

$$\Psi_x(f) = \lim_{T\to\infty} \frac{1}{T} \left| X_T(f) \right|^2$$

$$E(x(t)) = \int_{-\infty}^{+\infty} x\, p_x(x)\,dx$$

$$R_x(\tau) = E\left[x(t)\,x(t+\tau)\right] = \int_{-\infty}^{+\infty} x(t)\,x(t+\tau)\,p_x(x(t), x(t+\tau))\,dx$$

Se $R_x(\tau) = \phi$ non c'è ~~correlazione~~ fra $x(t)$ e $x(t+\tau)$.

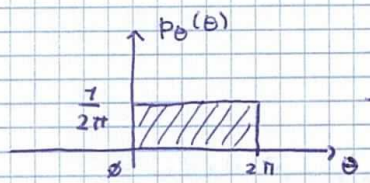
- Processo Stazionario in senso stretto
- " " " " lato $\bar{x}$, $R_x(\tau)$

Prendiamo un segnale:

$x(t) = V_0 \cdot \cos(2\pi f_0 t + \Theta)$, $\phi \le \phi \le 2\pi$ distribuita uniformemente fra $\phi \div 2\pi$.

$$R_x(\tau) = E\left\{ V_0 \cdot \cos(2\pi f_0 t + \Theta)\, V_0 \cos(2\pi f_0 (t+\tau) + \Theta) \right\}$$

$$= \int_{\phi}^{2\pi} V_0^2 \cos(2\pi f_0 t + \Theta) \cos\left[2\pi f_0 (t+\tau) + \Theta\right] p_\Theta(\theta)\,d\theta$$



$$\to \int_{\phi}^{2\pi} p_\Theta(\theta) = 1$$

$$= \frac{V_0^2}{2\pi} \int_{\phi}^{2\pi} \cos(2\pi f_0 t + \Theta) \cos\left[2\pi f_0 (t+\tau) + \Theta\right] d\theta$$

$$= \frac{V_0^2}{4\pi} \int_{\phi}^{2\pi} \left\{ \cos(2\pi f_0 \tau) + \cos\left[4\pi f_0 t + 2\pi f_0 t + 2\theta\right] \right\} d\theta$$

Valor medio nullo!

$$= \frac{V_0^2}{4\pi} \left\{ \cos(2\pi f_0 \tau) \int_{\phi}^{2\pi} d\theta + \int_{\phi}^{2\pi} \cos\left[4\pi f_0 t + 2\pi f_0 t + 2\theta\right] d\theta \right\}$$

$$= V_0^2 \cos(2\pi f_0 \tau) \cdot 2\pi + \frac{V_0^2}{4\pi}(\phi) \to \frac{V_0^2}{2} \cos(2\pi f_0 \tau) \longrightarrow \Psi_x(f)$$

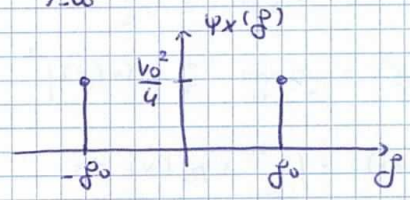
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$$\varphi_x(f) = \int_{-\infty}^{+\infty} \frac{V_0^2}{2} \cdot \cos(2\pi f_0 \tau)\, e^{-j2\pi f \tau}\, d\tau$$

$$= \frac{V_0^2}{2} \int_{-\infty}^{+\infty} \frac{e^{j2\pi f_0 \tau} + e^{-j2\pi f_0 \tau}}{2} \cdot e^{-j2\pi f \tau}\, d\tau$$

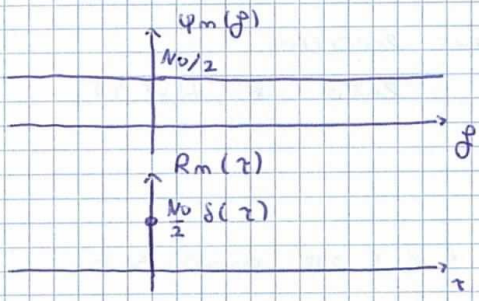
$$\varphi_x(f) = \frac{V_0^2}{4} \left\{ \int_{-\infty}^{+\infty} 1 \cdot e^{-j2\pi (f - f_0)\tau}\, d\tau + \int_{-\infty}^{+\infty} e^{-j2\pi (f + f_0)\tau}\, d\tau \right\}$$

$$= \frac{V_0^2}{4} \left\{ \delta(f + f_0) + \delta(f - f_0) \right\}$$

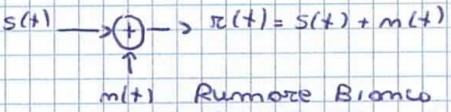


$$P_x = \frac{V_0^2}{4} + \frac{V_0^2}{4} = \frac{V_0^2}{2}.$$

Cambiando argomento:



← Rumore Bianco



Rumore AWGN: Additive White Gaussian Noise

spesso $\overline{n} = 0$.

$x(t)$ $R_x(\tau)$

$x(t), y(t)$

$$P_{x,y}(x,y) = Pr\{X \le x, Y \le y\}$$

$$p_{x,y}(x,y) = \frac{\partial^2}{\partial x \partial y} P_{x,y}(x,y) \quad , \quad \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} p_{x,y}(x,y)\, dx\, dy = 1.$$

$$\int_{-\infty}^{+\infty} p_{x,y}(x,y)\, dx = p_y(y).$$

Se x, y indipendenti (statisticamente):

$$p_{x,y}(x,y) = p(x) \cdot p(y).$$

$$P_{x/y} = P(x/y) \quad , \quad P_{x/y}(x/y)\, P_y(y) = P_{y/x}(y/x)\, P_x(x) = P_{x/y}(x/y).$$

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$$R_{x,y}(\tau) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) y(t+\tau)\, dt$$

$$\bar{E}\{x(t)\,y(t+\tau)\} = \iint_{-\infty}^{+\infty} x(t)\,y(t+\tau)\, p_{x,y}[x(t), y(t+\tau)]\, dx\, dy$$

Potenza P Watt

$1\ W \to 10 \log_{10} \frac{1}{1W} = 0\ dB\ W$

$10\ W \to 10 \log_{10} \frac{10}{1W} = 10\ dB\ W$

$100\ W \to 10 \log_{10} 100 = 20\ dB\ W$

$1000\ W \to 10 \log_{10} 1000 = 30\ dB\ W$

$0{,}1\ W \to 10 \log_{10} 0{,}1\ W = -10\ dB\ W$

$20\ W = 2 \cdot 10\ W \to 10 \log_{10} 2 + 10 \log_{10} 10 = 13\ dB\ W$

$4\ W \to 6\ dB\ W$

$2\ W \to 10 \log_{10} 2 = 3\ dB\ W.$

$1\ mW \to -30\ dB\ W$

$dB\ mW = dB_m \quad , P(mW) \to 10 \log_{10} \frac{P(mW)}{1\ mW}$

$1\ mW \to 0\ dB_m = -30\ dB\ W$

$10\ mW \to 10\ dB_m = -20\ dB\ W$

Trasm. $\longrightarrow$ [Canale] $\longrightarrow$ Ricev. $\qquad P_R < P_T$

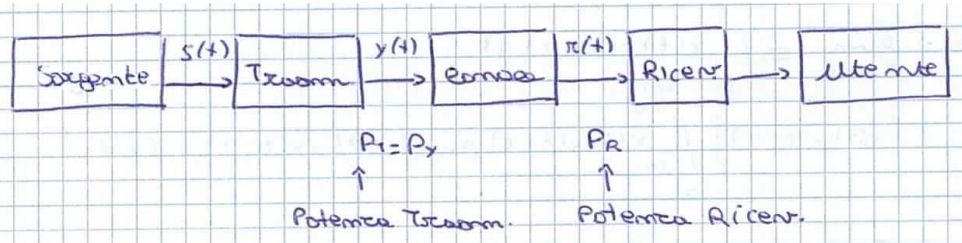
$P_T(W) \qquad P_R(W)$

$$A = \frac{P_R}{P_T} \to 10 \log_{10} \frac{P_T}{P_R} = 10 \log_{10} P_T - 10 \log_{10} P_R$$

$$A(dB) = P_T(dBW) - P_R(dBW)$$

$A(dB) = 10 \to P_T$ ha un ordine di grandezza superiore a P_R.

