FONDAMENTI DI TELECOMUNICAZIONI A

[Fotocopie di Appunti]

A CURA DI ALESSANDRO PAGHI

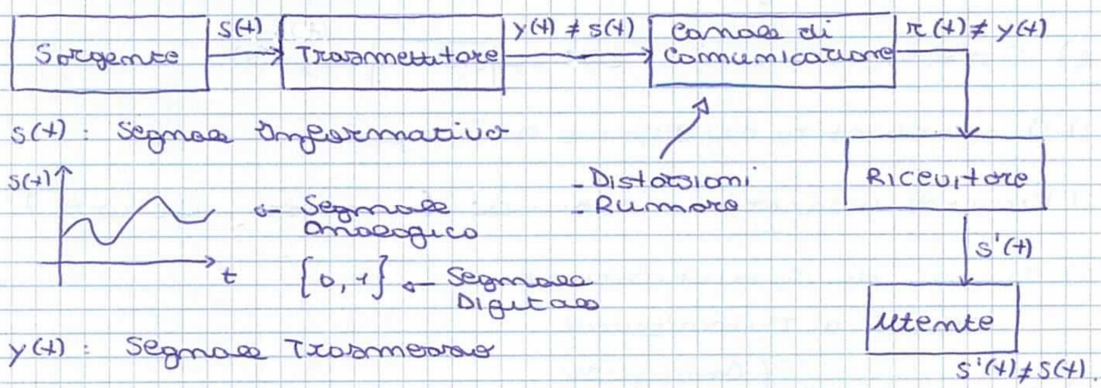
PROFESSORE: Giuliano Benelli (http://www3.diism.unisi.it/people/person.php?id=18)

LINK AL CORSO ANNO 2014/2015:
http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=54889&aa=2014

FREQUENTAZIONE: Consigliata.

SCHEMA A BLOCCHI GENERICO

02/03/2015



$s(t)$: Segnale Informativo

← Segnale Analogico

$\{0, 1\}$ ← Segnale Digitale

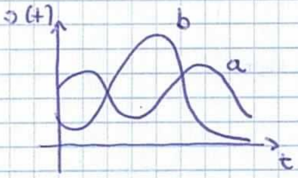
$y(t)$: Segnale Trasmesso

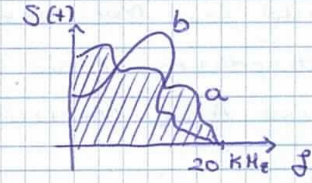
nelle prime lezioni:

$s(t) \rightarrow S(f)$: Spettro in frequenza di $s(t)$.

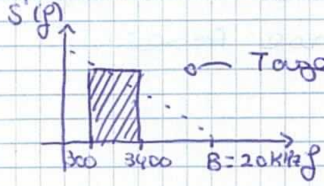
Serie di Fourier

Trasformata di Fourier





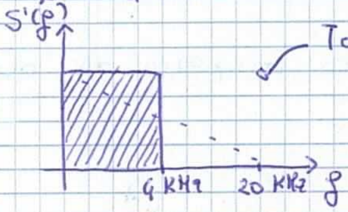
Il microfono usa solo la gamma di frequenze $300 \div 3400$ Hz



← Taglia le altre frequenze

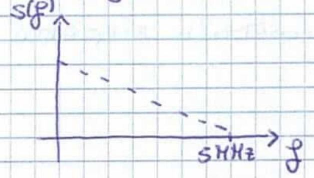
Segnale audio : $\varnothing \div 20$ KHz

Un microfono economico usa le frequenze $\varnothing \div 4$ KHz



Taglia da $4 \div 20$ KHz.

Segnale video : $\varnothing \div 5$ MHz



1) Segnali → Trasformazione in frequenza

2) Sistemi Lineari $s(t) \longrightarrow \boxed{\text{Sistema}} \longrightarrow y(t)$

3) Campionamento del segnale analogico.

4) Variabili aleatorie o casuali (rumore, disturbi)

5) Canali di Comunicazione

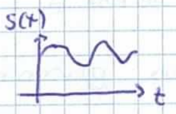
- Velocità di Trasmissione

6) Modulazioni $\begin{cases} \text{Analogiche} \\ \text{Digitali} \end{cases}$

Reti di comunicazione

Modello OSI: Open Standard Interconnection

Reti Dati:
- WAN Wide Area Network
- MAN Metropolitan Area Network
- LAN Local Area Network



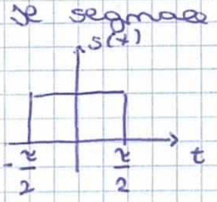
$\left\{ \begin{array}{ll} \text{Segnale analogico} & \\ \text{Segnale Digitale} & \{0, 1\} \end{array} \right\}$ Segnali Deterministici

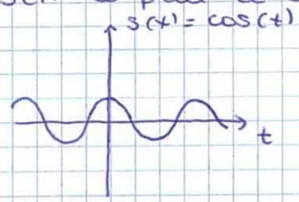
$s(t)$: Segnali Reali

$\boxed{\text{Sorgente}} \xrightarrow{s(t)} \boxed{\text{Trasmettitore}} \xrightarrow{y(t)}$

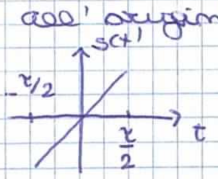
- Segnali in tempo reale: Telefono, video
- Segnali non in tempo reale

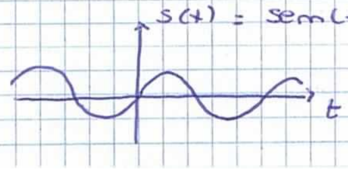
Il segnale $s(t)$ è pari se è simmetrico rispetto all'origine.





Il segnale $s(t)$ è dispari se è antisimmetrico rispetto all'origine





$s(t) \rightarrow s_p(t) = \dfrac{s(t)+s(-t)}{2}$

componente pari di $s(t)$.

$s_p(-t) = \dfrac{s(-t)+s(t)}{2} = s_p(t)$

$s_d(t) = \dfrac{s(t)-s(-t)}{2}$

componente dispari di $s(t)$.

$s_d(-t) = \dfrac{s(-t)-s(t)}{2} = -s_d(t)$.

$\Rightarrow s(t) = s_p(t) + s_d(t) = \dfrac{s(t)+s(-t)}{2} + \dfrac{s(t)-s(-t)}{2} = s(t)$.

E_s = Energia di $s(t) = \int_{-\infty}^{+\infty} |s(t)|^2 \, dt$

Se $E_s = M < \infty$

$s(t)$ segnale ad energia finita.

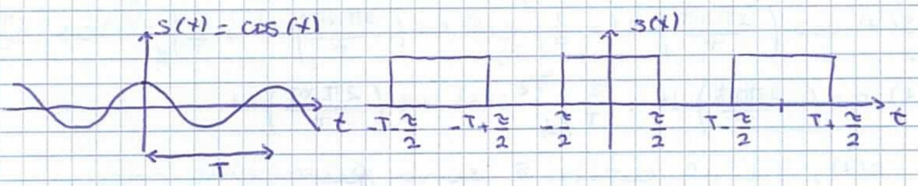
P_s: Potenza di $s(t) = \lim_{T \to \infty} \dfrac{1}{T} \int_{-T/2}^{+T/2} |s(t)|^2 \, dt$

se $s(t)$ ha energia finita, P_s nulla.

se $s(t)$ ha potenza finita, Energia infinita.

Segnale periodico $s(t)$ di periodo T se:

$s(t + mT) = s(t)$, $\forall m$ intero.



$s(t) = V_0 \cos(\theta(t))$

ampiezza

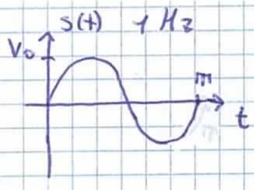
Fase Istantanea

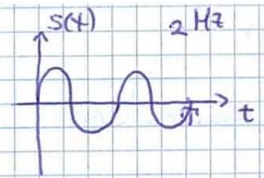
$\theta(t) \Rightarrow \omega(t) = \dfrac{d\theta(t)}{dt}$, $\omega = 2\pi f$

$f = \dfrac{1}{2\pi} \dfrac{d\theta(t)}{dt}$

T: Periodo di una funzione $\cos(t)$

f_0: Frequenza $= \dfrac{1}{T}$





$$f = f_0 \rightarrow \omega = 2\pi f_0 \rightarrow \theta(t) = \int_{\emptyset}^{t} 2\pi f_0 \, dt = 2\pi f_0 t.$$

$$\Rightarrow s(t) = V_0 \cdot \cos(2\pi f_0 t)$$
$$= V_0 \cdot \cos(2\pi f_0 t + \psi).$$

$$s(t) = s(t + mT)$$
$$s(t) = \frac{a_\emptyset}{2} + \sum_{m=1}^{\infty} \left[a_m \cos\left(\frac{2\pi m t}{T}\right) + b_m \operatorname{sen}\left(\frac{2\pi m t}{T}\right) \right]$$

$$\frac{d\theta(t)}{dt} = \omega = \frac{2\pi m}{T} \rightarrow f = \frac{\omega}{2\pi} = \frac{m}{T} = m f_0.$$

In quanto $\theta(t) = \frac{2\pi m t}{T}$.

$$a_\emptyset = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\, dt \rightarrow a_m = \frac{2}{T}\int_{-T/2}^{T/2} s(t) \cos\left(\frac{2\pi m t}{T}\right) dt$$
$$\rightarrow b_m = \frac{2}{T}\int_{-T/2}^{T/2} s(t) \operatorname{sen}\left(\frac{2\pi m t}{T}\right) dt$$

$s(t)$ pari, $s(t) = s(-t)$.

$$a_m = \frac{2}{T}\int_{-T/2}^{T/2} s(t) \cos\left(\frac{2\pi m t}{T}\right) dt = \frac{2}{T}\int_{-T/2}^{\emptyset} s(t)\cos\left(\frac{2\pi m t}{T}\right) dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t) \cos\left(\frac{2\pi m t}{T}\right) dt$$

$t \rightarrow -t$
$$= -\frac{2}{T}\int_{T/2}^{\emptyset} s(-t) \cos\left(-\frac{2\pi m t}{T}\right) dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t) \cos\left(\frac{2\pi m t}{T}\right) dt$$

ma $s(-t) = s(t)$, e il coseno è una funzione pari quindi $\cos\left(-\frac{2\pi m t}{T}\right) = \cos\left(\frac{2\pi m t}{T}\right)$.

$$= \frac{2}{T}\int_{\emptyset}^{T/2} s(t) \cos\left(\frac{2\pi m t}{T}\right) dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t) \cos\left(\frac{2\pi m t}{T}\right) dt.$$

$$= \frac{4}{T}\int_{\emptyset}^{T/2} s(t) \cos\left(\frac{2\pi m t}{T}\right) dt.$$

$$b_m = \frac{2}{T}\int_{-T/2}^{\emptyset} s(t) \operatorname{sen}\left(\frac{2\pi m t}{T}\right) dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t) \operatorname{sen}\left(\frac{2\pi m t}{T}\right) dt$$

$$t \rightarrow -t \Rightarrow = -\frac{2}{T}\int_{T/2}^{\emptyset} s(-t) \operatorname{sen}\left(-\frac{2\pi m t}{T}\right) dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t) \operatorname{sen}\left(\frac{2\pi m t}{T}\right) dt$$

$$= \frac{2}{T}\int_{\emptyset}^{T/2} s(-t) \operatorname{sen}\left(-\frac{2\pi m t}{T}\right) dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t) \operatorname{sen}\left(\frac{2\pi m t}{T}\right) dt$$

ma $s(t) = s(-t)$ e $\text{sen}\left(-\frac{2\pi m t}{T}\right) = -\text{sen}\left(\frac{2\pi m t}{T}\right)$

$$= -\frac{2}{T}\int_{\emptyset}^{T/2} s(t)\,\text{sen}\left(\frac{2\pi m t}{T}\right)dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t)\,\text{sen}\left(\frac{2\pi m t}{T}\right) = \emptyset$$

03/03/2015

Serie di Fourier

$s(t)$ periodico

$s(t + mT) = s(t) \quad \forall m$ intero

$T =$ periodo di $s(t)$.

$Es = \int_{-\infty}^{+\infty} |s(t)|^2 dt$ Segnali periodici hanno energia infinita

$$P_s = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2 dt.$$

$$s(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty}\left[a_m \cos\left(\frac{2\pi m t}{T}\right) + b_m\,\text{sen}\left(\frac{2\pi m t}{T}\right)\right]$$

$$a_0 = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\,dt$$

$$a_m = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\cos\left(\frac{2\pi m t}{T}\right)dt$$

$$b_m = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\,\text{sen}\left(\frac{2\pi m t}{T}\right)dt$$

$s(t)$ pari se $s(t) = s(-t)$

$s(t)$ dispari se $s(t) = -s(-t)$

$s(t)$ pari, $b_m = \emptyset \quad \forall m \geq 1$

$$a_m = \frac{4}{T}\int_{\emptyset}^{T/2} s(t)\cos\left(\frac{2\pi m t}{T}\right)dt$$

adesso $s(-t) = -s(t)$, $s(t)$ segnale dispari

$$a_0 = \frac{2}{T}\int_{-T/2}^{\emptyset} s(t)\,dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t)\,dt$$

$$\overset{t\to -t}{=} \frac{2}{T}\int_{T/2}^{\emptyset} \underbrace{s(-t)}_{-s(t)}\,\underbrace{d(-t)}_{-dt} + \frac{2}{T}\int_{\emptyset}^{T/2} s(t)\,dt = -\frac{2}{T}\int_{T/2}^{\emptyset} -s(t)\,dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t)\,dt$$

$$= -\frac{2}{T}\int_{\emptyset}^{T/2} s(t)\,dt + \frac{2}{T}\int_{\emptyset}^{T/2} s(t)\,dt = \emptyset.$$

$a_m = \emptyset$ in quanto S(t) dispari e cos(t) pari, prodotto fra pari e dispari fa dispari. Inoltre integrale fra $\int_a^b$ e $\int_b^a$ di una funzione dispari sono opposti.

$$b_m = \frac{4}{T}\int_{\emptyset}^{T/2} s(t)\, sen\left(\frac{2\pi m t}{T}\right) dt$$

a_0 è una sinusoide con frequenza $\emptyset$!

Per Fourier ogni segnale periodico può essere scritto come la somma di infinite sinusoidi con frequenza diverse multiple di $\frac{1}{T}$.

$$a_m \cos\left(\frac{2\pi m t}{T}\right) + b_m\, sen\left(\frac{2\pi m t}{T}\right) = A_m \cos\Big[\underbrace{\frac{2\pi m t}{T}}_{\alpha} - \underbrace{\phi_m}_{\beta}\Big]$$

$$\Rightarrow \cos(\alpha - \beta) = \cos\alpha \cdot \cos\beta + sen\,\alpha\, sen\,\beta$$

$$= A_m \cos\phi_m \cos\left(\frac{2\pi m t}{T}\right) + A_m\, sen\,\phi_m\, sen\left(\frac{2\pi m t}{T}\right)$$

$$\Rightarrow \begin{cases} a_m = A_m \cos\phi_m \\ b_m = A_m\, sen\,\phi_m \end{cases} \Rightarrow A_m = \sqrt{a_m^2 + b_m^2}$$

$$tg\,\phi_m = \frac{b_m}{a_m} \rightarrow \phi_m = arctg\,\frac{b_m}{a_m}$$

Se pongo $a_0 = A_0$

$$S(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty}\left[a_m \cos\left(\frac{2\pi m t}{T}\right) + b_m\, sen\left(\frac{2\pi m t}{T}\right)\right]$$

$$S(t) = \frac{A_0}{2} + \sum_{m=1}^{\infty} A_m \cos\left[\frac{2\pi m t}{T} - \phi_m\right]$$

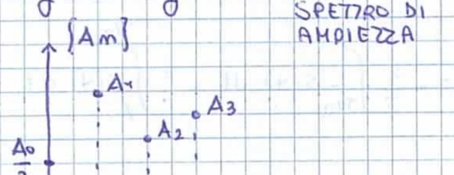
$\{A_0, A_1, \dots, A_m\}$ Spettro di Ampiezza

$\{\phi_0, \phi_1, \dots, \phi_m\}$ Spettro di fase

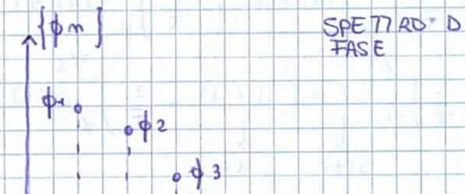
con $\phi_0 = \emptyset$.