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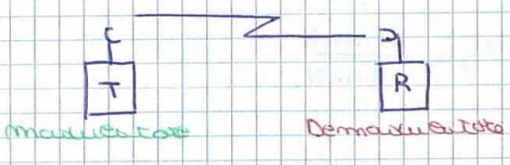


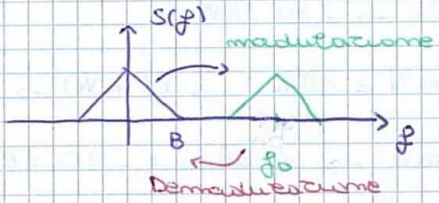
Sensibilità: Potenza minima della quale non posso andare.

Canale di Comunicazione: { Attenuazione (dB); Banda di Trasmissione B_T; Disturbi (Rumore, interferenza, ...) }

- Canali radio (Wireless)
- Canali Cablati: • Cavi coassiali;
  • Doppino Telefonico;
  • Fibre ottiche;

Trasmissione radio





Frequenze → Bande di Trasmissione	Frequenze	Lunghezza d'onda
ELF (Extremely Low Frequency)	30 ÷ 300 Hz	10.000 ÷ 1.000 Km
VF (Voice Frequency)	300 ÷ 3000 Hz	1000 ÷ 100 Km
VLF (Very Low Frequency)	3 ÷ 30 KHz	100 ÷ 10 Km
LF (Low Frequency)	30 ÷ 300 KHz	10 ÷ 1 Km
MF (Medium //)	3000 ÷ 3000 KHz	1000 ÷ 100 m
HF (High //)	3 ÷ 30 MHz	100 ÷ 10 m
VHF (Very High //)	30 ÷ 300 MHz	10 ÷ 1 m
UHF (Ultra High //)	300 ÷ 3000 MHz	1000 ÷ 10 mm
SHF (Super High //)	3 ÷ 30 GHz	10 ÷ 1 mm
EHF (Extremely High //)	30 ÷ 300 GHz	1 ÷ 0 mm

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Ci sono sempre delle attenuazioni nella trasmissione:

- attenuazione di spazio libero

$$L_s(f) = Loss = \left(\frac{4\pi f D}{c}\right)^2$$

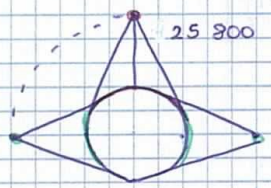
$c = 3 \cdot 10^8$ m/s

f: frequenza in Hz

D: distanza in m

lunghezza d'onda: $\lambda = \frac{c}{f}$.

Satelliti



Posto a 125 800 km così resta della stessa velocità della Terra.

3 satelliti per tutta la Terra posti a 120° di distanza.

$L_s(f) = 6.175$ GHz

$L_s(f) \cong -200$ dB

$$L_s(f)_{dB} = 10 \log_{10}\left[\frac{4\pi f D}{c}\right]^2$$

$$= 20\log_{10} 4\pi + 20\log_{10} f + 20\log_{10} D - 20\log_{10} c$$

$$= 21,97 - 9,54 - 160 + 20\log_{10} f_{Hz} + 20\log_{10} D_m \Big| - 20\log_{10} 3 - 20\log_{10} 10^8$$

$$= -147,57 + \dots$$

$f = 10$ MHz, $D = \dots$

$$= -147,57 + 20\log_{10}(10 \cdot 10^6) + 20\log_{10} D$$

$$= -147,57 + 20\log_{10}(10\ MHz) + 20\log_{10}(10^6) + 20\log_{10} D(m)$$

$$= -147,57 + 120 + 20\log_{10}(f\ MHz) + 20\log_{10} D(m)$$

$$= 32,4 + 20\log_{10}\left(f/_{MHz}\right) + 20\log_{10}\left(d/_{km}\right)$$

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Esercizi Esame. 13/04/2015

$x(t) \rightarrow \boxed{LTI} \rightarrow y(t)$

$x(t) = 8\,\mathrm{sinc}^2(4t)\, e^{-j4\pi t}$

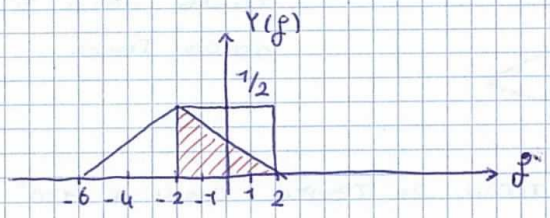
$h(t) = \mathrm{sinc}(2t)\cos(2\pi t)$

$E_y = ?$

$X(f) = 2\,\mathrm{tri}\left(\frac{f+2}{4}\right)$

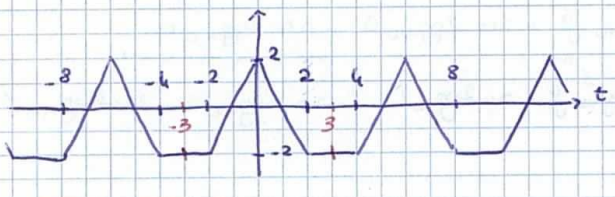
$H(f) = \frac{1}{4}\left[\mathrm{rect}\left(\frac{f-1}{2}\right) + \mathrm{rect}\left(\frac{f+1}{2}\right)\right]$

$Y(f) = \frac{1}{2}\,\mathrm{tri}\left(\frac{f+2}{4}\right)\left[\mathrm{rect}\left(\frac{f-1}{2}\right) + \mathrm{rect}\left(\frac{f+1}{2}\right)\right]$

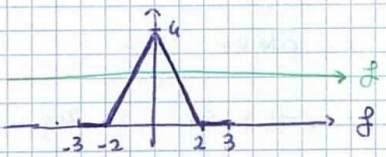


$$\int_{-\infty}^{+\infty} |Y(f)|^2 df = \frac{1}{16}\int_{-2}^{2}\left(1 - \frac{f}{2}\right)^2 df = \frac{1}{16}\int_{-2}^{2} 1 - f + \frac{f^2}{4}\, df = \frac{1}{16}\left[f - \frac{f^2}{2} + \frac{f^3}{12}\right]_{-2}^{2} = \frac{1}{3}.$$

Segnale Periodico



$T = -3 \div 3$



Formula di Poissom:

$$S_p(t) = \sum_{m=-\infty}^{+\infty} S_\tau(t - mT)$$

$$S_p(f) = \frac{1}{T}\sum_{m=-\infty}^{+\infty} S_\tau\left(\frac{m}{T}\right)\cdot\delta\left(f - \frac{m}{T}\right)$$

$S_\tau = 4\cdot \mathrm{tri}\left(\frac{t}{2}\right) \leftrightarrow 8\,\mathrm{sinc}^2(2f)$

$T = 6.$

$$\rightarrow S_p(t) = \left[4\cdot\sum_{m=-\infty}^{+\infty}\mathrm{tri}\left(\frac{t - m\cdot 6}{2}\right)\right] - 2$$

$$S_p(f) = \left[\frac{1}{6}\cdot\sum_{m=-\infty}^{+\infty} 8\,\mathrm{sinc}^2\left(2\cdot\frac{m}{6}\right)\delta\left(f - \frac{m}{6}\right)\right] - 2\delta(f) = \left[\frac{4}{3}\sum_{m=-\infty}^{+\infty}\mathrm{sinc}^2\left(\frac{1}{3}m\right)\cdot\delta\left(f - \frac{m}{6}\right)\right] - 2\delta(f)$$

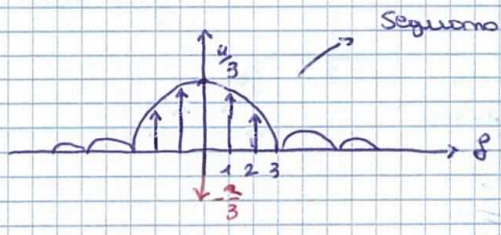
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$m = \not{0} \rightarrow Sp(f) = \frac{4}{3}$ - 2

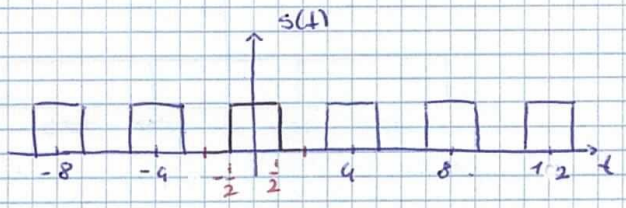
$m = \pm 1 \rightarrow Sp(f) = \frac{4}{3} \cdot sinc^2\left(\frac{1}{3}\right) = \frac{4}{3} \cdot \frac{sen^2\left(\frac{\pi}{3}\right)}{\left(\frac{\pi}{3}\right)^2}$

$m = \pm 2 \rightarrow Sp(f) = \frac{4}{3} sinc^2\left(\frac{2}{3}\right)$

$m = \pm 3 \rightarrow Sp(f) = \frac{4}{3} \frac{sen^2 \pi}{\pi^2}$



$s(t) = \sum_{m=-\infty}^{+\infty} rect(t - 4m)$

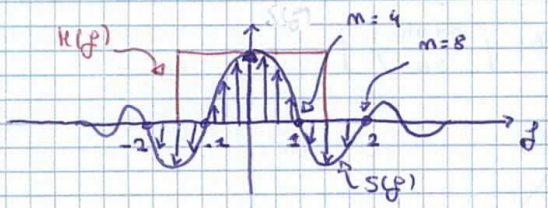


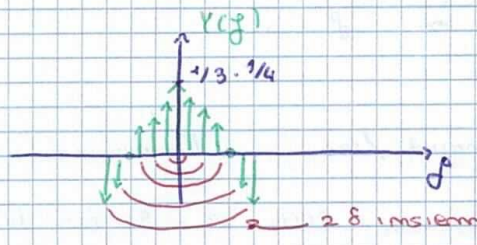
$R(t) = sinc(3t)$

$Y(f) = H(f) \cdot S(f)$

$H(f) = \frac{1}{3} rect\left(\frac{f}{3}\right)$.