$\frac{m}{T} = f_0 = m f_0$

SPETTRO DI AMPIEZZA



SPETTRO DI FASE



anche se non esiste nella realtà

$$a_m \cos\left(\frac{2\pi mt}{T}\right) = a_{-m} \cos\left(-\frac{2\pi mt}{T}\right).$$

DIM:

$$a_m = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\cos\left(\frac{2\pi mt}{T}\right) dt$$

$$a_{-m} = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\cos\left(-\frac{2\pi mt}{T}\right) dt = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\cos\left(\frac{2\pi mt}{T}\right) dt = a_m$$

$\hookrightarrow a_m = +a_{-m}$

$$b_m \operatorname{sen}\left(\frac{2\pi mt}{T}\right) = b_{-m} \operatorname{sen}\left(-\frac{2\pi mt}{T}\right)$$

DIM:

$$b_m = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\operatorname{sen}\left(\frac{2\pi mt}{T}\right) dt$$

$$b_{-m} = \frac{2}{T}\int_{-T/2}^{T/2} s(t)\operatorname{sen}\left(-\frac{2\pi mt}{T}\right) dt = -\frac{2}{T}\int_{-T/2}^{T/2} s(t)\operatorname{sen}\left(\frac{2\pi mt}{T}\right) = -b_m$$

$\hookrightarrow b_m = -b_{-m}$

$$s(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty}\left[a_m \cos\left(\frac{2\pi mt}{T}\right) + b_m \operatorname{sen}\left(\frac{2\pi mt}{T}\right)\right]$$

$$= \sum_{m=-\infty}^{+\infty}\left[\frac{a_m}{2}\cos\left(\frac{2\pi mt}{T}\right) + \frac{b_m}{2}\operatorname{sen}\left(\frac{2\pi mt}{T}\right)\right]$$

in quanto:

$$a_{-1}\cos\left(-\frac{2\pi mt}{T}\right) + b_{-1}\operatorname{sen}\left(-\frac{2\pi mt}{T}\right) = a_1\cos\left(\frac{2\pi mt}{T}\right) + b_1\operatorname{sen}\left(\frac{2\pi mt}{T}\right).$$

$$s(t) = \frac{A_0}{2} + \sum_{m=1}^{\infty} A_m \cos\left(\frac{2\pi mt}{T} - \phi_m\right)$$

$$A_m = \sqrt{a_m^2 + b_m^2} \quad , \quad \phi_m = \operatorname{arctg}\frac{b_m}{a_m}$$

$$A_{-m} = \sqrt{a_{-m}^2 + b_{-m}^2} = A_m$$

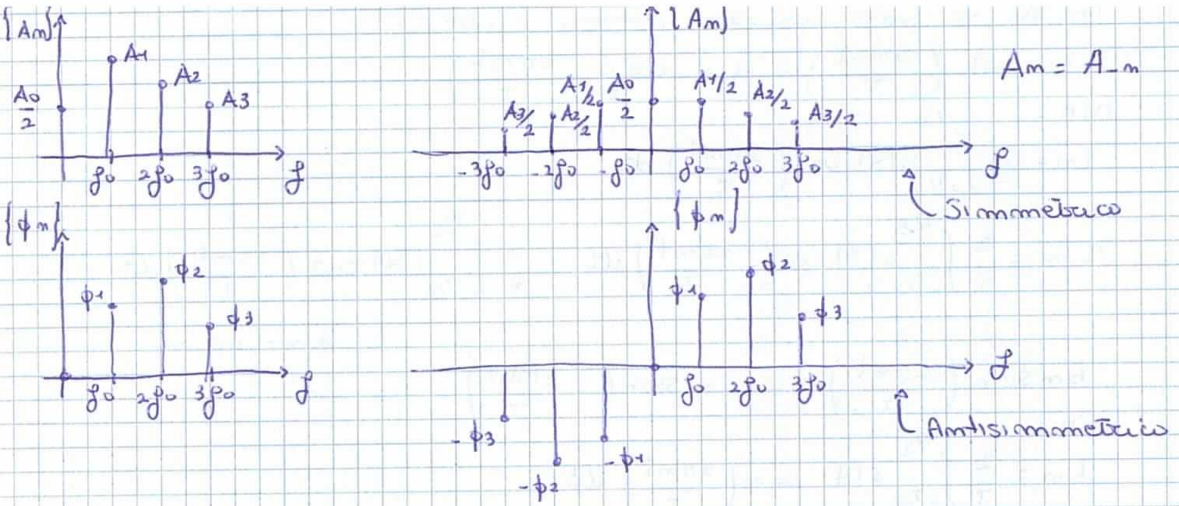
$$\phi_{-m} = \operatorname{arctg}\frac{b_{-m}}{a_{-m}} = \operatorname{arctg}\frac{-b_m}{a_m} = -\operatorname{arctg}\frac{b_m}{a_m} = -\phi_m$$

$$A_m \cos\left(\frac{2\pi mt}{T} - \phi_m\right) = A_{-m}\cos\left(-\frac{2\pi mt}{T} - \phi_{-m}\right)$$

$$\Rightarrow s(t) = \sum_{m=-\infty}^{+\infty}\frac{A_m}{2}\cos\left(\frac{2\pi mt}{T} - \phi_m\right).$$

$\{A_m\}$ Spettro di Ampiezza

$\{\phi_m\}$ Spettro di Fase



$$s(t) = \frac{A_0}{2} + \sum_{n=1}^{\infty} A_n \cos\left[\frac{2\pi n t}{T} - \phi_n\right]$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$A_n \cos\left[\frac{2\pi n t}{T} - \phi_n\right] = \frac{A_n}{2}\left\{ e^{-j\phi_n} e^{j\frac{2\pi n t}{T}} + e^{j\phi_n} e^{-j\frac{2\pi n t}{T}} \right\}$$

$$= C_n e^{j\frac{2\pi n t}{T}} + C_{-n} e^{-j\frac{2\pi n t}{T}}$$

con $C_n = \frac{A_n}{2} e^{-j\phi_n}$

$$C_{-n} = \frac{A_{-n}}{2} e^{-j\phi_{-n}} = \frac{A_n}{2} e^{j\phi_n}$$

$$C_0 = \frac{A_0}{2}$$

$$s(t) = C_0 + \sum_{n=1}^{\infty}\left[C_n e^{j\frac{2\pi n t}{T}} + C_{-n} e^{-j\frac{2\pi n t}{T}} \right]$$

$$= C_0 + \sum_{n=-\infty}^{+\infty} C_n e^{j\frac{2\pi n t}{T}} = \sum_{n=-\infty}^{+\infty} C_n e^{j\frac{2\pi n t}{T}}$$

con $n \neq 0$

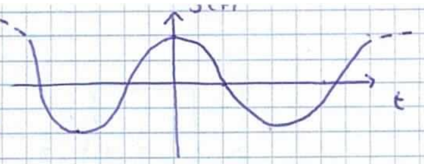
$$e^{-jx} = \cos x - j \,\text{sen}\, x$$

$$C_n = \underbrace{\frac{A_n}{2}\cos\phi_n}_{a_n} - j\underbrace{\frac{A_n}{2}\,\text{sen}\,\phi_n}_{b_n}$$

$$C_n = \frac{1}{2}\left[a_n - j b_n\right] = \frac{1}{2}\left[\frac{2}{T}\int_{-T/2}^{T/2} s(t)\cos\left(\frac{2\pi n t}{T}\right) dt - j\frac{2}{T}\int_{-T/2}^{T/2} s(t)\,\text{sen}\left(\frac{2\pi n t}{T}\right) dt\right]$$

$$= \frac{1}{\cancel{2}}\cdot\frac{\cancel{2}}{T}\int_{-T/2}^{T/2} s(t)\left[\cos\left(\frac{2\pi n t}{T}\right) - j\,\text{sen}\left(\frac{2\pi n t}{T}\right)\right] dt = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\, e^{-j\frac{2\pi n t}{T}} dt$$

$s(t) = V_0 \cos\left(2\pi f_0 t\right)$, $-\infty < t < +\infty$.



Un segnale con poche discontinuità ha uno spettro compatto.

$$s(t) = \frac{a_0}{2} + \sum_{m=1}^{\infty}\left[a_m \cos\left(\frac{2\pi m t}{T}\right) + b_m \,\text{sen}\left(\frac{2\pi m t}{T}\right)\right]$$

$a_0 = \emptyset$ in quanto la media di un coseno è $\emptyset$.

$b_m = \emptyset$ in quanto coseno è pari

$$a_m = \frac{4}{T}\int_{\emptyset}^{T/2} s(t)\cos\left(\frac{2\pi m t}{T}\right)dt = \frac{4}{T}V_0\int_{\emptyset}^{T/2}\cos\left(2\pi f_0 t\right)\cdot\cos\left(2\pi m f_0 t\right)dt$$

$$\Rightarrow \cos\alpha\cdot\cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$$

$$= \frac{\cancel{4}^{2}}{T}\cdot V_0\cdot\frac{1}{\cancel{2}}\left\{\int_{\emptyset}^{T/2}\cos\left(2\pi f_0 t\,(m+1)\right)dt + \int_{\emptyset}^{T/2}\cos\left(2\pi f_0 t\,(m-1)\right)dt\right\}$$

$$= \frac{\cancel{2}}{T}V_0\,\frac{1}{\cancel{2}\pi f_0(m+1)}\,\text{sen}\left[2\pi f_0 t(m+1)\right]_{\emptyset}^{T/2} + \frac{\cancel{2}}{T}V_0\cdot\frac{1}{\cancel{2}\pi f_0(m-1)}\,\text{sen}\left[2\pi f_0 t(m-1)\right]_{\emptyset}^{T/2}$$

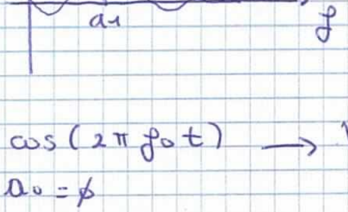
$$= \frac{V_0}{\cancel{T}\pi \cancel{f_0}(m+1)}\cdot\text{sen}\left[\cancel{2}\pi \cancel{f_0}\frac{\cancel{T}}{\cancel{2}}(m+1)\right] + \frac{V_0}{\cancel{T}\pi \cancel{f_0}(m-1)}\,\text{sen}\left[\cancel{2}\pi \cancel{f_0}\frac{\cancel{T}}{\cancel{2}}(m-1)\right]$$

$\to f_0\cdot T = 1$ per $m=1$ $\frac{\text{sen}\,x}{x} \to 1$

$$= \cancel{\frac{V_0\,\text{sen}\left(\pi(m+1)\right)}{\pi(m+1)}}^{\emptyset\ \forall m} + \frac{V_0\,\text{sen}\left(\pi(m-1)\right)}{\pi(m-1)}$$

$a_m = \emptyset$ per $m \neq 1$; $A_m = \emptyset$ $m\neq 1$

$a_1 = V_0$ ($m=1$) ; $b_m = \emptyset$; $A_1 = V_0$; $\phi_1 = \text{arctg}\,\frac{b_1}{a_1} = \emptyset$



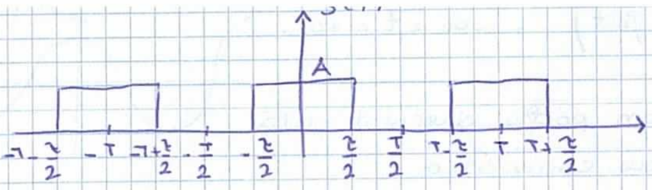
$V_0\cos(2\pi f_0 t) \longrightarrow V_0\,\text{sen}(2\pi f_0 t)$

$\to a_0 = \emptyset$

$\to a_m = \emptyset$

$$b_m = \frac{4}{T}V_0\int_{\emptyset}^{T/2}\text{sen}(2\pi f_0 t)\,\text{sen}(2\pi m f_0 t)\,dt$$

$\text{sen}\,\alpha\,\text{sen}\,\beta = \frac{1}{2}\left[\cos(\alpha-\beta) - \cos(\alpha+\beta)\right] \to b_1 = V_0$, $\phi_1 = \text{arctg}\,\frac{b_1}{}$



$b_m = 0$

$$a_m = \frac{4}{T}\int_0^{T/2} s(t)\cos\left(\frac{2\pi m t}{T}\right)dt = \frac{4}{T}\cdot A\int_0^{\tau/2}\cos\left(\frac{2\pi m t}{T}\right)dt$$

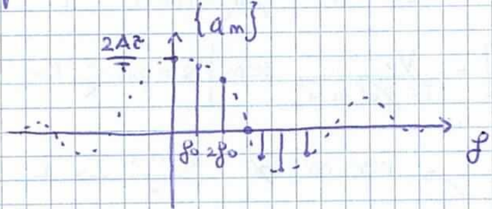
$$= \frac{4}{T}\cdot A \; \frac{1}{\frac{2\pi m}{T}} \; \mathrm{sen}\left(\frac{2\pi m t}{T}\right)\Bigg|_0^{\tau/2} = \frac{\cancel{4}^{2}}{T}\cdot A \cdot \frac{1}{\frac{\cancel{2}\pi m}{T}} \; \mathrm{sen}\left(\frac{\cancel{2}\pi m \tau}{\cancel{2} T}\right) \cdot \frac{\tau}{\tau}$$

$$= \frac{2A\tau}{T}\cdot\frac{\mathrm{sen}\left(\frac{\pi m \tau}{T}\right)}{\left(\frac{\pi m \tau}{T}\right)}$$

$$\frac{\mathrm{sen}(\pi x)}{\pi x} = \mathrm{sinc}(x)$$

$\pi x = k\pi$

$$A_m = \sqrt{a_m^2 + b_m^2} = |a_m|$$



Si azzera quando $\frac{\cancel{\pi} m \tau}{T} = k\cancel{\pi}$

$f = \frac{m}{T} = m f_0 = \frac{k}{\tau}$, k intero

05/03/2015

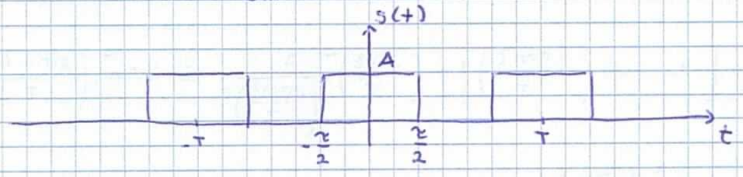
$$s(t) = \sum_{m=-\infty}^{+\infty} \left[\frac{a_m}{2} \cos\left(\frac{2\pi m t}{T}\right) + \frac{b_m}{2} \operatorname{sen}\left(\frac{2\pi m t}{T}\right) \right]$$

$$= \sum_{m=-\infty}^{+\infty} \frac{A_m}{2} \cos\left[\frac{2\pi m t}{T} - \phi_m\right] \quad , \quad A_m = \sqrt{a_m^2 + b_m^2}$$

$$\phi_m = \operatorname{arctg} \frac{b_m}{a_m}$$

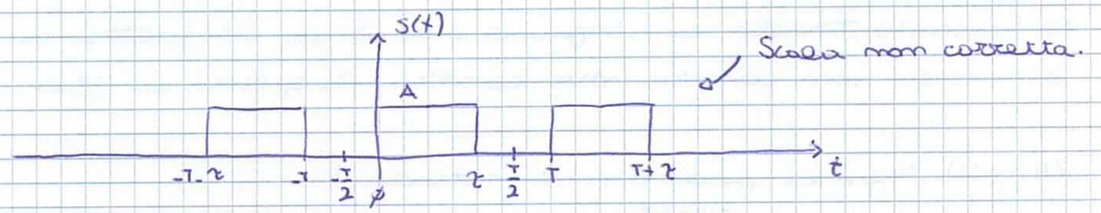
$$= \sum_{m=-\infty}^{+\infty} C_m \, e^{j\frac{2\pi m t}{T}}$$

$C_m \rightarrow S_m$



$$s(t) = \frac{2A\tau}{T} + \sum_{m=1}^{\infty} \frac{2A\tau}{T} \cdot \frac{\operatorname{sen}\left(\frac{m\pi\tau}{T}\right)}{\frac{m\pi\tau}{T}} \cdot \cos\left(\frac{2\pi m t}{T}\right)$$

con $\dfrac{\operatorname{sen}\left(\frac{m\pi\tau}{T}\right)}{\frac{m\pi\tau}{T}} = \operatorname{sinc}\left(\frac{m\tau}{T}\right)$



$$a_m = \frac{2}{T} \int_{-T/2}^{T/2} s(t) \cos\left(\frac{2\pi m t}{T}\right) dt = \frac{2A}{T} \int_0^{\tau} \cos\left(\frac{2\pi m t}{T}\right) dt = \frac{2A}{T} \frac{1}{\frac{2\pi m}{T}} \operatorname{sen}\left(\frac{2\pi m t}{T}\right)\Bigg|_0^{\tau}$$

$$= \frac{2A}{T} \frac{1}{\frac{2\pi m}{T}} \operatorname{sen}\left(\frac{2\pi m \tau}{T}\right) \cdot \frac{\tau}{\tau} = \frac{2A\tau}{T} \operatorname{sinc}\left(\frac{2m\tau}{T}\right)$$

$$b_m = \frac{2}{T} \int_{-T/2}^{T/2} s(t) \operatorname{sen}\left(\frac{2\pi m t}{T}\right) dt = \frac{2A}{T} \int_0^{\tau} \operatorname{sen}\left(\frac{2\pi m t}{T}\right) dt =$$

$$= \frac{2A}{T} \frac{1}{\frac{2\pi m}{T}} \left(-\cos\left(\frac{2\pi m t}{T}\right)\Bigg|_0^{\tau} \right) = \frac{2A}{T} \frac{1}{\frac{2\pi m}{T}} \left(-\cos\left(\frac{2\pi m \tau}{T}\right) + 1 \right)$$

$$\Rightarrow \frac{1 - \cos x}{2} = \operatorname{sen}^2\left(\frac{x}{2}\right)$$

$$= \frac{2A}{T} \cdot \frac{1}{\frac{\cancel{2}\pi m}{T}} \cdot \cancel{2} \operatorname{sen}^2\left(\frac{\pi m \tau}{T}\right) \cdot \frac{\tau}{\tau}$$

Quindi:

$$a_n = \frac{2A\tau}{T} \cdot \frac{\operatorname{sen}\left(\frac{2\pi n\tau}{T}\right)}{\frac{2\pi n\tau}{T}}$$

$$b_n = \frac{2A\tau}{T} \cdot \frac{1}{\frac{2\pi n\tau}{T}} \left[1 - \cos\left(\frac{2\pi n\tau}{T}\right) \right]$$

$$a_n^2 + b_n^2 = \left(\frac{2A\tau}{T}\right)^2 \left\{ \frac{\operatorname{sen}^2\left(\frac{2\pi n\tau}{T}\right)}{\left(\frac{2\pi n\tau}{T}\right)^2} + \frac{1}{\left(\frac{2\pi n\tau}{T}\right)^2} \left[1 + \cos^2\left(\frac{2\pi n\tau}{T}\right) - 2\cos\left(\frac{2\pi n\tau}{T}\right) \right] \right\}$$

$$A_n^2 = \left(\frac{2A\tau}{T}\right)^2 \frac{2}{\left(\frac{2\pi n\tau}{T}\right)^2} \left[1 - \cos\left(\frac{2\pi n\tau}{T}\right) \right] = \left(\frac{2A\tau}{T}\right)^2 \frac{2}{\left(\frac{2\pi n\tau}{T}\right)^2} \cdot 2 \operatorname{sen}^2\left(\frac{\cancel{2}\pi n\tau}{T \cdot \cancel{2}}\right)$$

$$= \left(\frac{2A\tau}{T}\right)^2 \cdot \frac{\cancel{4}}{\cancel{4} \cdot \left(\frac{\pi n\tau}{T}\right)^2} \operatorname{sen}^2\left(\frac{\pi n\tau}{T}\right)$$

$$A_n = \frac{2A\tau}{T} \cdot \frac{\operatorname{sen}\left(\frac{\pi n\tau}{T}\right)}{\frac{\pi n\tau}{T}}$$

=> Quando trasla un segnale nel tempo lo spettro di ampiezza non cambia ma cambia lo spettro di fase.

$s(t)$, $g(t)$ segnali periodici di periodo T.

$$s(t) = \sum_{n=-\infty}^{+\infty} S_n \, e^{j\frac{2\pi n t}{T}}$$

$$g(t) = \sum_{n=-\infty}^{+\infty} G_n \, e^{j\frac{2\pi n t}{T}}$$

$h(t) = s(t) \cdot g(t)$ è un segnale periodico di periodo T.

$$h(t) = \sum_{n=-\infty}^{+\infty} H_n \, e^{j\frac{2\pi n t}{T}}$$

$$H_n = \frac{1}{T}\int_{-T/2}^{T/2} h(t)\, e^{-j\frac{2\pi n t}{T}} dt = \frac{1}{T}\int_{-T/2}^{T/2} s(t) \cdot g(t)\, e^{-j\frac{2\pi n t}{T}} dt$$

$$= \frac{1}{T}\int_{-T/2}^{T/2} \sum_{k=-\infty}^{+\infty} S_k\, e^{j\frac{2\pi k t}{T}} \cdot \sum_{m=-\infty}^{+\infty} G_m\, e^{j\frac{2\pi m t}{T}} \cdot e^{-j\frac{2\pi n t}{T}} dt.$$

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} S_k G_m \int_{-T/2}^{T/2} e^{-j\frac{2\pi t}{T}\cdot(n-k-m)} dt.$$

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} S_k G_m \frac{1}{-j\frac{2\pi}{T}(n-k-m)} \cdot e^{-j\frac{2\pi t}{T}\cdot(n-k-m)} \Bigg|_{-T/2}^{T/2}$$

$$= \frac{1}{T} \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} S_k G_m \frac{1}{-j\frac{2\pi}{T}(n-k-m)} \left[e^{-\frac{j\cancel{2}\pi\cancel{T}}{\cancel{2T}}(n-k-m)} - e^{\frac{j\cancel{2}\pi\cancel{T}}{\cancel{2T}}(n-k-m)} \right]$$

Cambio segno coe segno $-2j$!

$$\Rightarrow \operatorname{sen} x = \frac{e^{jx} - e^{-jx}}{2j}$$

$$= \frac{1}{\cancel{T}} \sum_{k=-\infty}^{+\infty} \sum_{m=-\infty}^{+\infty} S_k G_m \frac{\operatorname{sen}(\pi(n-k-m))}{\frac{\pi}{\cancel{T}}(n-k-m)} = H_n$$

$n - k - m = \emptyset$

$m = n - k \longrightarrow \frac{\operatorname{sen}(\emptyset)}{\emptyset} = 1$!

$$H_n = \sum_{k=-\infty}^{+\infty} S_k G_{n-k}$$

$s(t)$, Periodico di periodo T.

$$P_s = \lim_{T' \to \infty} \frac{1}{T'} \int_{-T'/2}^{T'/2} s^2(t)\, dt = \lim_{T' \to \infty} \frac{1}{T'} \int_{-T'/2}^{T'/2} |s(t)^2|\, dt \quad , \text{con } T' = nT$$

nell'intervallo T: $P_s = \lim_{T \to \infty} \frac{1}{T} \int_{-T/2}^{T/2} |s(t)|^2\, dt$

Ma P_s ha lo stesso valore in ogni T, quindi: $\cancel{n} \frac{1}{T'(=\cancel{n}T)} = \frac{1}{T}$

$$P_s = \frac{1}{T} \int_{-T/2}^{T/2} s^2(t)\, dt$$

$$s(t) = \sum_{n=-\infty}^{+\infty} S_n e^{\frac{j2\pi nt}{T}}$$

$$\int_{-T/2}^{T/2} s^2(t)\, dt = \int_{-T/2}^{T/2} \sum_{n=-\infty}^{+\infty} S_n e^{\frac{j2\pi nt}{T}} \cdot \sum_{k=-\infty}^{+\infty} S_k e^{\frac{j2\pi kt}{T}}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} S_n S_k \int_{-T/2}^{T/2} e^{\frac{j2\pi t}{T}(n+k)}\, dt$$

$$= \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} S_n S_k \frac{1}{\frac{2j\pi(n+k)}{T}} \left[e^{\frac{j\cancel{2}\pi(n+k)}{\cancel{T}} \cdot \frac{\cancel{T}}{\cancel{2}}} - e^{-\frac{j\cancel{2}\pi(n+k)}{\cancel{T}} \cdot \frac{\cancel{T}}{2}} \right]$$

$\operatorname{sen}(\pi(n+k))$

$$= \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} S_n S_k \frac{1}{\frac{\pi(n+k)}{T}} \cdot \operatorname{sen}(\pi(n+k))$$

$$P_s = \frac{1}{T} \int_{-T/2}^{T/2} s^2(t)\, dt = \frac{1}{\cancel{T}} \sum_{n=-\infty}^{+\infty} \sum_{k=-\infty}^{+\infty} S_n S_k \cdot \frac{1}{\frac{\pi(n+k)}{\cancel{T}}} \cdot \operatorname{sen}(\pi(n+k))$$

$n = -k \Rightarrow P_s = \sum_{n=-\infty}^{+\infty} S_n S_{-n}$, altrimenti $P_s = \emptyset$.

$$S_m = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\, e^{-j\frac{2\pi m t}{T}}\, dt$$

$$S_{-m} = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\, e^{j\frac{2\pi m t}{T}}\, dt$$

$$S_m^* = \frac{1}{T}\int_{-T/2}^{T/2} s^*(t)\, e^{j\frac{2\pi m t}{T}}\, dt = \frac{1}{T}\int_{-T/2}^{T/2} s(t)\, e^{j\frac{2\pi m t}{T}}\, dt = S_{-m}$$

Vero per i segnali reali.

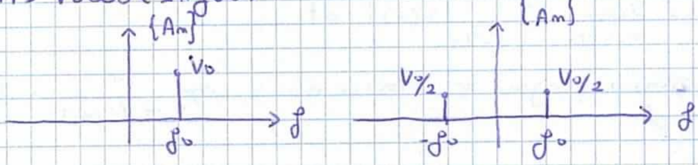
$$\Rightarrow P_s = \sum_{m=-\infty}^{+\infty} S_m S_{-m} = \sum_{m=-\infty}^{+\infty} |S_m|^2 = \frac{1}{T}\int_{-T/2}^{T/2} |s(t)|^2\, dt$$

Teorema di Parseval per i Segnali Periodici

$$S_m = |S_m|\, e^{-j\theta_m}$$

$$|S_m|^2 = S_m S_m^* = |S_m|\, \cancel{e^{-j\theta_m}}\, |S_m|\, \cancel{e^{j\theta_m}}$$

$$s(t) = V_0 \cos(2\pi f_0 t)$$



$$P_s = \sum_{m=-\infty}^{+\infty} |S_m|^2$$

$$C_0 = \frac{1}{2} A_0 = \emptyset$$

$$C_m = S_m = \frac{1}{2} A_m\, e^{-j\phi_m}$$

$$S_1 = \frac{1}{2} A_1\, e^{-j\phi_1} = \frac{1}{2} V_0\, e^{-j\phi_1}$$

$$S_{-1} = S_1^* = \frac{1}{2} A_1\, e^{j\phi_1}$$

Se un segnale è periodico e va da $-\infty$ a $+\infty$:

$$P_s = \frac{V_0^2}{4} + \frac{V_0^2}{4} = \frac{V_0^2}{2}$$

Prendiamo $s(t)$ tale che:

$$\int_{-\infty}^{+\infty} s^2(t)\,dt = M < \infty$$, energia finita!

$$\int_{-\infty}^{+\infty} |s(t)^2|\,dt = M < \infty$$ $s(t)$ è a quadrato integrabile.

Se è vero questo posso definire la Trasformata di Fourier.

→ Trasformata di Fourier $S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt$.

Adesso da $S(f)$ posso risolvere a $s(t)$.

$$S(f) \rightarrow s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{j2\pi f t}\, df$$

$$S(f) = \underbrace{|S(f)|}_{\text{Spettro di Ampiezza}}\, e^{-j\underbrace{\theta(f)}_{\text{Spettro di Fase}}}$$

$$\overset{s(t)?}{S(f)} = \int_{-\infty}^{+\infty} |S(f)|\, e^{j[2\pi f t \overset{!}{\oplus} \theta(f)]}\, df.$$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt$$

$$S^*(f) = \int_{-\infty}^{+\infty} s^*(t)\, e^{j2\pi f t}\, dt$$, $s(t)$ reale.

$$S^*(f) = \int_{-\infty}^{+\infty} s(t)\, e^{j2\pi f t}\, dt$$

$$S(-f) = \int_{-\infty}^{+\infty} s(t)\, e^{j2\pi f t}\, dt = S^*(f)$$

$$S(f) = |S(f)|\, e^{-j\theta(f)}$$

$$S^*(f) = |S(f)|\, e^{j\theta(f)}$$

$$S(-f) = |S(-f)|\, e^{-j\theta(f)}$$

$\Rightarrow$ $S^*(f) = S(-f)$

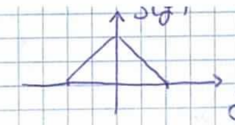
$|S^*(f)| = |S(-f)|$

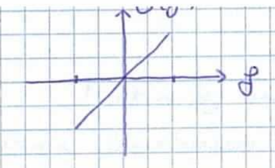
ovvero $|S(f)| = |S(-f)|$

Lo spettro di ampiezza $|S(f)|$ di un segnale reale è simmetrico rispetto all'origine

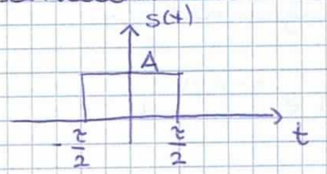
$\Rightarrow \theta(f) = -\theta(-f)$

Lo spettro di fase $\theta(f)$ di un segnale reale è antisimmetrico





Prendiamo un segnale non periodico:



$s(t) = A \, rect\left(\frac{t}{\tau}\right)$ → per il Disegno (grafico).