$S(f) = \frac{1}{4} \sum_{m=-\infty}^{+\infty} sinc\left(\frac{m}{4}\right) \cdot \delta\left(f - \frac{m}{4}\right)$





$\delta(\not{0})$: componente in continua.

$Y(t) = \frac{1}{12} + \frac{1}{6} \cdot \frac{\sqrt{2}}{\pi} \cdot \cos\left(2\pi \frac{t}{4}\right) + \frac{1}{6} \cdot \ldots \cos\left(2\pi \frac{2}{4} t\right) + \frac{1}{6} \ldots \cos\left(2\pi \frac{3}{4} t\right) +$

$+ \frac{1}{6} \ldots \cos\left(2\pi \frac{5}{4} t\right) + \frac{1}{6} \ldots \cos\left(2\pi \frac{6}{4} t\right)$

$\frac{1}{3} \cdot \frac{1}{4}$ → $\frac{1}{3} \cdot \frac{1}{4} \cdot (2)$ → antitrasformata di $\frac{1}{2}[\delta(f - f_0) + \delta(f + f_0)]$

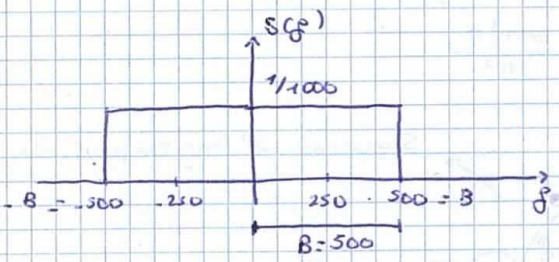
$\frac{sen\ \pi/4}{\pi/4}$ $\qquad$ $\frac{sen\ \pi/2}{\pi/2}$

Campionamento

$s(t) = \text{sinc}(500t)\cos(500\pi t)$

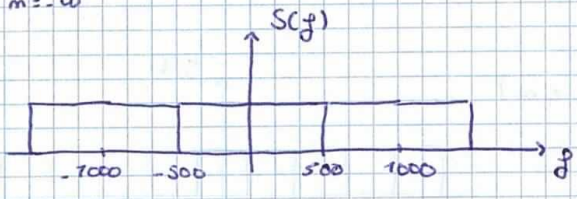
$f_c = 2B$: frequenza minima di campionamento

$$S(f) = \frac{1}{1000}\cdot\left[\text{rect}\left(\frac{f-250}{500}\right) + \text{rect}\left(\frac{f+250}{500}\right)\right]$$



$f_c = 1000$ Hz.

$$S(f) = f_c \cdot \sum_{m=-\infty}^{+\infty} S_i(f - mf_c)$$



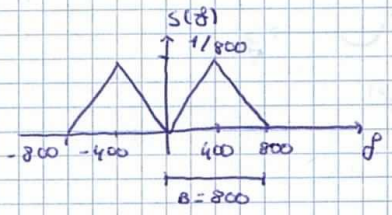
$s(t) = \text{sinc}^2(400t)\cdot\cos(800\pi t)$

Campionato il segnale alla min. frequenza: $f_c = 2B$.

Ogni campione è rappresentato da 12 bit: $N_{BIT/CAMPIONE} = 12$ bit

$T_s = 5s$

$$S(f) = \frac{1}{800}\left[\text{tri}\left(\frac{f-400}{400}\right) + \text{tri}\left(\frac{f+400}{400}\right)\right]$$



$f_c = 1600$ Hz $\rightarrow$ 1600 campioni/sec $\rightarrow N_{CAMPIONI/SS} = 1600 \cdot 5 = 8000$

$N_{BIT/SS} = N_{CAMPIONI/SS} \cdot N_{BIT/CAMPIONE} = 8000 \cdot 12 = 96.000$ bit

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$s(t) = \text{sinc}^2(60t) \cdot \cos(100\pi t)$

$f_c = 2B$

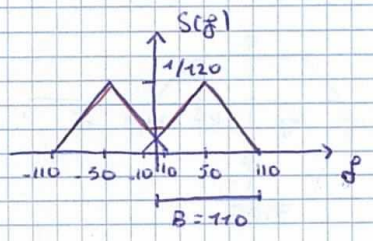
$n_{bit/campione} = 12\ bit$

Linea di trasmissione con velocità: $V_T = 2\ Mbit/s$

Tempo necessario per trasmettere 45 min di segnale.

$T_S = 45\ min$

$$S(f) = \frac{1}{120}\left[rect\left(\frac{f-50}{60}\right) + rect\left(\frac{f+50}{60}\right)\right]$$



$f_c = 2B = 220\ Hz$.

$n_{c/1s} = 220$

$n_{c/45min} = 220 \cdot 45 \cdot 60 = 594.000$

$n_{bit/45min} = 594.000 \cdot 12 = 7.128.000\ bit$

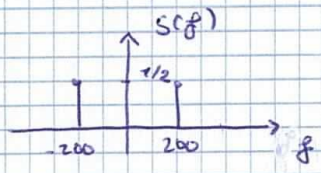
$T_{TRASMISSIONE} = n_{bit/45min} / V_T = 3,564\ sec$

$s(t) = \cos(400\pi t) \longleftrightarrow S(f) = \frac{1}{2}\cdot[\delta(f-200) + \delta(f+200)]$

$f_c = 200\ Hz$

$B_{LP} = 250\ Hz$ ← Banda filtro passa basso

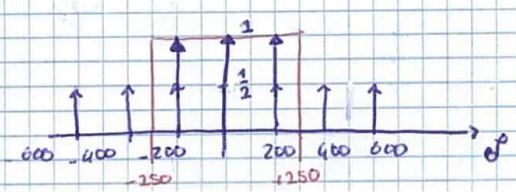
Determinare l'uscita del filtro



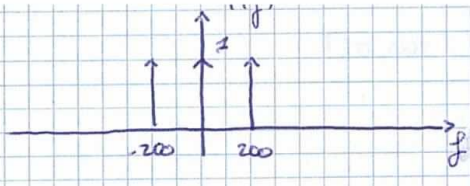
$B = 200\ Hz$

$f_c = 400\ Hz \neq 200\ Hz$

$$S_c(f) = f_c \sum_{m=-\infty}^{+\infty} S(f - mf_c)$$



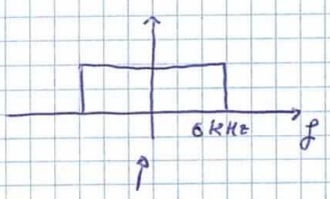
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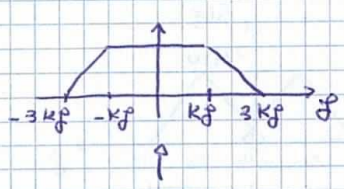
$$y(t) = f_c\left[1 + 2\cos(400\pi t)\right] = 200 + 400\cos(400\pi t).$$

$$y(t) \neq s(t)$$

Esercizio



Spettro Segnale



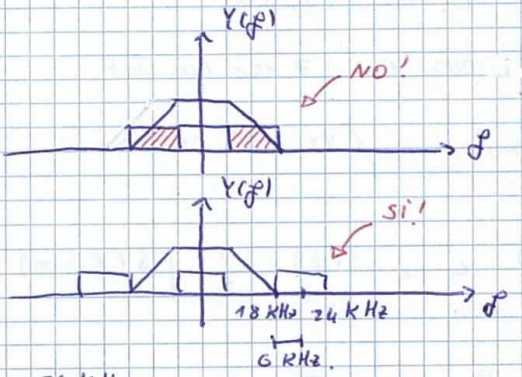
Filtro

$f_c = ?$

$k_f = ?$

Per una corretta ricostruzione: $k_f \geq 6\ KHz \rightarrow k_f = 6\ KHz$

Per Shannon: $f_c = 12\ KHz$, ma non va bene!



Se segnale viene ricostruito male!

$f_c = 18\ KHz + 6\ KHz = 24\ KHz$

14/04/2015

$P_T(W) \rightarrow 10 \log_{10} \frac{P_T}{1W}$ (dBW)

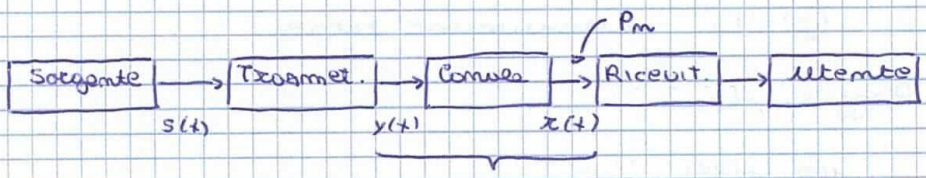
$P_T = 1\,W \rightarrow 0\,dBW$ $\quad$ $P_T = 2\,W \rightarrow 3\,dBW$

$10\,W \rightarrow 10\,dBW$ $\quad$ $12\,W \rightarrow 13\,dBW$

$100\,W \rightarrow 20\,dBW$ $\quad$ $14\,W \rightarrow 16\,dBW$

$0{,}1\,W \rightarrow -10\,dBW$

$W \rightarrow mW \rightarrow 10 \log_{10} \frac{P_T(mW)}{1\,mW} = P_T(dBm)$



con $P_R < P_y$ ↑ attenuazione

Calcolare $\frac{P_y}{P_R} \rightarrow 10 \log_{10} \frac{P_y}{P_R} = \frac{P_y}{P_R}(dB)$

↑ numero puro!

$\frac{P_y}{P_R}(dB) \rightarrow P_y(dBW) - P_R(dBW)$.

$P_R \geq S$: Sensibilità del ricevitore.

Se $P_R < S$ il sistema non funziona.

P_m: Potenza Rumore

Comunicazione Radio

- Attenuazione del segnale:
  - attenuazione di spazio libero, $L_S(f) = \left(\frac{4\pi f D}{c}\right)^2$;

    con f (Hz) frequenza

    D (m) distanza

    $c = 3 \cdot 10^8$ m/s

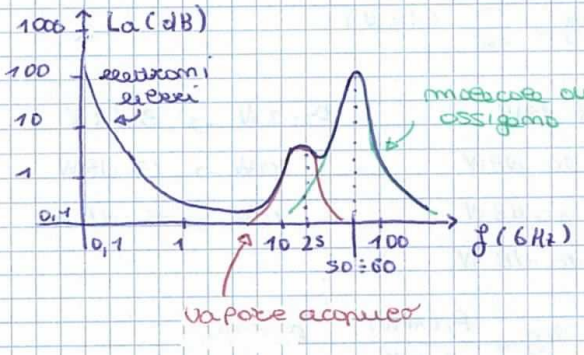
    $L_S(f) \rightarrow 10 \log_{10} L_S(f) = L_S(f)$ in dB

$P_A(dBW) = P_T(dBW) - L_S(f)|_{dB}$, $P_A = \frac{P_T}{L_S(f)}$

↑ Equazione della tratta

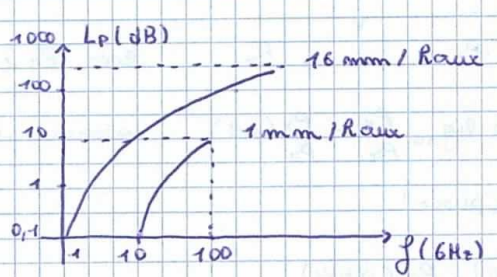
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- attenuazione dovuta all'atmosfera, L_a



$L_T = L_s(f) + L_a$: attenuazione totale

- Pioggia, L_P



→ $P_R = P_T - L_s(f) - L_a - L_P$ ← Per trasmissioni Broadcast

Satellite geostazionario
Posto a distanza 25800 Km dalla Terra.

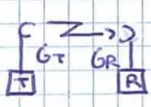
$f = 6,175$ GHz

$L_s(f) = 200$ dB



→ Trasmissione Punto Punto (Direttive)

$P_R \neq P_T - L_s(f) - L_a - L_P$



la potenza viene concentrata in una sola direzione!

G_T: Guadagno antenna Trasmissione
G_R: Guadagno antenna Ricevitore

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nei ponteggi geostazionari:

$$G_T = \eta \cdot \left(\frac{\pi f d}{c} \right)^2 = G_R$$

η : efficienza antenna , $0 < \eta < 1 \rightarrow \eta = 0,5 \div 0,6$

f : frequenza (Hz)

d : diametro antenna (m)

$$10 \log_{10} G_T = 10 \log_{10} \eta + 20 \log_{10} \pi + 20 \log_{10} f + 20 \log_{10} d - 20 \log_{10} c$$

$$G_T(dB) = -159,64 + 10 \log_{10} \eta + 20 \log_{10} f + 20 \log_{10} d$$
$$= 20,4 + 10 \log_{10} \eta + 20 \log_{10} f|_{GHz} + 20 \log_{10} d|_m$$

$f = 6,175$ GHz

$\eta = 0,55$, $d = 30$ m $\rightarrow G_T = 63,2$ dB

Quindi: $P_R = P_T + G_T - L_S(f) - L_a - L_p + G_R$ (trasmissioni Broadcast)

Se $L_S(f) = 200$ dB, $L_a = L_p = \emptyset \rightarrow P_R = P_T - 200$ dB

$P_R = P_T - 200 + 63,2 + 63,2 = P_T - 74$ dB (trasmissioni Direzione)

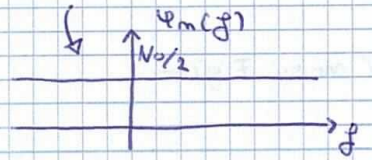
$P_R \geq S \rightarrow$ Canale $\xrightarrow{P_R}$ Ricevitore

$P_N = KTW$

$\frac{P_R}{P_N}$

Distorsioni: { rumore (termico), altri rumori, interferenze }

$n(t)$ rumore termico



$N_0 = KT$

K : Costante di Boltzmann

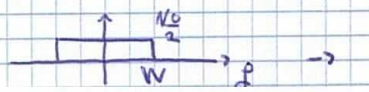
$K = 1,38 \cdot 10^{-23} \frac{W_S}{^\circ K}$

con T espressa in $^\circ K$.

$\rightarrow$ Canale $\rightarrow$

$\uparrow n(t)$, $\Psi_n(f) = \frac{n_0}{2}$

con W: Banda del canale



$$\rightarrow P_n = \int_{-W}^{W} \frac{N_0}{2} df = N_0 W = KTW$$

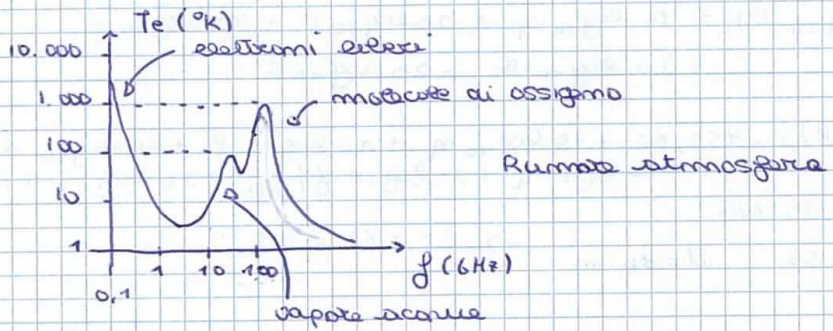
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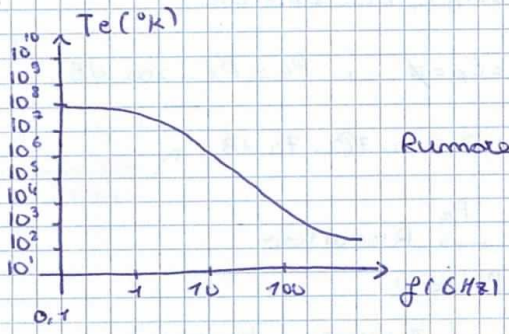
Rapporto segnale - rumore: $\frac{P_R}{P_m} = \frac{S_i}{N_i}$ (segnale / noise)

$P_m = KTW$

Temperatura Equivalente di Rumore Te

$Te = \frac{P_m}{KW}$ Spesso Te viene dato in funzione di f.