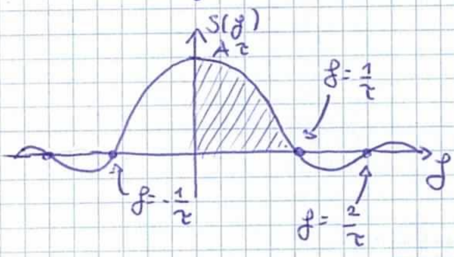
Ampiezza

Durata Segnale

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt = \int_{-\frac{\tau}{2}}^{\tau/2} A \cdot e^{-j2\pi f t}\, dt = \frac{A}{-j2\pi f} \cdot e^{-j2\pi f t}\Big|_{-\tau/2}^{\tau/2}$$

$$= \frac{A}{-j2\pi f}\left\{ e^{-j2\pi f\frac{\tau}{2}} - e^{j2\pi f\frac{\tau}{2}} \right\} = \frac{A}{\pi f} \, sen(\pi f \tau) \cdot \frac{\tau}{\tau}$$

$$= A\tau \cdot \frac{sen(\pi f \tau)}{\pi f \tau} = A\tau \cdot sinc(f\tau).$$



Quindi lo spettro di Ampiezza di un rettangolo è infinito.
Nessun dispositivo è in grado di trasmettere tutte le frequenze.
RETTANGOLO non è riproducibile idealmente.

assume valore $\emptyset$ quando:

$k\cancel{\pi} = \cancel{\pi} f \tau$, $k \neq \emptyset$ interi

$f = \frac{k}{\tau}$

Per conoscere il segno del rettangolo basta che trasmetta solo le frequenze da $\emptyset$ a $\frac{1}{\tau}$.

TRASM. — CANALE — RICEV.

B_T: Banda di Trasmissione $= \left(0 \div \frac{1}{\tau}\right)$

$B_T = \frac{1}{\tau}$

Voglio trasmettere 1000 bit al secondo

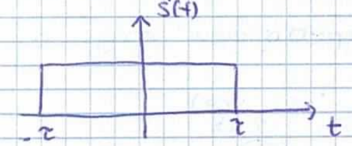
$$\frac{1\ sec}{1000} = \tau \Rightarrow B_\tau = \frac{1}{\tau} = \frac{1}{\frac{1}{1000}} = 1000\ Hz.$$

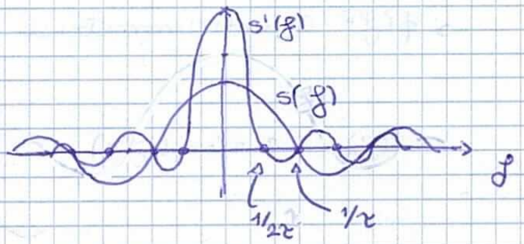
La banda deve avere almeno 1 kHz di banda!

Banda = 1000 Hz

Velocità trasmissione = 1000 bit/sec.

Prendiamo un rettangolo $s'(t)$ doppio del precedente $s(t)$.





$$S(f) = A\tau \cdot \frac{sen(\pi f \tau)}{\pi f \tau}$$

$$S'(f) = A 2\tau \cdot \frac{sen(2\pi f \tau)}{\pi f 2\tau}$$

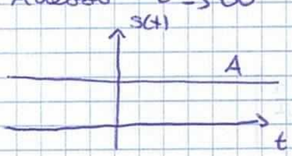
Zeri in: $\cancel{\pi} f 2\tau = k\cancel{\pi} \Rightarrow f = \frac{k}{2\tau}$

$A\phi$ di $s'(t)$ è doppio di quello di $s(t)$.

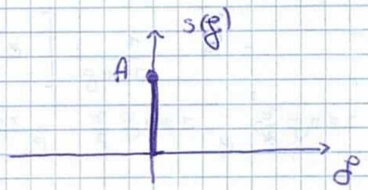
$s'(t)$ ha il doppio degli zeri.

E' richiesta la metà della banda

Adesso $\tau \to \infty$



$\Rightarrow$



adesso il segnale è tutto descritto dalla frequenza ϕ.

Più le discontinuità sono rapide più lo spettro è ampio!

alta frequenza per variazioni brusche, bassa frequenza per variazioni lente!

09/03/2015

$s(t) \longrightarrow \int_{-\infty}^{+\infty} s^2(t)\,dt = M < \infty$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt = |S(f)|\, e^{-j\phi(f)}$$

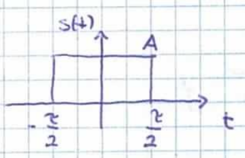
$|S(f)|$: spettro di ampiezza

$\phi(f)$: spettro di fase

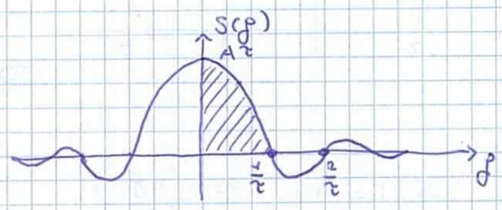
$$s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{j2\pi f t}\, df$$

$s(t)$ reale $\rightarrow$ $|S(f)|$ è pari, simmetrico rispetto all'origine

$\rightarrow$ $\phi(f)$ è antisimmetrico rispetto all'origine



$$S(f) = A\tau\, \text{sinc}(f\tau)$$

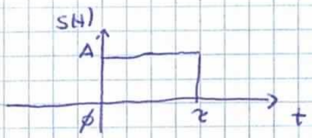


$\cancel{\pi} f\tau = k\cancel{\pi}$, k intero

$f = \frac{k}{\tau}$ zero!

1000 bit/sec = 1 Kb/s

$\tau = \frac{1}{1000}$, $B = \frac{1}{\tau} = 1000\ Hz$



$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt = A \int_{0}^{\tau} e^{-j2\pi f t}\, dt$$

$$= A \cdot \frac{1}{-j2\pi f} \left[e^{-j2\pi f \tau} - 1 \right]$$

$$= A \frac{1}{-j2\pi f} \left[e^{-j\cancel{2}\pi f \frac{\tau}{\cancel{2}}} e^{-j\cancel{2}\pi f \frac{\tau}{\cancel{2}}} - e^{-j\cancel{2}\pi f \frac{\tau}{\cancel{2}}} e^{j\cancel{2}\pi f \frac{\tau}{\cancel{2}}} \right]$$

$$= \frac{A}{-j2\pi f} e^{-j\cancel{2}\pi f \frac{\tau}{\cancel{2}}} \left[e^{-j\pi f \tau} - e^{j\pi f \tau} \right] = \frac{\tau}{\tau} \cdot \frac{A}{\pi f} e^{-j\pi f \tau} \left(\frac{e^{+j\pi f\tau} - e^{-j\pi f \tau}}{2j} \right)$$

$$= A\tau\, e^{-j\pi f \tau} \cdot \frac{\text{sen}(\pi f \tau)}{\pi f \tau} = S(f) = |S(f)|\, e^{-j\phi(f)}$$

$$|S(f)| = A\tau \cdot \frac{\text{sen}(\pi f \tau)}{\pi f \tau} \quad , \quad \phi(f) = \pi f \tau$$

$$S(f) = |S(f)|\, e^{-j\phi(f)}$$

$$s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{j2\pi f t}\, df = \int_{-\infty}^{+\infty} |S(f)|\, e^{j[2\pi f t - \theta(f)]}\, df$$

$s(t)$ ~~reale~~ $\rightarrow$ $|S(f)| = |S(-f)|$

$\theta(f) = -\theta(-f)$

$$= \int_{-\infty}^{0} |S(f)|\, e^{j[2\pi f t - \theta(f)]}\, df + \int_{0}^{+\infty} |S(f)|\, e^{j[2\pi f t - \theta(f)]}\, df$$

$f \rightarrow -f$

$$= \int_{+\infty}^{0} |S(-f)|\, e^{j[-2\pi f t - \theta(-f)]}\, d(-f) + \dots$$

$$= \int_{0}^{+\infty} |S(f)|\, e^{-j[+2\pi f t - \theta(f)]}\, df + \dots$$

$$= \int_{0}^{+\infty} |S(f)|\, e^{-j[2\pi f t - \theta(f)]}\, df + \int_{0}^{+\infty} |S(f)|\, e^{j[2\pi f t - \theta f]}\, df$$

$$= \int_{0}^{+\infty} |S(f)| \left[\underbrace{\frac{e^{j[2\pi f t - \theta(f)]} + e^{-j[2\pi f t - \theta(f)]}}{2}} \right] df \cdot \frac{2}{2}$$

$$= 2 \cdot \int_{0}^{+\infty} |S(f)| \cos(2\pi f t - \theta(f))\, df$$

$$= \int_{-\infty}^{+\infty} S(f)\, e^{j2\pi f t}\, df$$

$s(t) \rightarrow S(f)$

$$s(t - t_0) \rightarrow S_1(f) = \int_{-\infty}^{+\infty} s(t - t_0)\, e^{-j2\pi f t}\, dt$$

$t - t_0 = \tau$

$$= \int_{-\infty}^{+\infty} s(\tau)\, e^{-j2\pi f (t_0 + \tau)}\, d\tau = \int_{-\infty}^{+\infty} s(\tau)\, e^{-j2\pi f t_0}\, e^{-j2\pi f \tau}\, d\tau$$

$$= e^{-j2\pi f t_0} \underbrace{\int_{-\infty}^{+\infty} s(\tau)\, e^{-j2\pi f \tau}\, d\tau}_{S(f)} = S(f)\, e^{-j2\pi f t_0} = S_1(f)$$

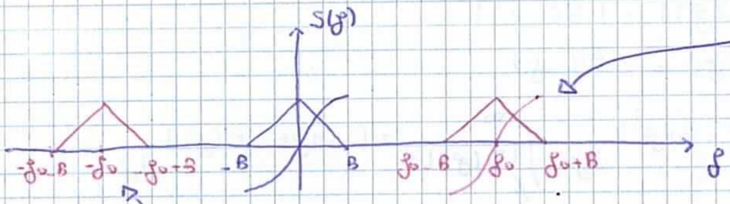
$|S_1(f)| = |S(f)|$

$\phi_1(f) = \phi(f) + 2\pi f t_0$

$s(t) \to S(f)$

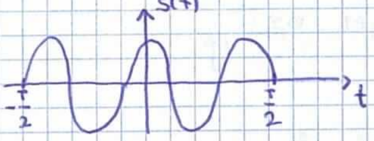
$s_1(t) = s(t)\, e^{j2\pi f_0 t} \to S_1(f) = \int_{-\infty}^{+\infty} s(t)\, e^{j2\pi f_0 t}\, e^{-j2\pi f t}\, dt$

$= \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi (f-f_0) t}\, dt \underset{f-f_0=\lambda}{=} \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi \lambda t} = S(\lambda) = S(f-f_0)$



$s(t)\, e^{-j2\pi f_0 t} \to S_1(f) = S(f+f_0)$

Consideriamo un coseno troncato:



$$s(t) = \begin{cases} V_0 \cos(2\pi f_0 t) & -\frac{T}{2} < t < \frac{T}{2} \\ \emptyset & \text{altrove} \end{cases}$$

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt = V_0 \int_{-\frac{T}{2}}^{\frac{T}{2}} \cos(2\pi f_0 t)\, e^{-j2\pi f t}\, dt$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$= \frac{V_0}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} \left[e^{j2\pi f_0 t} + e^{-j2\pi f_0 t} \right] e^{-j2\pi f t}\, dt$$

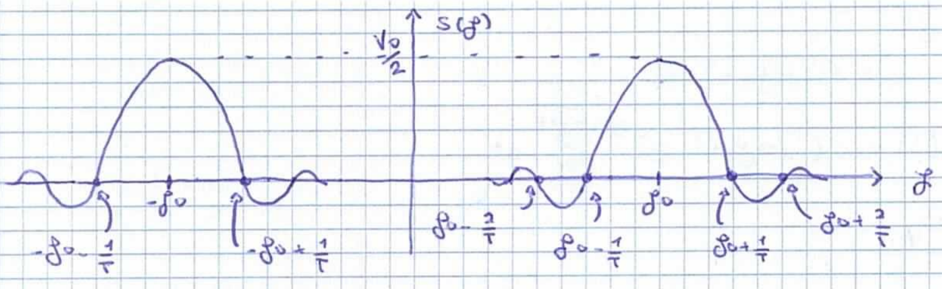
$$= \frac{V_0}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-j2\pi (f-f_0) t}\, dt + \frac{V_0}{2} \int_{-\frac{T}{2}}^{\frac{T}{2}} e^{-j2\pi (f+f_0) t}\, dt$$

$$= \frac{V_0}{2} \frac{1}{-j2\pi (f-f_0)} \left[e^{-j\cancel{2}\pi (f-f_0)\frac{T}{\cancel{2}}} - e^{j\cancel{2}\pi (f-f_0)\frac{T}{\cancel{2}}} \right] +$$

$$+ \frac{V_0}{2} \frac{1}{-j2\pi (f+f_0)} \left[e^{-j\cancel{2}\pi (f+f_0)\frac{T}{\cancel{2}}} - e^{j\cancel{2}\pi (f+f_0)\frac{T}{\cancel{2}}} \right]$$

$$= \frac{V_0}{2} \cdot \frac{1}{\pi (f-f_0)} \operatorname{sen}[\pi (f-f_0) T] \cdot \frac{T}{T} + \frac{V_0}{2} \frac{1}{\pi (f+f_0)} \operatorname{sen}[\pi (f+f_0) T] \cdot \frac{T}{T}$$

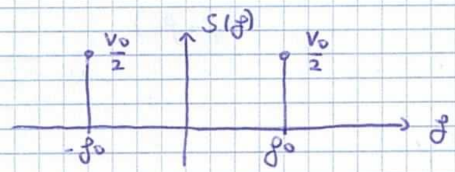
$$= \frac{V_0}{2} \cdot T \operatorname{sinc}[\pi (f-f_0) T] + \frac{V_0}{2} \cdot T \operatorname{sinc}[(f+f_0) T]$$



$\cancel{\pi}(f-f_0)T = K\cancel{\pi} \Rightarrow f - f_0 = \frac{K}{T} \Rightarrow f = f_0 + \frac{K}{T}$

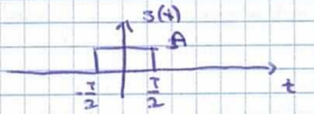
Mentre:

$s(t) = V_0 \cdot \cos(2\pi f_0 t) \quad , \quad -\infty < t < +\infty$



Se moltiplico:

$s(t) = A \cdot rect\left(\frac{t}{T}\right)$



per un coseno che va da $-\infty$ a $+\infty$

$$s(t) = V_0 \cdot rect\left(\frac{t}{T}\right) \cos(2\pi f_0 t) = \begin{cases} s(t) = V_0 \cos(2\pi f_0 t) & -\frac{T}{2} < t < \frac{T}{2} \\ \emptyset & \text{altrove} \end{cases}$$

Quando trasla raddoppio la banda di frequenze da trasmettere in quanto io trasmetto solo frequenze positive!

Se non moltiplico per un coseno io trasmetto solo il segnale considerato fra $\emptyset \div +\infty$.