Trasm. $\xrightarrow{P_T}$ Canale $\xrightarrow{\frac{P_R}{P_m}}$ Ricevit. $\xrightarrow{\frac{S_u}{N_u}}$ Utente

Canale: P_m, $\frac{S_i}{N_i}$

Figura di Rumore: $F = \frac{\frac{S_i}{N_i}}{\frac{S_u}{N_u}}$: NF (Noise Figure).

$F = 0$ dB $\rightarrow \frac{S_i}{N_i} = \frac{S_u}{N_u}$

$F > 0$ dB $\rightarrow \frac{S_i}{N_i} > \frac{S_u}{N_u}$, $F = 3$ dB è un buon apparato!

$\frac{S_i}{N_i} = 50$ dB , $F = 10$ dB $\rightarrow \frac{S_u}{N_u} = 40$ dB

$F(dB) = \left(\frac{S_i}{N_i}\right)\Big|_{dB} - \left(\frac{S_u}{N_u}\right)\Big|_{dB}$

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$N_i = KT_i W$, T_u : Temperatura equivalente di

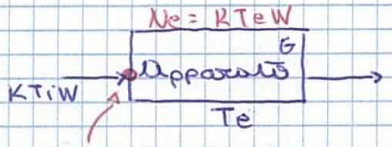
$N_u = KT_u W$ Rumore all'uscita

$N_e = KT_e W$

$T_0 = T_i = 290°K$, temperatura di ingresso

$N_i = 1,38 \cdot 10^{-23} \cdot 290 \cdot W$

$\rightarrow 10 \log_{10} 1,38 + 10 \log_{10} 10^{-23} + 10 \log_{10} 290$



Ne applicato solo e tutto all'ingresso

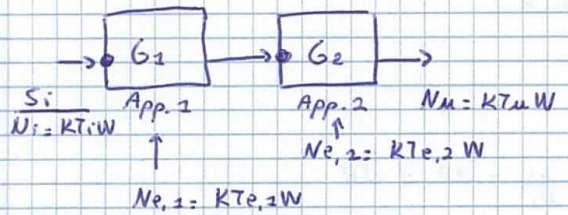
$N_i, S_i \rightarrow S_u = G \cdot S_i$

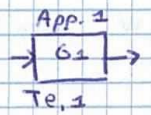
$N_u = G \cdot KT_i W + GKT_e W$

$$F = \frac{\frac{S_i}{N_i}}{\frac{S_u}{N_u}} = \frac{\frac{S_i}{KT_i W}}{\frac{GS_i}{GK \cdot W (T_i + T_e)}} = \frac{T_i + T_e}{T_i} = 1 + \frac{T_e}{T_i}$$

$F, T_i \rightarrow T_e = (F-1) T_i = (F-1) \cdot 290°K$

Due apparati in cascata





$$F_1 = 1 + \frac{T_{e,1}}{T_0}$$



$$F_2 = 1 + \frac{T_{e,2}}{T_0}$$

$S_u = G_1 G_2 S_i$

$N_u = KT_i W G_1 G_2 + KT_{e,1} W G_1 G_2 + KT_{e,2} W G_2$

$$F = \frac{\frac{S_i}{KT_i W}}{\frac{G_1 G_2 W \cdot S_i}{KW G_1 G_2 \left[T_i + T_{e,1} + \frac{T_{e2}}{G_1}\right]}} = \frac{T_i + T_{e,1} + \frac{1}{G_1} \cdot T_{e,2}}{T_i}$$

$$= 1 + \frac{T_{e,1}}{T_i} + \frac{1}{G_1} \cdot \frac{T_{e,2}}{T_i}$$

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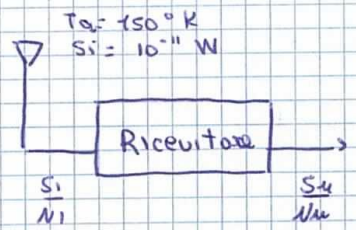
Quindi: $F = F_1 + (F_2 - 1) \cdot \frac{1}{G_1}$

F_2 ha meno peso di F_1!
Se G_1 è alto, $F_2 \to \emptyset$.

Se avessimo:

F_1, G_1 F_2, G_2 F_3, G_3

$$F = F_1 + \frac{1}{G_1}(F_2 - 1) + \frac{1}{G_1 G_2}(F_3 - 1)$$



$F = 10\ dB \to F = 10$
$G = 80\ dB \to G = 10^8$
$W = 6\ MHz$

Ta: Temperatura Reale.
$T_\emptyset$: Temperatura ambiente.

$$T_e = (F - 1)\,T_0 = 9 \cdot 290 = 2610\ ^\circ K$$

(T_0 = 290° K)

$N_u = G \cdot N_i = KT_i W G + KT_e W G$

$KWT_a G = 1{,}2\ \mu W$ con $T_a = 150^\circ K$
$KWT_e G = 21{,}6\ \mu W$ con $T_e = 2610^\circ K$

$N_u = 22{,}8\ \mu W$

$$\frac{S_i}{N_i} = \frac{10^{-11}}{KT_a W} = 806{,}5 \qquad \frac{S_i}{N_i} = 29{,}1\ dB$$

$$\frac{S_u}{N_u} = \frac{GS_i}{22{,}8 \cdot 10^{-6}} = 43{,}9 \qquad \frac{S_u}{N_u} = 16{,}4\ dB$$

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$F_1 = 3\,dB$ $\quad$ $F_2 = 10\,dB \rightarrow F_2 = 10$

$W = 6\,MHz$ $\quad$ $G_2 = 80\,dB \rightarrow G_2 = 10^8$

$G_1 = 13\,dB$ $\quad$ $W = 6\,MHz$

$T_{e,1} = (F_1 - 1)\,T_0 = (2-1)\,T_0 = T_0 = 290\,°K$

$T_{e,2} = (F_2 - 1)\cdot T_0 = 2610\,°K$

$F = F_1 + \frac{F_2 - 1}{G_1} = 2 + \frac{9}{20} = 2{,}45 \rightarrow 3{,}89\,dB = F$

$N_u = KT_a W G_1 G_2 + KT_{e,1} G_1 G_2 + KT_{e,2} G_1 G_2$

$KT_a W G_1 G_2 = 24{,}8\ \mu W$

$KT_{e,1} G_1 G_2 + KT_{e,2} G_1 G_2 = 69{,}6\ \mu W$

$S_u = S_i G_1 G_2 = 10^{-11} \cdot 20 \cdot 10^8 = 20 \cdot 10^{-3}$

$N_u = 24{,}8 \cdot 10^{-6} + 69{,}6 \cdot 10^{-6} = 94{,}4 \cdot 10^{-6}\ \mu W$

$\frac{S_u}{N_u} = 212 \rightarrow 23{,}3\,dB$

Se rapporto Segnale/Rumore è migliorato!

Se Rumore è più basso del Segnale!

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16/04/2015

Wireless Radio

- Wi-Fi (rete locale) → 2,4 GHz ISM (Industrial, Scientific, Medical)
- UWB (Ultra Wide Band)
- Zigbee IEEE 802.16
- Bluetooth → i Beacon
  Mercato per applicazioni Marketing di Prossimità.
  Portata di circa 100 m.
- NFC (Near Field communication)
  Portata di circa 5 ÷ 6 cm.

Proprietà:

- Basso costo;
- Banda larga;
- "attenuazioni piccole";

ci sono più cammini del segnale fra trasm. e ricev.



Multipath!

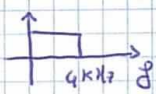
- Disturbi;
- Interferenze;

Sistemi cablati:

1) Doppino telefonico;
2) Cavo coassiale;
3) Fibre ottiche;

1) attenuazione molto elevata

Bt = 4 kHz → Doppino cat. 1



Cat. 1 → Bt = 4 kHz.
ISDN Cat. 2 → Bt = 64 kHz.
Cat. 3 → Bt = 10 MHz.
Token Ring Cat. 4 → Bt = 16 MHz.
Fast ETH Cat. 5 → Bt = 100 MHz.