$s(t) \to S(f)$

$g(t) = s(t) \cos(2\pi f_0 t)$

$$G(f) = \int_{-\infty}^{+\infty} s(t) \cos(2\pi f_0 t)\, e^{-j2\pi f t}\, dt$$

$$\cos x = \frac{e^{jx} + e^{-jx}}{2}$$

$$= \frac{1}{2} \int_{-\infty}^{+\infty} s(t) \left[e^{+j2\pi f_0 t} + e^{-j2\pi f_0 t} \right] e^{-j2\pi f t}\, dt$$

$$= \underbrace{\frac{1}{2} \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi(f-f_0)t}\, dt}_{S(f-f_0)} + \underbrace{\frac{1}{2} \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi(f+f_0)t}\, dt}_{S(f+f_0)}$$

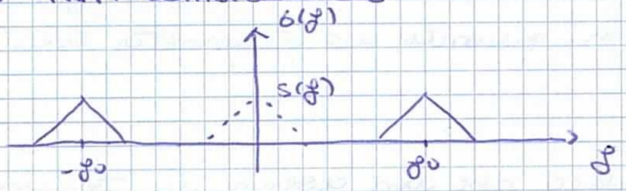
$$G(f) = \frac{1}{2} \left[S(f-f_0) + S(f+f_0) \right]$$

$$\operatorname{sen} x = \frac{e^{jx} - e^{-jx}}{2j}$$

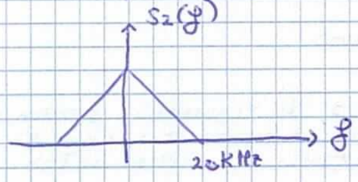
Se uso il seno lo spettro di ampiezza è lo stesso ma cambia quello di fase.

modulazione: moltiplicare un segnale per un coseno
→ traslare lo spettro in frequenza

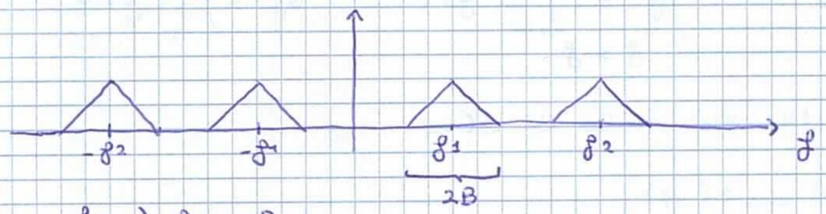
Se una costante moltiplica tutte le frequenze essa è un'attenuazione ma non una distorsione
→ Posso non considerarla



Ho due segnali $s_1(t)$ e $S_2(t)$



Per trasmettere due segnali insieme li trasmetto a frequenze diverse in modo che non si sovrappongono.
(FM)



$f_2 \geq f_1 + 2B$

$s(t) \rightarrow g(t) = s(t) \cdot \cos(2\pi f_0 t)$

adesso sono dalla parte del ricevitore che vuole recuperare il messaggio.

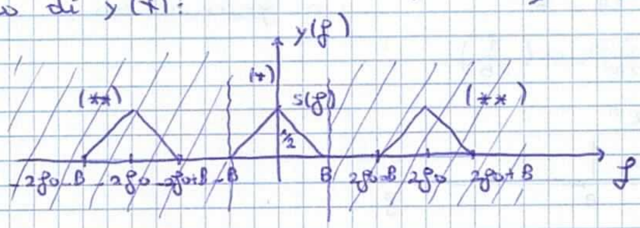
$y(t) = s(t)\cos(2\pi f_0 t) \cdot \cos(2\pi f_0 t)$

$\rightarrow \cos\alpha \cdot \cos\beta = \frac{1}{2}\left[\cos(\alpha+\beta) + \cos(\alpha-\beta)\right]$

$= \underbrace{\frac{s(t)}{2} \cdot \cos\left[4\pi f_0 t\right]}_{(**)} + \underbrace{\frac{s(t)}{2} \cos \cancel{0}^{\,1}}_{(*)}$

il fattore $\frac{1}{2}$ non interessa

Spettro di $y(t)$:



Inserendo un filtro passa-basso elimino le frequenze che non mi interessano

$s(t) \longrightarrow \otimes \longrightarrow \dots$
$\uparrow \cos(2\pi f_0 t)$ ← Siamo dalla parte del trasmettitore.

Modulatore

Teorema della Dualità o Reciprocità

$s(t) \to S(f)$
$S(t) \to s(-f)$

Consente l'uso di risultati ottenuti in un senso per derivare senza calcoli i risultati ottenuti nell'altro.

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt$$

$$S(t) = \int_{-\infty}^{+\infty} s(f)\, e^{-j2\pi f t}\, df \underset{f \to -f}{=} -\int_{+\infty}^{-\infty} s(-f)\, e^{j2\pi f t}\, df = \int_{-\infty}^{+\infty} s(-f)\, e^{j2\pi f t}\, df$$

$S(t)$ è l'antitrasformata di $s(-f)$.

$$S(t) \leftrightarrow s(-f)$$

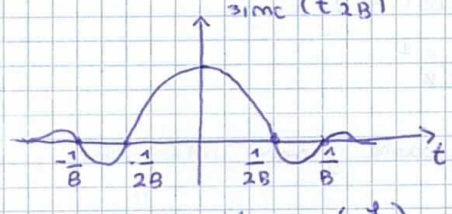
$$A\,\mathrm{rect}\left(\frac{t}{T}\right) \leftrightarrow A\,T\,\mathrm{sinc}(fT)$$

$$\cancel{A}\,T\,\mathrm{sinc}(tT) \leftrightarrow A\cdot\mathrm{rect}\left(\frac{-f}{T}\right) = \cancel{A}\,\mathrm{rect}\left(\frac{f}{T}\right)$$

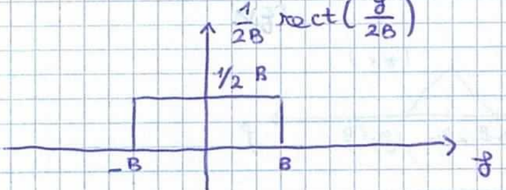
$T = 2B$

$$\mathrm{sinc}(t\,2B) \leftrightarrow \frac{1}{2B}\,\mathrm{rect}\left(\frac{f}{2B}\right)$$

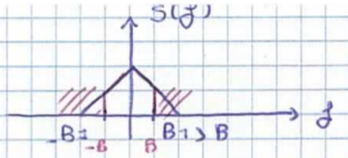
$$\frac{\mathrm{sen}(\pi t \cdot 2B)}{\pi t\, 2B} \Rightarrow \cancel{\pi} t\, 2B = k\cancel{\pi} \Rightarrow t = \frac{k}{2B}$$



La trasformata di un $\frac{\mathrm{sen}\,x}{x}$ nel tempo è un rettangolo in frequenza.



Il rettangolo nel tempo ha come trasformata la sinc in frequenza. Quindi la sinc nel tempo ha come trasformata il rettangolo (con $-f$) in frequenza. Ma rettangolo è pari, quindi [illegible].



Supponiamo di avere un segnale s(t) con questo spettro di ampiezza

Un filtro passa basso fa passare solo le frequenze vicine allo $\emptyset$.

$$s(t) \to S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt$$

$$S^*(f) = \int_{-\infty}^{+\infty} s^*(t)\, e^{j2\pi f t}\, dt$$

$$t \to -t$$

$$= -\int_{+\infty}^{-\infty} s^*(-t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty} s^*(-t)\, e^{-j2\pi f t}\, dt$$

$$S^*(f) \leftrightarrow s^*(-t)$$

se $s(t)$ reale $\to s^*(-t) = s(-t)$

$\Rightarrow s(-t) \leftrightarrow S^*(f)$

Quindi: $s(t) \leftrightarrow S(f)$

$s(-t) \leftrightarrow S^*(f)$

10/03/2015

PROPRIETÀ

$s(t) \to S(f)$

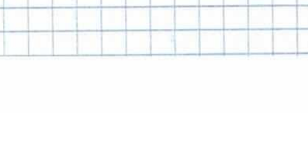
$s(t-t_0) \to S(f)\, e^{-j2\pi f t_0}$

$$s(t) = \begin{cases} \cos(2\pi f_0 t) & -\frac{T}{2} \le t \le \frac{T}{2} \\ \emptyset & \text{altrove} \end{cases} \qquad g(t) = s(t)\cos(2\pi f_0 t)$$

$s(t) \to S(f)$

$S(t) \to s(-f)$

Esercizio:



Sicuramente lo spettro delle ampiezze è più compatto di quello del rettangolo.

$$S(f) = \int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi f t}\, dt = \int_{-\tau}^{\tau} s(t) e^{-j2\pi f t}\, dt$$

Per la retta $t > \phi$

Passa per (ϕ, A) e (τ, ϕ)

$s(t) = mt + n$

$n = A$

$s(t) = mt + A$

$\phi = m\tau + A \Rightarrow m = -\frac{A}{\tau}$

Per $t < \phi$

$s(t) = mt + A$

$\phi = -m\tau + A \Rightarrow m = \frac{A}{\tau}$

$$s(t) = \begin{cases} \frac{A}{\tau} \cdot t + A \cdot \frac{\tau}{\tau} & se \quad -\tau \le t \le \phi \\ -\frac{A}{\tau} \cdot t + A \cdot \frac{\tau}{\tau} & se \quad \phi \le t \le \tau \end{cases}$$

$$= \begin{cases} \frac{A}{\tau}(t+\tau) & se \quad -\tau \le t \le \phi \\ \frac{A}{\tau}(-t+\tau) & se \quad \phi \le t \le \tau \end{cases}$$

$$\Rightarrow \int_{-\tau}^{\phi} \frac{A}{\tau}(\tau+t) e^{-j2\pi ft} dt + \int_{\phi}^{\tau} \frac{A}{\tau}(\tau-t) e^{-j2\pi ft} dt$$

$t \to -t$

$$= \int_{\tau}^{\phi} \frac{A}{\tau}(\tau-t) e^{j2\pi ft} d(-t) + \dots$$

$$= \int_{\phi}^{\tau} \frac{A}{\tau}(\tau-t) e^{j2\pi ft} dt + \int_{\phi}^{\tau} \frac{A}{\tau}(\tau-t) e^{-j2\pi ft} dt$$

$$= \frac{A}{\tau} \cdot 2 \int_{\phi}^{\tau} (\tau-t) \left[e^{j2\pi ft} + e^{-j2\pi ft} \right] \cdot \frac{1}{2} dt$$

$$S(f) = \frac{2A}{\tau} \int_{\phi}^{\tau} (\tau-t) \cos(2\pi ft) dt =$$

Per parti

$$= \frac{2A}{\cancel{\tau}} \cdot \cancel{\tau} \int_{\phi}^{\tau} \cos(2\pi ft) dt - \frac{2A}{\tau} \cdot \int_{\phi}^{\tau} t \cdot \cos(2\pi ft) dt$$

$$= 2A \cdot \frac{1}{2\pi f} \cdot sen(2\pi ft) - \frac{2A}{\tau} \cdot t \frac{1}{2\pi f} \cdot sen(2\pi ft) \Big|_{\phi}^{\tau} + \frac{2A}{\tau} \cdot \frac{1}{2\pi f} \int_{\phi}^{\tau} sen(2\pi ft) dt$$

$$= \cancel{2A \cdot \frac{sen(2\pi f\tau)}{2\pi f}} \cdot \frac{\tau}{\tau} - \cancel{\frac{2A}{\tau} \cdot \tau \frac{1}{2\pi f} sen(2\pi f\tau)} + \frac{2A}{\tau} \frac{1}{(2\pi f)^2} \left[-\cos(2\pi f\tau) + 1 \right]$$

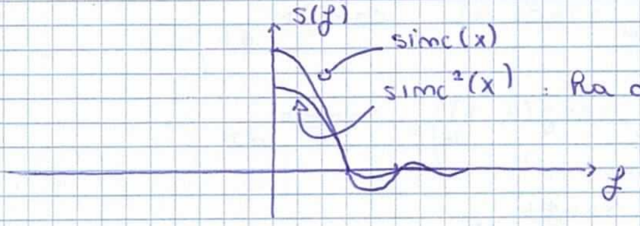
$$\Rightarrow \frac{1-\cos x}{2} = sen^2 \frac{x}{2}$$

$$= \frac{2A}{\tau} \frac{1}{(2\pi f)^2} \, 2\,\text{sen}^2(\pi f \tau) \cdot \frac{\tau}{\tau}$$

$$= \frac{\cancel{4}A\tau}{(\cancel{2}\pi f \tau)^2} \text{sen}^2(\pi f \tau) = A\tau \cdot \frac{\text{sen}^2(\pi f \tau)}{(\pi f \tau)^2} = A\tau \, \text{sinc}^2(f\tau)$$

Solo $f > \cancel{0}$!

$$\cancel{\pi} f \tau = k\cancel{\pi} \Rightarrow f = \frac{k}{\tau}$$



Dati due segnali:

$s(t) \longleftrightarrow S(f)$

$g(t) \longleftrightarrow G(f)$

$h(t) = s(t) \cdot g(t)$

$$H(f) = \int_{-\infty}^{+\infty} h(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty} s(t) \cdot g(t)\, e^{-j2\pi f t}\, dt$$

$$g(t) = \int_{-\infty}^{+\infty} G(f')\, e^{j2\pi f' t}\, df'$$

$$= \int_{-\infty}^{+\infty} s(t) \left\{ \int_{-\infty}^{+\infty} G(f')\, e^{j2\pi f' t}\, df' \right\} \cdot e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} G(f') \underbrace{\int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi (f - f')t}\, dt}_{f - f' = \lambda} \cdot df'$$

$$\int_{-\infty}^{+\infty} s(t)\, e^{-j2\pi t\lambda}\, dt = S(\lambda)$$

$$= \int_{-\infty}^{+\infty} G(f')\, S(\lambda)\, df' = -\int_{+\infty}^{-\infty} S(\lambda)\, G(f-\lambda)\, d\lambda = \int_{-\infty}^{+\infty} S(\lambda)\, G(f-\lambda)\, d\lambda = H(f).$$

$$S(f) * G(f) = \int_{-\infty}^{+\infty} S(\lambda)\, G(f-\lambda)\, d\lambda = H(f)$$

è convoluzione fra $S(f)$ e $G(f)$.

$s(t)$, $g(t)$ reali

$R(t)$ convoluzione tra $s(t)$ e $g(t) = \int_{-\infty}^{+\infty} s(\tau)\, g(t-\tau)\, d\tau$

$$H(f) = \int_{-\infty}^{+\infty} R(t)\, e^{-j2\pi f t}\, dt = \int_{-\infty}^{+\infty} \left(\int_{-\infty}^{+\infty} s(\tau)\, g(t-\tau)\, d\tau \right) e^{-j2\pi f t}\, dt$$

$$= \int_{-\infty}^{+\infty} s(\tau) \left(\int_{-\infty}^{+\infty} g(t-\tau)\, e^{-j2\pi f t}\, dt \right) d\tau \quad . \; e^{-j2\pi f \tau}\; e^{j2\pi f \tau}$$

$$= \int_{-\infty}^{+\infty} s(\tau) \left(\int_{-\infty}^{+\infty} g(t-\tau)\, e^{-j2\pi f (t-\tau)} \cdot e^{-j2\pi f \tau}\, dt \right) d\tau$$

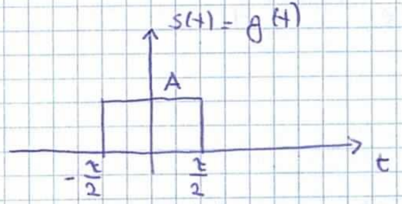
$$= \int_{-\infty}^{+\infty} s(\tau) \left(\int_{-\infty}^{+\infty} g(t-\tau)\, e^{-j2\pi f (t-\tau)}\, dt \right) e^{-j2\pi f \tau}\, d\tau$$

$$\hookrightarrow \; g(t')\, e^{-j2\pi f t'}\, dt'$$

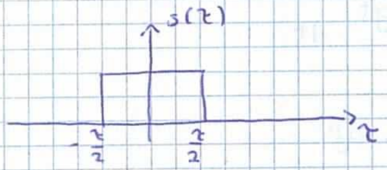
$$= G(f) \int_{-\infty}^{+\infty} s(\tau)\, e^{-j2\pi f \tau}\, d\tau = G(f) \cdot S(f)$$

$s(t)$, $g(t)$ complessi

$$R(t) = \int_{-\infty}^{+\infty} s(\tau)\, g^*(t-\tau)\, d\tau$$

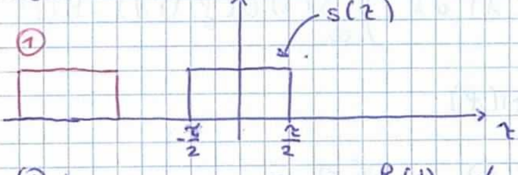


$$R(t) = \int_{-\infty}^{+\infty} s(\tau)\, g(t-\tau)\, d\tau$$



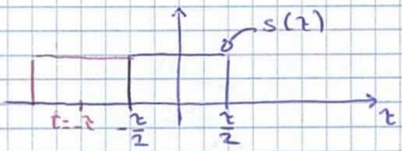
mentre $g(t)$ da cui deriva $g(t-\tau)$ trasla sull'asse avendo centro in $\tau = t$!

Se :

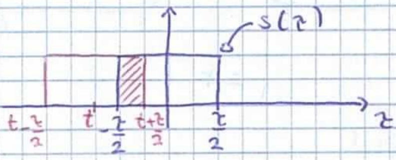


● → $g(t-\tau)$

① In questo caso $R(t) = \emptyset$



Fino a $t < -\tau \Rightarrow R(t) = \emptyset$.



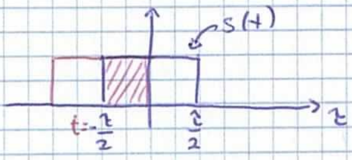
Adesso $R(t) \neq \emptyset$

$$\int_{-\frac{\tau}{2}}^{t+\frac{\tau}{2}} \underbrace{s(t)}_{A} \cdot \underbrace{g(t-\tau)}_{A} \, d\tau = A^2 \int_{-\frac{\tau}{2}}^{t+\frac{\tau}{2}} d\tau$$

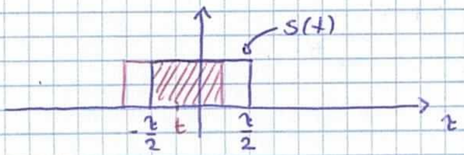
$$= A^2 \left[\left(t + \frac{\tau}{2}\right) + \frac{\tau}{2} \right] = A^2 (\tau + t)$$

$R(t) = \emptyset$ per $-\infty < t \leq -\tau$

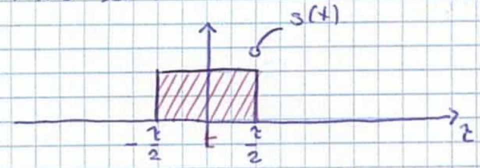
$R(t) = A^2 (t+\tau)$ $\quad -\tau \leq t \leq \emptyset$



Poi:

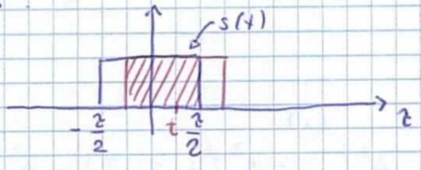


Fino a:



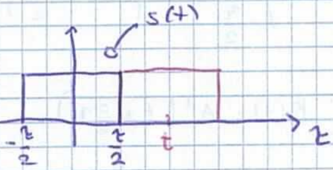
$$\rightarrow A^2 \int_{-\frac{\tau}{2}}^{\frac{\tau}{2}} d\tau = A^2 \cancel{2} \frac{\tau}{\cancel{2}} = A^2 \tau$$

Dopo:



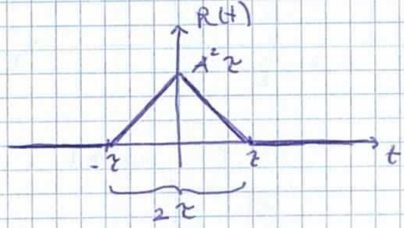
$$\rightarrow A^2 \int_{t-\frac{\tau}{2}}^{\frac{\tau}{2}} d\tau = A^2 \left(\frac{\tau}{2} - t + \frac{\tau}{2} \right) = A^2 (\tau - t)$$

Fino a:



In conclusione:

$$\begin{cases} R(t) = \emptyset & -\infty \le t \le -\tau \\ R(t) = A^2(\tau + t) & -\tau \le t \le \emptyset \\ R(t) = A^2(\tau - t) & \emptyset \le t \le \tau \\ R(t) = \emptyset & t \ge \tau \end{cases}$$



↳ Rappresenta un triangolo!

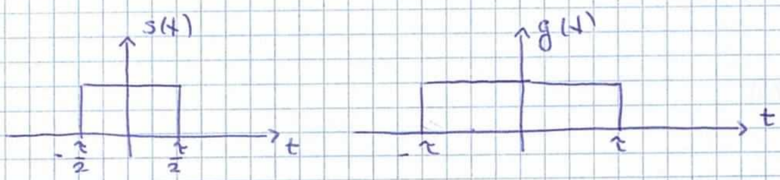
La convoluzione fra due rettangoli dà un triangolo.

La convoluzione tende a diminuire le discontinuità ma ad allargare il segnale.

$$R(t) = \int_{-\infty}^{+\infty} s(\tau)\, g(t-\tau)\, d\tau \rightarrow H(f) = S(f) \cdot G(f)$$

$$s(t),\ g(t) \rightarrow S(f) = G(f) = A \cdot \tau \ \mathrm{sinc}\,(f\tau)$$

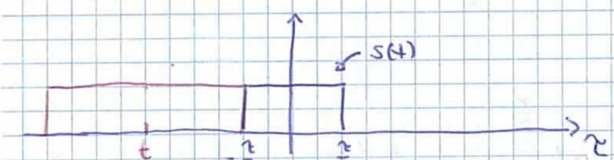
$$H(f) = A^2 \cdot \tau^2 \cdot \mathrm{sinc}^2(f\tau)$$



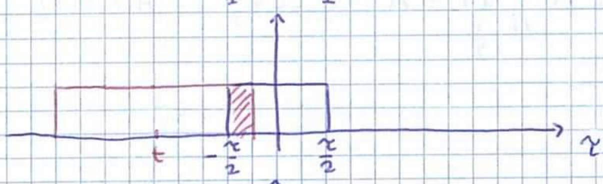
$$R(t) = \int_{-\infty}^{+\infty} s(\tau)\, g(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} g(\tau)\, s(t-\tau)\, d\tau$$

$$t - \tau = t'$$

$$= \int_{-\infty}^{+\infty} s(t-t')\, g(t')\, dt'$$

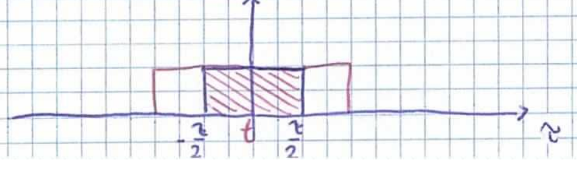


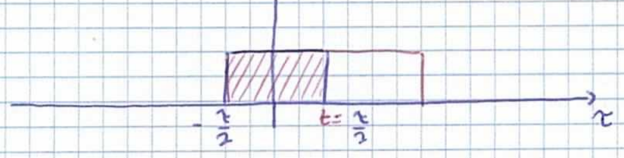
$$R(t) = \emptyset \quad , \ t \le -\frac{3}{2}\tau$$



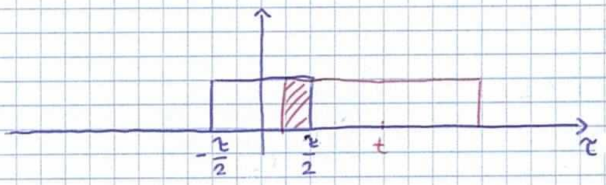
$$A^2 \int_{-\frac{\tau}{2}}^{t+\tau} = A^2\left(t + \tau + \frac{\tau}{2}\right) = A^2\left(t + \frac{3}{2}\tau\right)$$

$$R(t) = A^2\left(t + \frac{3}{2}\tau\right) \quad , \ -\frac{3}{2}\tau \le t \le -\frac{\tau}{2}$$





$R(t) = A^2\tau \,, \quad -\frac{\tau}{2} \leq t \leq \frac{\tau}{2}$

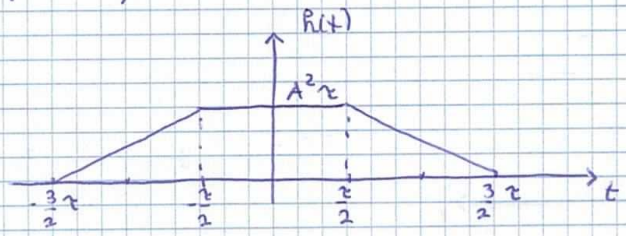


$$A^2 \int_{t-\tau}^{\tau/2} dt = A^2\left(\frac{\tau}{2} - t + \tau\right) = A^2\left(\frac{3}{2}\tau - t\right)$$

$R(t) = A^2\left(\frac{3}{2}\tau - t\right) \,, \quad \frac{\tau}{2} \leq t \leq \frac{3}{2}\tau$

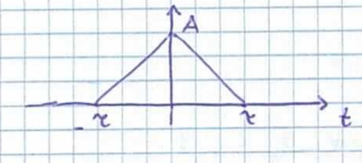
$R(t) = \emptyset \,, \quad t \geq \frac{3}{2}\tau$

$$\begin{cases} R(t) = \emptyset & t \leq -3/2\,\tau \\ R(t) = A^2(t + 3/2\,\tau) & -3/2\,\tau \leq t \leq -\tau/2 \\ R(t) = A^2\tau & -\tau/2 \leq t \leq \tau/2 \\ R(t) = A^2(-t + 3/2\,\tau) & \tau/2 \leq t \leq 3/2\,\tau \\ R(t) = \emptyset & t \geq 3/2\,\tau \end{cases}$$



Banda : 3τ.

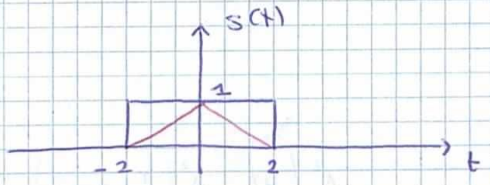
Il segnale convoluto ha banda uguale alla somma delle bande dei segnali.



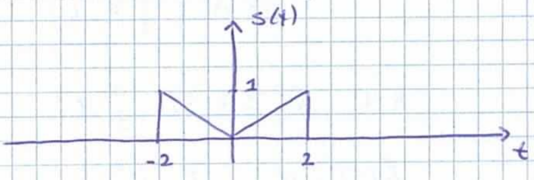
$tri\left(\frac{t}{\tau}\right)$ $\quad \tau$ è la metà della durata.

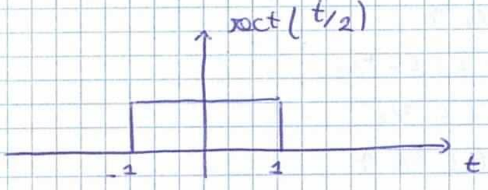
$s(t) = \text{rect}\left(\frac{t}{4}\right) - \text{tri}\left(\frac{t}{2}\right)$

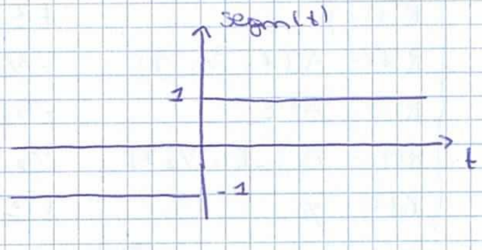
$g(t) = \text{rect}\left(\frac{t}{2}\right) \text{segn}(t)$



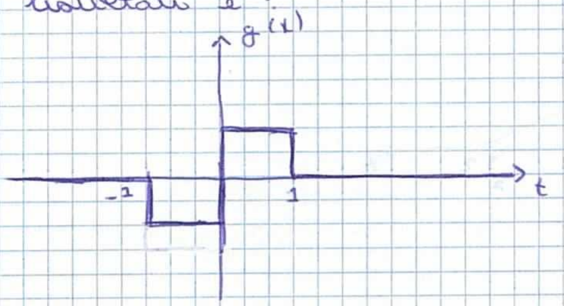
Il risultato è:





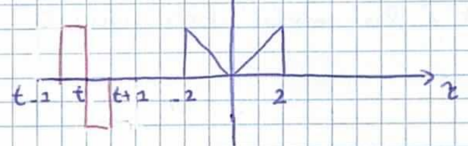


Il risultato è:

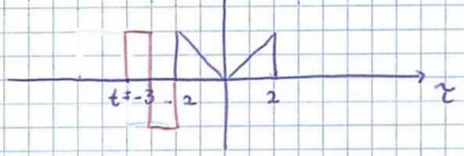


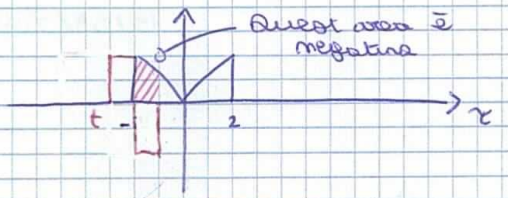
Sposto il segnale antisimmetrico: $s(\tau) \cdot g(t-\tau)$.

Ribalto $g(t)$ in quanto è $g(t-\tau)$.