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Cat. 5e → Vt = 1 Gb/s max

Gigabit ETH Cat. 6 → Vt = 1 Gb/s

10 Gigabit ETH Cat. 7 → Vt = 10 Gb/s

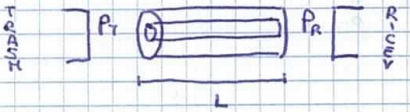
2) Primo sistema di trasmissione a lunga distanza.

mm	d_1	d_2	
	2,6	9,5	→ attenuazione 2÷3 dB/Km max 140 Mbit/s
	1,2	4,4	
	0,7	2,9	→ max 2 Mbit/s



- d_1
- d_2

Equazione della Tratta



α = attenuazione dB/Km

$P_R = P_T \cdot A$, $A = 4L$

3) Attenuazioni di :

0,1 dB/Km

0,01 dB/Km

0,001 dB/Km

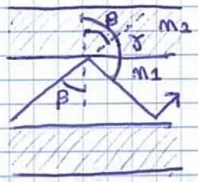
Bande di :

100 GHz

... THz (10^{12})



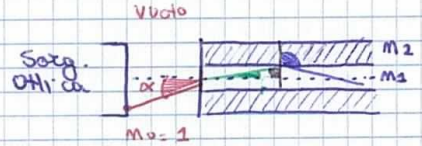
- d_2
- d_1



$\gamma \geq 90°$

$m_1 \operatorname{sen}\beta = m_2 \operatorname{sen}\gamma$

$\gamma = 90° \rightarrow m_1 \operatorname{sen}\beta = m_2$



- γ ↗
- β ↗
- α ↗
- β_2 ↗

$\gamma \geq 90°$

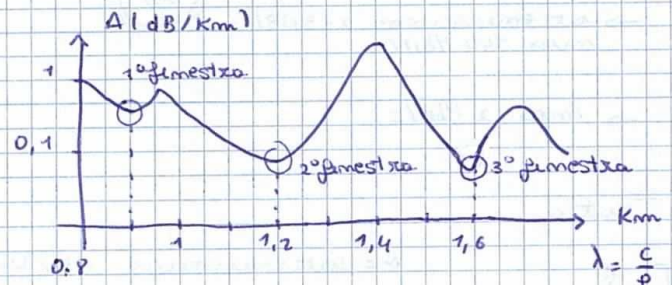
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$$\begin{cases} n_0 \operatorname{sen}\alpha = n_1 \operatorname{sen}\beta \\ n_1 \operatorname{sen}\beta_1 = n_2 \operatorname{sen}\gamma \end{cases} \quad \begin{cases} \operatorname{sen}\alpha = n_1 \operatorname{sen}\beta \\ n_1 \operatorname{sen}\beta_1 = n_2 \end{cases}$$

$\beta_1 = 90 - \beta$ $\qquad \operatorname{sen}\beta_1 = \frac{n_2}{n_1}$ $\qquad \alpha = \operatorname{arc\,sen}\sqrt{n_1^2 - n_2^2}$

$\operatorname{sen}\beta_1 = \cos\beta$

$\operatorname{sen}\alpha = n_1\sqrt{1-\left(\frac{n_2}{n_1}\right)^2}$ $\qquad \sqrt{n_1^2 - n_2^2}$: NA (Numerical Aperture).

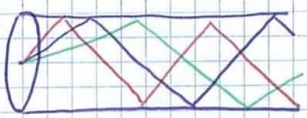


Sorg. Ottico]  [Ricev. Ottico

Diodi PIN : Positive Intrisic Negative

APD : Avalanche Photo Diode

Modi di Propagazione



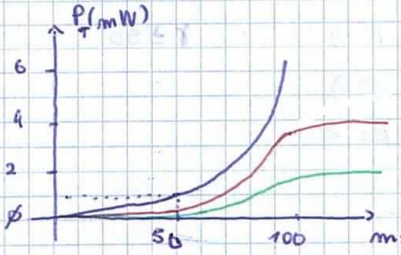
modi di propagazione per uno stesso segnale!

m modi , $m \geq 1$ $\qquad M = \frac{1}{2}\left(\frac{\pi d_1 NA}{\lambda}\right)^2$

Esistono due categorie di fibre:

- Fibre Multimodali : $M > 1$, $d_1 = 25\,\mu m$ / $50\,\mu m$
- Fibre Monomodali : $M = 1$, $d_1 = 9\,\mu m$

LASER



dopo 10.000 h di funzionamento

Con il tempo la P_T si riduce!

→ $P_R \geq S$ (sensibilità)

$P_R = P_T - \alpha \cdot \ell - \alpha_g \cdot L - A_1 - A_2 + mg$

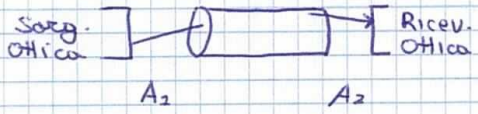
α = attenuazione in dB/Km

ℓ = lunghezza fibra

α_g = attenuazione dovuta ai giunti dB/Km

mg = margine di sicurezza ($P_R \geq S$).



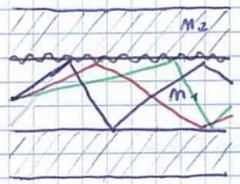


$\alpha = \arccos \sqrt{m_1^2 - m_2^2}$

Banda nella Fibra

Dispersione:





Separazione Reale

Mute

← Ci sono rifrazioni diverse

N.B: da banda si calcola in funzione della distanza da coprire

Dispersione dei materiali

$B_c^2 = \frac{0,44 \cdot 10^6}{\mu \cdot L \cdot \Delta\lambda}$ ← Banda dovuta alla dispersione del materiale.

μ: dipende dalla fibra

$\Delta\lambda$: larghezza del fascio ottico emesso dalla sorgente

LASER: $\Delta\lambda = 5 \div 6$ LED: $\Delta\lambda = 100$

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$B_m^2 = \frac{B_0}{L^\delta}$

B_0 : Banda su 1 km

δ : dipende dalla fibra $(0,5 \div 1)$

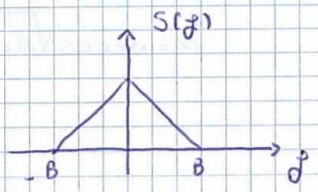
↑ Banda dovuta alla dispersione modale

$Bt \Rightarrow \frac{1}{Bt^2} = \frac{1}{Bc^2} + \frac{1}{Bm^2}$

↑ Banda Totale

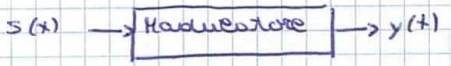
MODULAZIONE

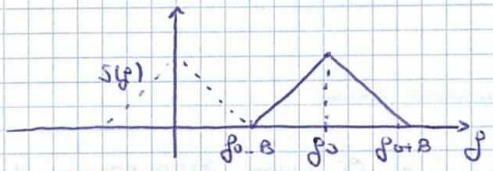
$s(t) \rightarrow S(f)$



Banda Base

Modulazione





$s(t)$ segnale informativo o segnale modulante

$y(t)$ segnale modulato

$v(t) = V_0 \cdot \cos(2\pi f_0 t)$ Portante

- Modulazione di Ampiezza (AM)
  l'ampiezza della portante $v(t)$ viene fatta variare proporzionalmente a $s(t)$.
- Modulazione di Frequenza (FM)
  la frequenza della portante $v(t)$ viene fatta variare proporzionalmente a $s(t)$.

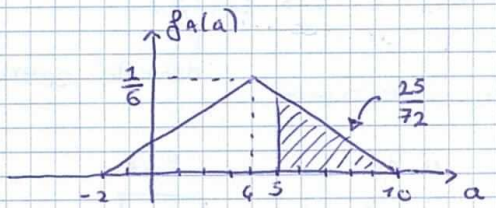
- Modulazione di Fase (PM)
la fase di $v(t)$ varia proporzionalmente a $s(t)$.

Esercizi Esame 20/04/15

10/05/2011

$A \Rightarrow f_A(a) = \frac{1}{6} tri\left(\frac{a-4}{6}\right)$

$E = \{A \leq 5\}$



$E(a) = 4$

$E(a^2) = \int_{-\infty}^{+\infty} a^2 \cdot f_A(a)\, da = \frac{1}{6}\int_{-2}^{4} a^2 \cdot \left(1 + \frac{a-4}{6}\right) da + \frac{1}{6}\int_{4}^{10} a^2 \cdot \left(1 - \frac{a-4}{6}\right) da$

$= \frac{1}{36}\int_{-2}^{4} a^2(2+a)\, da + \frac{1}{36}\int_{4}^{10} a^2(10-a)\, da = \dots = 22$

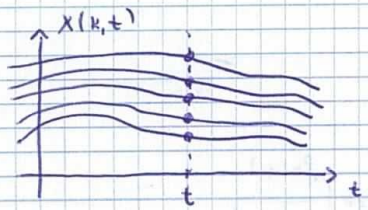
$\sigma^2(a) = E(a^2) - E(a)^2 = 22 - 16 = 6.$

$P(A \leq 5) = \int_{-\infty}^{5} f_A(a)\, da = \underbrace{\int_{-2}^{4} 1 + \frac{a-4}{6}\, da}_{1/2} + \frac{1}{6}\int_{4}^{5} 1 - \frac{a-4}{6}\, da = \frac{47}{72}.$

$P(A \geq 5) = 1 - \frac{25}{72} = \frac{47}{72}.$

Processo stocastico è sinonimo di segnale aleatorio.

Segnale aleatorio: $X(k,t)$



t: tempo

k: scelgo la funzione (indice)

Se fisso t: $x(k)$ v.a.

$X(k,t)$

$\mu_X(t) = E[x(k,t)]$

$R_{xx}(t_1, t_2) = E[x(k,t_1) \cdot x(k,t_2)]$

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Processo stocastico SSL:

$\mu_x(t)$ = costante

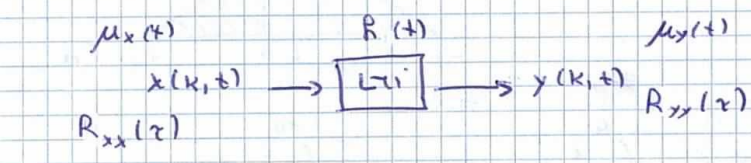
$R_{xx}(t_1, t_2) = R_{xx}(t_1 - t_2)$ ← Segnale T.I.

$= R_{xx}(\tau)$

$R_{xx}(\tau) \leftrightarrow \Psi_{xx}(f)$

↑ autocorrelazione

↑ densità spettrale di potenza media



$$\mu_y = \mu_x \int_{-\infty}^{+\infty} R(t)\,dt = \mu_x \cdot H(0)$$

$$R_{yy}(\tau) = R_{xx}(\tau) * R(t) * R(-t)$$

$R_{yy}(\tau) \leftrightarrow \Psi_{yy}(f)$

$\Psi_{yy}(f) = \Psi_{xx}(f) \cdot |H(f)|^2$

$P_x = R_{xx}(0)$ ← Potenza media di $x(k,t)$

$$= \int_{-\infty}^{+\infty} \Psi_{xx}(f)\,df$$

1/4/05

Processo stocastico $x(k,t)$

$\Psi_{xx}(f) = \text{rect}\left(\frac{f}{2B}\right)$

$y(k,t) = x(k,t) \cdot \cos(2\pi f_0 t - \theta)$

θ v.a. uniforme in $0 \div 2\pi$.



$\Psi_{yy}(f)$?

$R_{yy}(t_1, t_2) = E[y(k,t_1) \cdot y(k,t_2)]$

$= E[x(k,t_1) \cdot \cos(2\pi f_0 t_1 - \theta) \cdot x(k,t_2) \cdot \cos(2\pi f_0 t_2 - \theta)]$

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$= E[X(k,t_1) \cdot X(k,t_2) \cdot \cos(2\pi f_0 t_1 - \theta) \cdot \cos(2\pi f_0 t_2 - \theta)]$

θ e t sono indip.

$= \underbrace{E[X(k,t_1) \cdot X(k,t_2)]}_{R_{xx}(t_1,t_2)} \cdot E[\cos(2\pi f_0 t_1 - \theta) \cdot \cos(2\pi f_0 t_2 - \theta)]$

$= \frac{1}{2} R_{xx}(t_1,t_2) \cdot E\left[\frac{1}{2}\left[\cos(2\pi f_0 (t_1+t_2) - 2\theta) + \cos(2\pi f_0 (t_1 - t_2))\right]\right]$

$= \frac{1}{2} R_{xx}(t_1,t_2) \cdot E[\cos(2\pi f_0 (t_1+t_2) - 2\theta)] + E[\cos(2\pi f_0 (t_1 - t_2))]$

$\cancel{\phi}$

Sto integrando il cos sul periodo

$\int_{\phi}^{T} \cos(\dots) = \phi$.

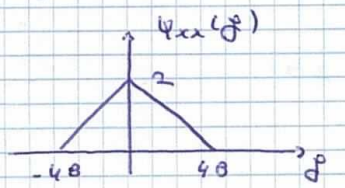
$R_{yy}(\tau) = \frac{1}{2} R_{xx}(\tau) \cdot \cos(2\pi f_0 \tau)$ con $\tau = t_1 - t_2$.

$\updownarrow$

$\psi_{yy}(f) = \frac{1}{4} \cdot \left[\text{rect}\left(\frac{f-f_0}{2B}\right) + \text{rect}\left(\frac{f+f_0}{2B}\right)\right]$.

$X(k,t)$, $\psi_{xx}(f) = 2\,\text{tri}\left(\frac{f}{4B}\right)$ P_x ?

$P_x = \int_{-\infty}^{+\infty} \psi_{xx}(f)\, df = R_{xx}(\phi)$.



$P_x = \int_{-4B}^{4B} 2\,\text{tri}\left(\frac{f}{4B}\right) df = 8B$.

$R_{xx}(\tau) = 8B \cdot \text{sinc}^2(4B\tau)$

$P_x = R_{xx}(\tau)\Big|_{\phi} = 8B$

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5/09/05

$C_{xx}(\tau) = \text{tri}(g\tau)$ $\qquad R_{xx}(\tau) = C_{xx}(\tau) + \mu_x^2$

$\mu_x = 1$ $\qquad$ ↑ costante

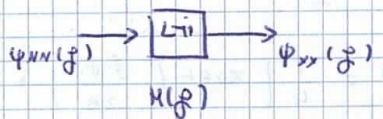
$\Psi_{xx}(f) = ?$ $\qquad = \text{tri}(g\tau) + 1$

$$\Psi_{xx}(f) = \frac{1}{g}\,\text{sinc}^2\left(\frac{f}{g}\right) + \delta(f).$$

$P_y = ?$

$R(t) = \text{sinc}(t)$, $H(f) = \text{rect}(f)$.

$\Psi_{NN}(f) = \frac{N_0}{2}$ $\rightarrow \mu_N = \not{0}$



$R_{NN}(\tau) = \frac{N_0}{2}\cdot\delta(\tau)$.

$\Psi_{yy}(f) = \Psi_{xx}(f)\cdot|H(f)|^2 = \frac{N_0}{2}\cdot\text{rect}(f)$

$$P_y = \int_{-\infty}^{+\infty} \frac{N_0}{2}\,\text{rect}(f) = \frac{N_0}{2}.$$

$x(K,t) = (A-B)\cdot\cos(2\pi f_0 t - 2\Theta)$

A, B v.a.

$f_A(a) = \frac{1}{2}\,\text{rect}\left(\frac{a}{2}\right)$

$f_B(b) = \frac{1}{6}\,\text{rect}\left(\frac{b-2}{6}\right)$ ← distribuzioni unif.

$f_\Theta(b) = \frac{1}{4\pi}\,\text{rect}\left(\frac{\Theta - 2\pi}{4\pi}\right)$ ← unif. distr. in $\not{0} \div 4\pi$.

v.a. indip. fra loro.

Studiare SSL e calcolare autocorrelazione !?

SSL: $\mu_x = c$

$R_{xx}(t_1, t_2) = R_{xx}(t_1 - t_2)$

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$E[|A-B|\cos(2\pi f_0 t - 2\theta)] =$

$E[A\cos(2\pi f_0 t - 2\theta) - B\cos(2\pi f_0 t - 2\theta)] =$

$E[A] \cdot E[\cos(2\pi f_0 t - 2\theta)] - E[B] \cdot E[\cos(2\pi f_0 t - 2\theta)]$

↓ media di $\frac{1}{2}\text{rect}\left(\frac{t}{2}\right)$ → $\frac{1}{2}$; ↓ $\emptyset$; ↓ 2 ; ↓ $\emptyset$

$= \emptyset \quad \rightarrow \mu_x = \text{costante}$ ✓

$R_{xx}(t_1, t_2) = E[X(k,t_1) \cdot X(k,t_2)]$

$= E[(A-B)\cos(2\pi f_0 t_1 - 2\theta) \cdot (A-B) \cdot \cos(2\pi f_0 t_2 - 2\theta)]$

$= \left\{E[A]^2 - 2E[A]E[B] + E[B]^2\right\} \cdot \frac{1}{2}\left\{E[\cos(2\pi f_0 (t_1+t_2) - 4\theta)] + E[\cos(2\pi f_0 (t_1 - t_2))]\right\}$

($E[A]E[B] \rightarrow \emptyset$; $\cos(2\pi f_0 (t_1+t_2) - 4\theta) \rightarrow \emptyset$)

A e B non dipendono da t

$= \frac{1}{2}\left\{A_1 + B_1\right\}\left\{E[\cos(2\pi f_0 (t_1 - t_2))]\right\}$

$= \frac{1}{2}\left\{A_1 + B_1\right\} \cdot \cos(2\pi f_0 \tau)$ dipende solo da τ ✓

PROCESSO SSL !