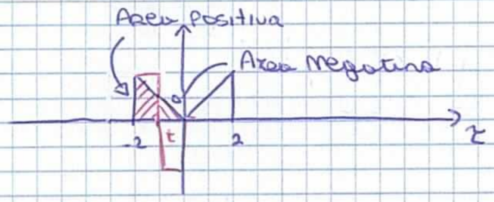


$R(t) = \emptyset$ per $t \leq -3$





$$\int_{-2}^{t+1} (-1)\cdot\left(-\frac{1}{2}\tau\right) d\tau$$



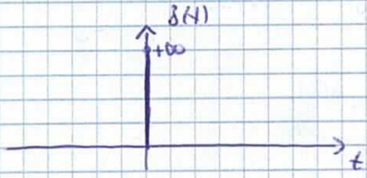
$$\int_{-2}^{t} s(\tau)\, g(t-\tau)\, d\tau + \int_{t}^{t+1} s(\tau)\, g(t-\tau)\, d\tau$$

Formula generica

nel nostro caso c'è $-$

DELTA DI DIRAC

$$\delta(t) = 0 \quad \text{per } t \neq 0 \quad , \quad \int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

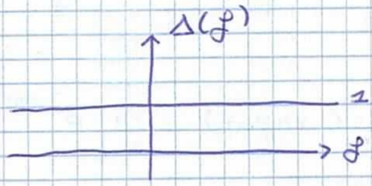


$$\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

$$\int_{-\infty}^{+\infty} g(t)\,\delta(t)\, dt = g(0)$$

$$\int_{-\infty}^{+\infty} g(t)\,\delta(t-t_0)\, dt = g(t_0)$$

$$\int_{-\infty}^{+\infty} \delta(t)\, e^{j 2\pi f t}\, dt = 1$$



12/03/2015

$\delta(t) = 0$, per $t \neq 0$

$$\int_{-\infty}^{+\infty} \delta(t)\, dt = 1$$

$\delta(t - t_0) = 0$, per $t \neq t_0$

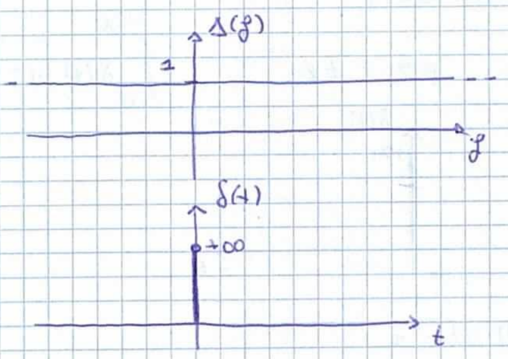
$$\int_{-\infty}^{+\infty} \delta(t - t_0)\, dt = 1$$

$$\int_{-\infty}^{+\infty} s(t)\, \delta(t)\, dt = s(0)$$

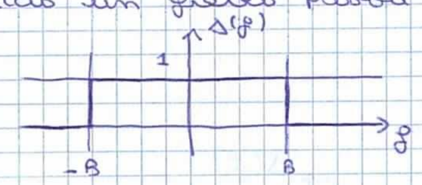
$$\int_{-\infty}^{+\infty} s(t)\, \delta(t - t_0)\, dt = s(t_0)$$

$\delta(t) = \delta(-t)$

$$\int_{-\infty}^{+\infty} \delta(t)\, e^{-j2\pi f t}\, dt = 1$$



Se faccio un filtro passa basso



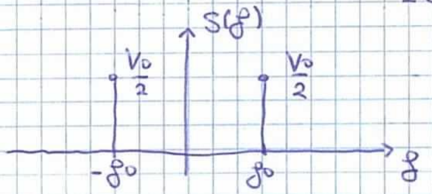
=> nel tempo ottengo un sinc.

$$s(t) = V_0 \cos(2\pi f_0 t) = \frac{V_0}{2}\left[e^{j2\pi f_0 t} + e^{-j2\pi f_0 t}\right] \longleftrightarrow S(f) = \frac{V_0}{2}\left[\delta(f+f_0) + \delta(f-f_0)\right]$$

$s(t) \rightarrow S(f)$

$s(t)\, e^{j2\pi f_0 t} \rightarrow S(f - f_0)$

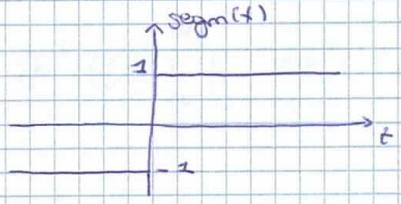
$$s(t) = V_0 \,\mathrm{sen}(2\pi f_0 t) = \frac{V_0}{2j}\left[e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}\right] \longleftrightarrow S(f) = \frac{V_0}{2j}\left[\delta(f-f_0) - \delta(f+f_0)\right]$$



Sen e Cos hanno lo stesso spettro di ampiezza.

$\delta(t) \to 1$

$1 \to \delta(-f) = \delta(f)$



$$\text{segn}(t) = \begin{cases} 1 & \text{se } t > \emptyset \\ -1 & \text{se } t < \emptyset \end{cases}$$

$\text{segn}(t) \to \dfrac{1}{j\pi f}$: Trasformata di segn(t).

Dato $s(t)$, $\displaystyle\int_{-\infty}^{+\infty} |s(t)|^2 dt = M < \infty$ Energia finita.

$$E_S = \int_{-\infty}^{+\infty} s^2(t)\, dt$$

allora, $s(t) \to S(f)$

Dati $s(t)$, $g(t)$ complessi.

Facciamo $s(t) * g(t) = \displaystyle\int_{-\infty}^{+\infty} s(\tau)\, g^*(t-\tau)\, d\tau$

$$S(f) \qquad G^*(-f)$$

$$= \int_{-\infty}^{+\infty} S(f)\, G^*(-f)\, e^{j2\pi f t}\, df$$

Devo avere $G^*(f)$! $\Rightarrow$ $G^*(f) \leftarrow g^*(-t)$

Facciamo la convoluzione di:

$s(t) * g(-t) = \displaystyle\int_{-\infty}^{+\infty} s(\tau)\, g^*(-t+\tau)\, d\tau =$

$g^*(t) \to G^*(-f)$ $\Rightarrow$ DIM: $g^*(t) = \displaystyle\int_{-\infty}^{+\infty} G^*(-f)\, e^{j2\pi f t}\, df$

$g^*(-t) \to G^*(f)$

$$g^*(-t) = \int_{-\infty}^{+\infty} G^*(-f)\, e^{-j2\pi f t}\, df$$

$$= -\int_{+\infty}^{-\infty} G^*(f)\, e^{j2\pi f t}\, df$$

$$= \int_{-\infty}^{+\infty} S(f)\, G^*(f)\, e^{j2\pi f t}\, df$$

Adesso va bene

Ma se $t = \emptyset$ (calcolo in un istante)

$$= \int_{-\infty}^{+\infty} s(\tau)\, g^*(\tau)\, d\tau = \int_{-\infty}^{+\infty} S(f)\, G^*(f)\, df$$

Con $s(t) = g(t)$

$$\int_{-\infty}^{+\infty} s(\tau)\, s^*(\tau)\, d\tau = \int_{-\infty}^{+\infty} S(f)\, S^*(f)\, df$$

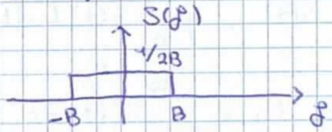
$$E_s = \int_{-\infty}^{+\infty} |s(\tau)|^2 d\tau = \int_{-\infty}^{+\infty} |S(f)|^2 df$$ ← TEOREMA DI PARSEVAL.

Prendiamo

$s(t) = A \cdot \text{sinc}(t \cdot T)$ T costante

$$\int_{-\infty}^{+\infty} A^2 \text{sinc}^2(t \cdot T)\, dt = A^2 \int_{-\infty}^{+\infty} \left(\frac{\text{sen}(\pi t T)}{\pi t T} \right)^2 dt$$

ma il sinc in frequenza è:

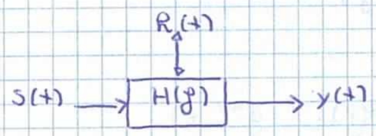


$A\,\text{rect}\left(\frac{t}{T}\right) \rightarrow AT\,\text{sinc}(fT)$

$AT\,\text{sinc}(t\,T) \rightarrow A \cdot \text{rect}\left(\frac{f}{T}\right) \rightarrow \text{sinc}(tT) \rightarrow \frac{1}{T}\,\text{rect}\left(\frac{f}{T}\right)$

$T = 2B$, $\text{sinc}(t \cdot 2B) \rightarrow \frac{1}{2B}\,\text{rect}\left(\frac{f}{2B}\right)$

$$E_s = \int_{-\infty}^{+\infty} |S(f)|^2 df = \int_{-B}^{B} \frac{1}{(2B)^2}\, df = \frac{1}{(2B)^{\cancel{2}}} \cdot \cancel{2B} = \frac{1}{2B}$$



Sistema

R(t): Risposta impulsiva o Indicia.

H(t): Funzione di Trasferimento o Caratteristica.

R(t) è il segnale in uscita quando all'ingresso viene applicato un segnale $s(t) = \delta(t)$.

Sistema Lineare:

Se da $s_1(t)$ ottengo $y_1(t)$

Se da $s_2(t)$ ottengo $y_2(t)$

applicando $s(t) = a_1 s_1(t) + a_2 s_2(t)$, a_1 e a_2 costanti

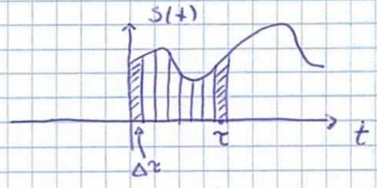
ottengo $y(t) = a_1 y_1(t) + a_2 y_2(t)$

Sistema Tempo-Invariante

Se applicando $s(t)$ ottengo $y(t)$

applicando $s(t - t_0)$ ottengo $y(t - t_0)$

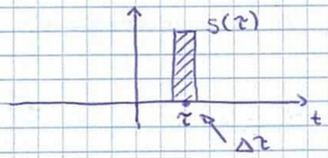
Se :



s(t) generico

Posso pensare all'uscita come l'uscita data dalla somma di infiniti impulsi, se s(t) è lineare.

$\Rightarrow s(\emptyset) \cdot \Delta\tau$



$\Rightarrow s(\tau) \cdot \Delta\tau$

$y_1(t) = h(t-\tau)\, s(\tau) \cdot \Delta\tau$

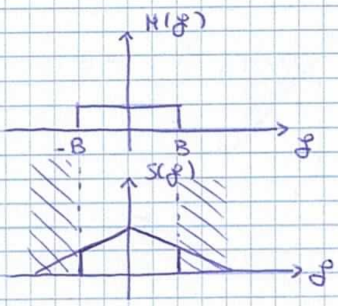
$$y(t) = \int_{-\infty}^{+\infty} h(t-\tau)\, s(\tau)\, d\tau = \int_{-\infty}^{+\infty} s(\tau)\, h(t-\tau)\, d\tau = \int_{-\infty}^{+\infty} h(\tau)\, s(t-\tau)\, d\tau$$

Se il sistema è lineare e tempo-invariante l'uscita è data dalla convoluzione dell'ingresso e la risposta impulsiva del sistema.

$Y(f) = H(f) \cdot S(f)$

Il sistema è causale se $h(t) = \emptyset$ per $t < \emptyset$.

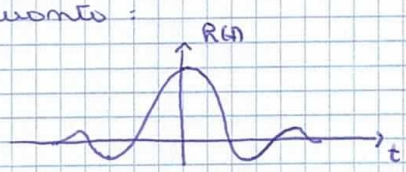
Se:



È un filtro passa-basso.

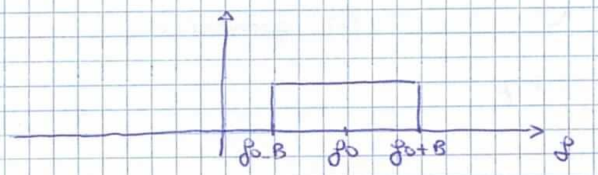
ma questo è un filtro non causale!

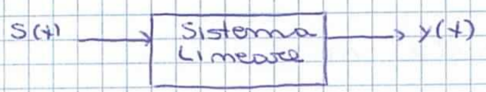
In quanto:



applicando il segnale a $t = \emptyset$, viene generato il segnale anche per $t < \emptyset$.

Filtro Passa - Banda





y(t) è non distorto rispetto a s(t) se:

$y(t) = A \cdot s(t - t_0)$, A e t_0 costanti

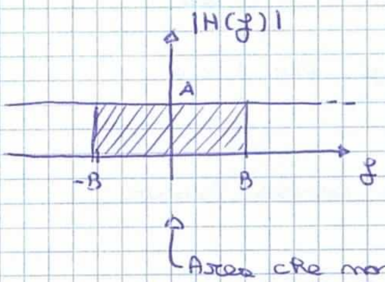
$Y(f) = S(f) \cdot H(f)$

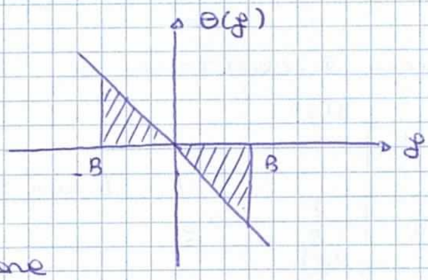
$Y(f) = A \cdot S(f) \cdot e^{-j2\pi f t_0}$

Quindi $H(f) = A \cdot e^{-j2\pi f t_0}$, in questo caso non si ha distorsione!

$$\begin{cases} |H(f)| = A \\ \Theta(f) = 2\pi f t_0 \end{cases}$$ in questo caso il segnale non viene distorto

condizione di non distorsione





Sistema non lineare

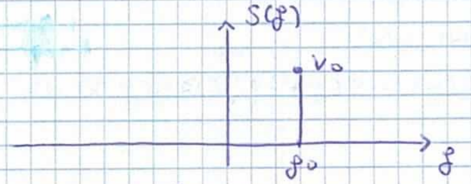
$s(t) \to y(t) = a_0 + a_1 s(t) + a_2 s^2(t) + \dots$

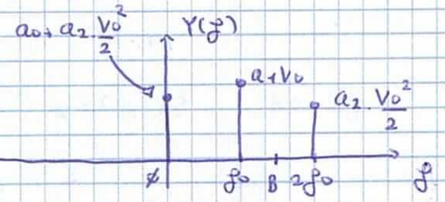
$s(t) = V_0 \cos(2\pi f_0 t)$

$$y(t) = a_0 + a_1 V_0 \cos(2\pi f_0 t) + a_2 V_0^2 \cos^2(2\pi f_0 t)$$

$$= a_0 + \underbrace{a_1 V_0 \cos(2\pi f_0 t)}_{f_0} + \underbrace{a_2 \frac{V_0^2}{2} \cos(4\pi f_0 t)}_{2f_0} + a_2 \frac{V_0^2}{2}$$

in quanto $\cos^2 x = \dfrac{1+\cos(2x)}{2}$





Se inserisco un filtro che fa passare solo le armoniche $< 2f_0$ potrei recuperare il segnale in ingresso! ($B < 2f_0$)

$s(t) \to y(t) = a_0 + a_1 s(t) + a_2 s^2(t)$

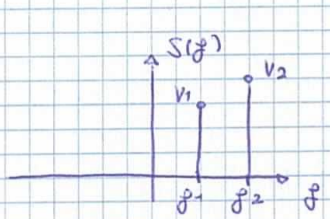
$s(t) = V_1 \cdot \cos(2\pi f_1 t) + V_2 \cdot \cos(2\pi f_2 t)$

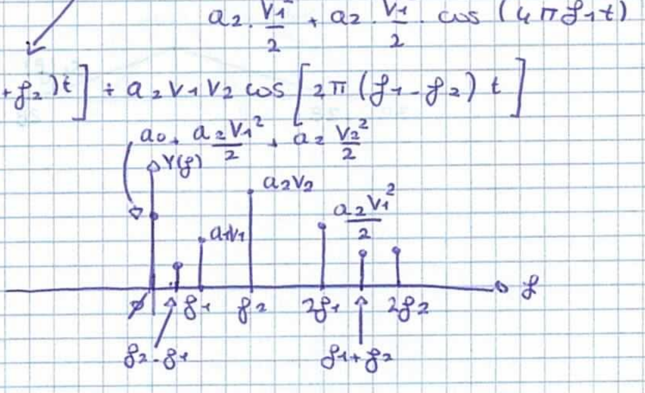
$$y(t) = a_0 + a_1 V_1 \cos(2\pi f_1 t) + a_1 V_2 \cos(2\pi f_2 t) + \underline{a_2 V_1^2 \cos^2(2\pi f_1 t)}$$
$$+ \underline{a_2 V_2^2 \cos^2(2\pi f_2 t)} + \underline{2 a_2 V_1 V_2 \cos(2\pi f_1 t) \cos(2\pi f_2 t)}$$

$a_2 \dfrac{V_2^2}{2} + a_2 \dfrac{V_2^2}{2} \cdot \cos(4\pi f_2 t)$

$a_2 \dfrac{V_1^2}{2} + a_2 \dfrac{V_1^2}{2} \cdot \cos(4\pi f_1 t)$

$a_2 V_1 V_2 \cos\left[2\pi (f_1 + f_2) t\right] + a_2 V_1 V_2 \cos\left[2\pi (f_1 - f_2) t\right]$





Quindi nei sistemi non lineari le frequenze in uscita si mescolano!