21/04/2015

AM: Amplitude Modulation

FM: Frequency //

PM: Phase //

$s(t)$: segnale modulante

$y(t)$: segnale modulato

$v(t) = V_0 \cdot \cos(2\pi f_0 t)$ portante

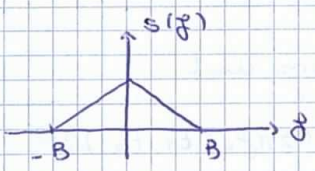
AM

$y(t) = V_0 [1 + K s(t)] \cos(2\pi f_0 t)$

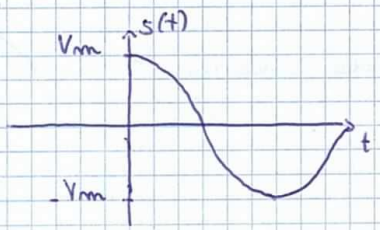
K costante

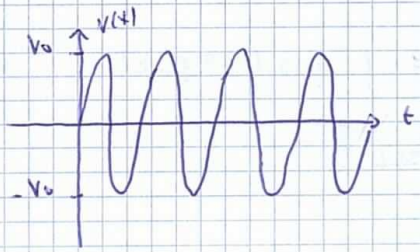
$s(t) = V_m \cdot \cos(2\pi f_m t) \rightarrow y(t) = V_0 [1 + K V_m \cos(2\pi f_m t)] \cos(2\pi f_0 t)$

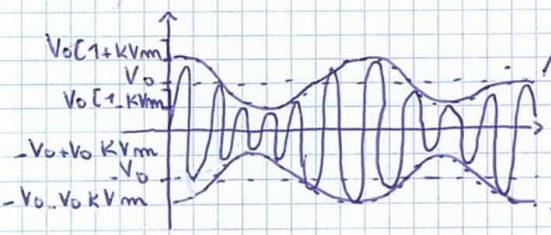
$f_0 \gg f_m$.



$f_0 \gg B$.







→ Inviluppo superiore → centrato in V_0.
$V_0[1 - KV_m] \div V_0[1 + KV_m]$.

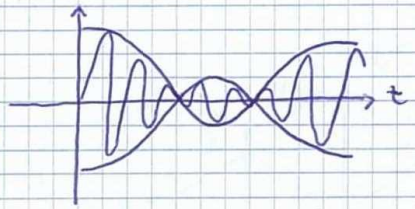
→ Inviluppo inferiore → centrato in $-V_0$.

Se salgo max K può darsi che gli inviluppi si intersichino! ma devono rimanere separati!

condizione separatezza: $\begin{cases} 1 - KV_m \geq 0 \rightarrow KV_m \leq 1 & \text{inv. sup.} \\ -1 + KV_m \leq 0 \rightarrow KV_m \leq 1 & \text{inv. inf.} \end{cases}$

$KV_m \leq 1$

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← non va bene!

$m = KV_m$ = Indice di Modulazione di ampiezza ≤ 1.

$y(t) = V_0 [1 + Ks(t)] \cos(2\pi f_0 t)$ In generale:

$1 + Ks(t) \rightarrow 1 + KS_M$

$1 - KS_m \geq 0 \qquad KS_m \leq 1$

$\rightarrow -1 + KS_M \geq 0 \qquad KS_m \leq 1$

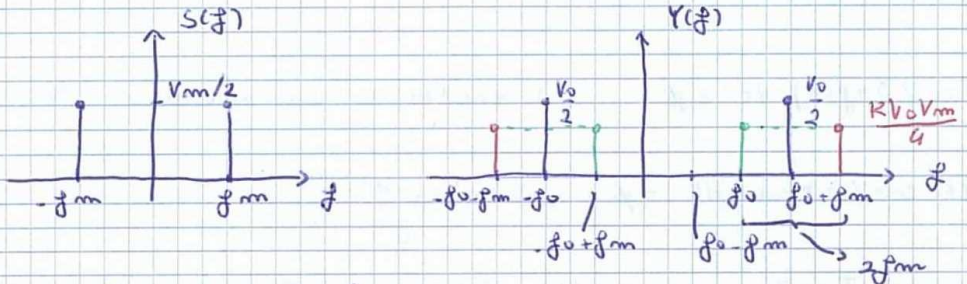
$-1 - KS_m$

$\boxed{k \cdot \max |s(t)| \leq 1} = m.$

$y(t) = V_0 \cdot \cos(2\pi f_0 t) + KV_0V_m \cos(2\pi f_m t) \cos(2\pi f_0 t)$

$= V_0 \cdot \cos(2\pi f_0 t) + \frac{KV_0V_m}{2} \cdot \cos[2\pi (f_0 + f_m) t]$

$+ \frac{KV_0V_m}{2} \cdot \cos[2\pi (f_0 - f_m) t]$



$P_y = \frac{V_0^2}{4} \cdot 2 + \frac{K^2 V_0^2 V_m^2}{16} \cdot 2 \cdot 2 = P_T$

$= \frac{V_0^2}{2} + \frac{K^2 V_0^2 V_m^2}{4} = \frac{V_0^2}{2} + \frac{V_0^2 m^2}{4}$

$KV_m = m \leq 1$

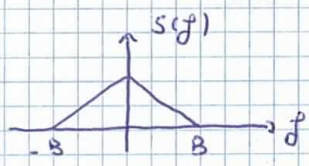
con $m = 1 \rightarrow P_y = \frac{V_0^2}{2} + \frac{V_0^2}{4}$

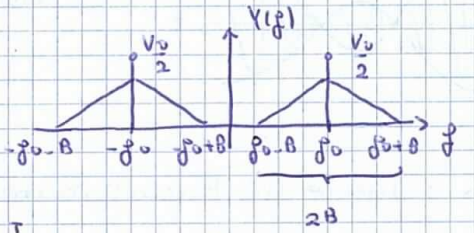
$\frac{V_0^2}{2}$: Portante

$\frac{V_0^2}{4}$: Potenza per ogni riga laterale

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$$y(t) = V_0 [1 + K s(t)] \cos(2\pi f_0 t)$$
$$= V_0 \cos(2\pi f_0 t) + V_0 K s(t) \cos(2\pi f_0 t)$$





$f_0 >> B$.

$$P_y = \frac{1}{T}\int_\phi^T y^2(t)\,dt = \frac{1}{T} \cdot V_0^2 \cdot \int_\phi^T [1 + K s(t)]^2 \cos^2(2\pi f_0 t)\,dt$$

$$\cos^2 x = \frac{1+\cos 2x}{2}$$

$$= \frac{V_0^2}{2T}\left\{\int_\phi^T [1 + K s(t)]^2\,dt + \int_\phi^T [1 + K s(t)]^2 \cos(4\pi f_0 t)\,dt\right\}$$

$$= \frac{V_0^2}{2T}\left\{\int_\phi^T dt + K^2\int_\phi^T s^2(t)\,dt + 2K\int_\phi^T s(t)\,dt + \int_\phi^T \cos(4\pi f_0 t)\,dt + \right.$$
$$\left. + K^2\int_\phi^T s^2(t)\cos(4\pi f_0 t)\,dt + 2K\int_\phi^T s(t)\cos(2\pi f_0 t)\,dt\right\}$$

$$= \frac{V_0^2}{2} + \frac{K^2 V_0^2}{2}\,\frac{1}{T}\int_\phi^T s^2(t)\,dt + \frac{V_0^2}{2} 2K\,\frac{1}{T}\int_\phi^T s(t)\,dt + \dots$$

$$= \frac{V_0^2}{2} + \frac{K^2 V_0^2}{2}\,\overline{s^2(t)} + K V_0^2\,\overline{s(t)} + \dots$$

$\int_\phi^T \cos(4\pi f_0 t)\,dt = \phi$, T multiplo del periodo : $T = n T_0$

$\int_\phi^T s(t)\cos(2\pi f_0 t)\,dt = \phi$, $T >> T_0 = \frac{1}{f_0}$

Siccome $T >> T_\phi$ → $s(t)$ è considerato costante in T_0

e l'integrale fra $\phi \div T$ è la somma degli integrali negli intervalli T_0 che fanno ϕ!

$$\int_\phi^T s^2(t)\cos(4\pi f_0 t)\,dt = \phi$$

$$\Rightarrow P_y = \frac{V_0^2}{2}\left[1 + K^2\,\overline{s^2(t)} + 2K\,\overline{s(t)}\right].$$

$\overline{s(t)} = \phi$ → $P_y = \frac{V_0^2}{2}\left[1 + K^2\,\overline{s^2(t)}\right]$

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