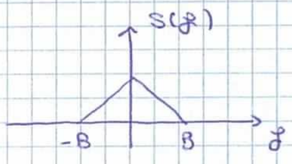
Prendiamo $s(t)$ segnale qualsiasi:

$s(t) \to S(f)$

Uso un sistema non lineare:

$y(t) = a_0 + a_1 s(t) + a_2 s^2(t) + a_3 s^3(t)$

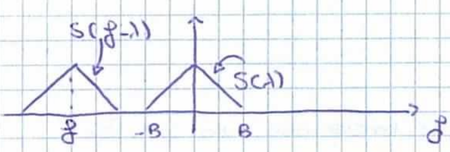
$Y(f) = a_0 + a_1 S(f) + \dots$



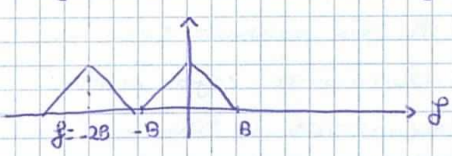
$s^2(t) \to S_1(f)$

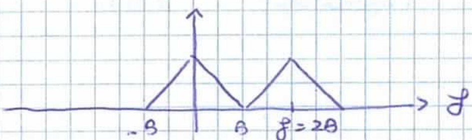
$s^2(t) = s(t) \cdot s(t)$

$S_1(f) = S(f) * S(f) = \int_{-\infty}^{+\infty} S(\lambda) S(f-\lambda)\, d\lambda$



$f \leq -2B \quad , \quad S_1(f) = \emptyset$

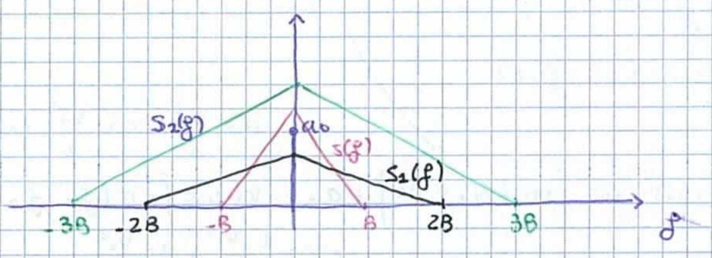




$f \geq 2B \quad , \quad S_1(f) = \emptyset$

$s^3(t) = s^2(t) \cdot s(t) \to S_1(f) * S(f)$

$S_2(f) = S_1(f) * S(f)$



$$E_s = \int_{-\infty}^{+\infty} |s(t)|^2 dt$$

Tramite Parseval

$$E_s = \int_{-\infty}^{+\infty} |S(f)|^2 df$$

Densità Spettrale di Energia : $\psi_s(f) = |S(f)|^2$

Indica come l'energia è distribuita nelle varie frequenze!

$\psi_s(f) \geq \emptyset$

$s(t)$ segnale reale : $|S(f)|$ pari

$\psi_s(f)$ è pari.

$$s(t) \longrightarrow \boxed{R(t)} \longrightarrow y(t)$$

$Y(f) = H(f) \cdot S(f)$

$$\psi_y(f) = |Y(f)|^2 = |H(f) \cdot S(f)|^2 = |H(f)|^2 \cdot |S(f)|^2$$
$$= |H(f)|^2 \cdot \psi_s(f)$$

Dato $s(t)$, $s(t) \longrightarrow R_s(\tau)$ funzione di autocorrelazione di $s(t)$

$$= \int_{-\infty}^{+\infty} s(t)\, s(t+\tau)\, dt$$

$t + \tau = t_1$

$$\Rightarrow \int_{-\infty}^{+\infty} s(t_1 - \tau) \cdot s(t_1)\, dt_1 = R_s(-\tau)$$

R_s è una funzione pari!

$[s(t) - s(t+\tau)]^2 \geq \emptyset$

$s^2(t) + s^2(t+\tau) - 2 s(t)\, s(t+\tau)$

$$\int_{-\infty}^{+\infty} \left[s^2(t) + s^2(t+\tau) - 2 s(t) s(t+\tau) \right] dt \geq \emptyset$$

$$= \int_{-\infty}^{+\infty} s^2(t)\, dt + \int_{-\infty}^{+\infty} s^2(t+\tau)\, dt - 2\int_{-\infty}^{+\infty} s(t)\, s(t+\tau)\, dt \geq \emptyset$$

$$R_s(\emptyset) + R_s(\emptyset) - 2 \cdot R_s(\tau) \geq \emptyset$$

$$\int_{-\infty}^{+\infty} s^2(t+\tau) \cdot d(t+\tau) \quad , \text{con } t+\tau = t_1$$

in quanto τ è una costante

$$= \int_{-\infty}^{+\infty} s^2(t_1)\, dt_1 = R_s(\emptyset)$$

$$\Rightarrow R_s(\tau) \leq R_s(\emptyset)$$

$g(t)$ complesso

$$\int_{-\infty}^{+\infty} s(t) \cdot g^*(\tau - t)\, dt = \int_{-\infty}^{+\infty} S(f) G^*(-f)\, e^{j2\pi f t}\, df$$

$$\Rightarrow \int_{-\infty}^{+\infty} s(t)\, g^*(-\tau + t)\, dt = \int_{-\infty}^{+\infty} S(f)\, G^*(f)\, e^{j2\pi f t}\, df$$

prendo $g(t) = s(t)$

$$\Rightarrow \int_{-\infty}^{+\infty} s(t)\, s^*(-\tau + t)\, dt = \int_{-\infty}^{+\infty} S(f)\, S^*(f)\, e^{j2\pi f t}\, df$$

$s(t)$ reale

$$R_s(-\tau) = \int_{-\infty}^{+\infty} s(t)\, s(t - \tau)\, dt = \int_{-\infty}^{+\infty} |S(f)|^2\, e^{j2\pi f t}\, df = R_s(\tau)$$

$$|S(f)|^2 = \psi_s(f)$$

$$R_s(\tau) = \int_{-\infty}^{+\infty} s(t) \cdot s(t+\tau)\, d\tau$$

La funzione di autocorrelazione è l'antitrasformata della densità spettrale di energia.

16/03/2013

Trasformata di Hilbert

$$s(t) \rightarrow \hat{s}(t) = \text{t. di Hilbert di } s(t) = \frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{s(\tau)}{t - \tau}\, d\tau$$

$$s(t) = -\frac{1}{\pi} \int_{-\infty}^{+\infty} \frac{\hat{s}(\tau)}{t - \tau}\, d\tau$$

$$\hat{s}(t) = s(t) * \frac{1}{\pi t} \rightarrow \hat{S}(f) = S(f) \cdot G(f) = -j \cdot \text{segn}(f) \cdot S(f) = \hat{S}(f)$$

$S(f) \longleftrightarrow s(t)$

$G(f) \longleftrightarrow \frac{1}{\pi t} \quad , \quad G(f) = -j\,\text{segn}(f)$

$\text{segn}(t) \rightarrow \frac{1}{j\pi f} \quad , \quad s(t) \rightarrow S(f)$

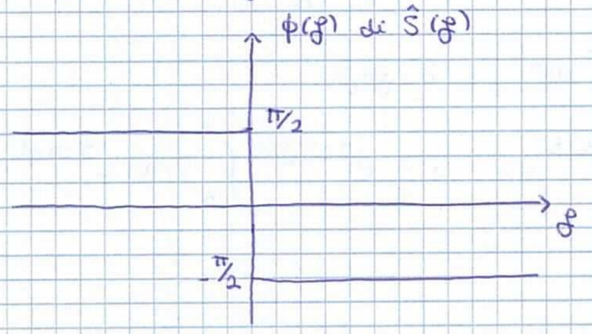
$\qquad\qquad\qquad\qquad\quad S(t) \rightarrow s(-f)$

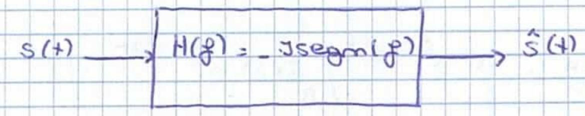
$\frac{1}{j\pi t} \rightarrow \text{segn}(-f) = -\text{segn}(f)$

$|\hat{S}(f)| = |S(f)|$ La Trasformata di Hilbert non cambierà lo spettro di ampiezze.

$f < \emptyset$, $\hat{S}(f) = j \cdot S(f)$ sfasate di $+90°$

$f > \emptyset$, $\hat{S}(f) = -j \cdot S(f)$ sfasate di $-90°$





Trasformatore di Hilbert

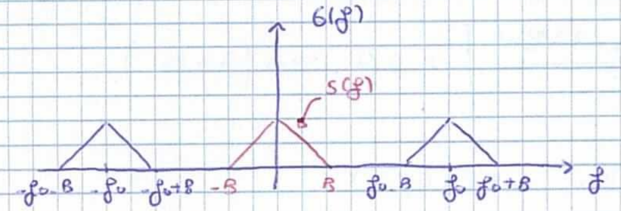
$|H(f)| = K$

ma viene introdotta una distorsione in quanto

$\phi(f) \neq 2\pi f t_0$

$s(t) \rightarrow S(f) \neq \emptyset$ in $(-B, B)$

$g(t) = s(t) \cdot \cos(2\pi f_0 t) \rightarrow G(f) = \frac{1}{2}\left[S(f+f_0) + S(f-f_0)\right]$



$\hat{g}(t) \rightarrow \hat{G}(f) = -j\,\text{segn}(f) \cdot G(f) = -\frac{j}{2}\,\text{segn}(f)\left[S(f+f_0) + S(f-f_0)\right]$

$S(f+f_0) \neq \emptyset$ in $(-f_0 - B, -f_0 + B)$ con $\text{segn}(f) = -1$

$S(f-f_0) \neq \emptyset$ in $(f_0 - B, f_0 + B)$ con $\text{segn}(f) = 1$

$$\hat{G}(f) = \begin{cases} \frac{j}{2} S(f+f_0), & -f_0 - B \leq f \leq -f_0 + B \\ -\frac{j}{2} S(f-f_0), & f_0 - B \leq f \leq f_0 + B \\ \emptyset & \text{altrove} \end{cases}$$

$$\hat{g}(t) = \int_{-\infty}^{+\infty} \hat{G}(f)\, e^{j2\pi f t}\, df$$

$$= \underbrace{\int_{-f_0-B}^{-f_0+B} \frac{j}{2} S(f+f_0)\, e^{j2\pi f t}\, df}_{A} + \underbrace{\int_{f_0-B}^{f_0+B} -\frac{j}{2} S(f-f_0)\, e^{j2\pi f t}\, df}_{B}$$

$$A = \frac{j}{2}\int_{-f_0-B}^{-f_0+B} S(f+f_0)\, e^{j2\pi f t}\, df \quad , \quad \lambda = f+f_0$$

$$= \frac{j}{2}\int_{-B}^{B} S(\lambda)\, e^{j2\pi(\lambda-f_0)t}\, d\lambda = \frac{j}{2}\, e^{-j2\pi f_0 t}\int_{-B}^{B} S(\lambda)\, e^{j2\pi\lambda t}\, d\lambda$$

$$= \frac{j}{2} \cdot e^{-j2\pi f_0 t}\, s(t)$$

$$B = -\frac{j}{2}\int_{+f_0-B}^{f_0+B} S(f-f_0)\, e^{j2\pi f t}\, df \quad , \quad \lambda = f-f_0$$

$$= -\frac{j}{2}\int_{-B}^{B} S(\lambda)\, e^{j2\pi(\lambda+f_0)t}\, d\lambda = -\frac{j}{2}\, e^{j2\pi f_0 t}\int_{-B}^{B} S(\lambda)\, e^{j2\pi\lambda t}\, d\lambda$$

$$= -\frac{j}{2}\, e^{j2\pi f_0 t}\, s(t)$$

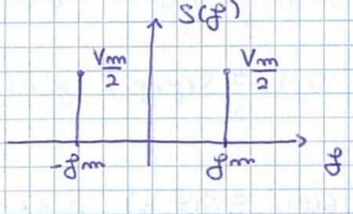
$$\hat{g}(t) = \frac{j}{2} \cdot s(t) \cdot \left[e^{-j2\pi f_0 t} - e^{j2\pi f_0 t} \right] \cdot \frac{j}{j}$$

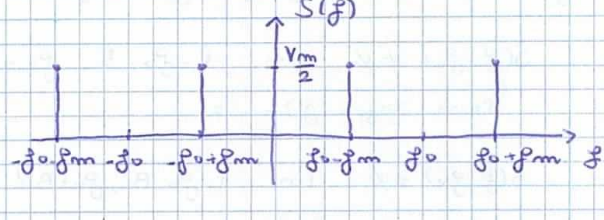
al numeratore viene $j \cdot j = j^2 = -1$!

$$= s(t) \cdot \left[\frac{e^{j2\pi f_0 t} - e^{-j2\pi f_0 t}}{2j} \right] = s(t) \cdot \mathrm{sen}\,(2\pi f_0 t).$$

Supponiamo: $s(t) = V_m \cdot \cos(2\pi f_m t)$

$\hat{s}(t) = V_m\, \mathrm{sen}\,(2\pi f_m t)$

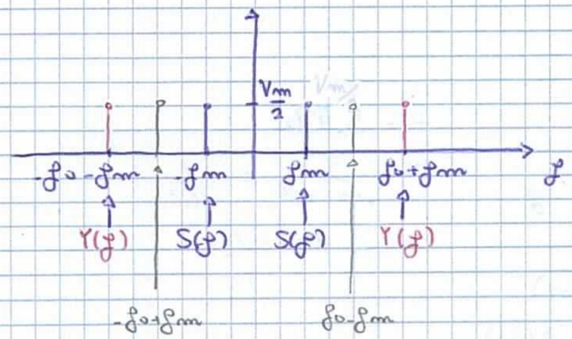




$y(t) = s(t) \cdot \cos(2\pi f_0 t) \pm \hat{s}(t) \,\text{sen}(2\pi f_0 t)$

$s(t) = V_m \cdot \cos(2\pi f_m t)$

$y(t) = V_m \cdot \cos(2\pi f_m \cdot t) \cdot \cos(2\pi f_0 t) - V_m \,\text{sen}(2\pi f_m t) \cdot \text{sen}(2\pi f_0 t)$
$= V_m \cdot \cos\left[2\pi (f_0 \pm f_m) \cdot t\right]$



Ho trovato solo una riga! Quindi trasferisco solo la parte positiva.

SSB = Single Side Band (tipo di modulazione)

$s(t) \quad \hat{s}(t)$

$S(f) \quad \hat{S}(f) = -j \,\text{segn}(f) \cdot S(f)$

$$E_s = \int_{-\infty}^{+\infty} s^2(t)\, dt = \int_{-\infty}^{+\infty} |S(f)|^2\, df$$

La trasformata di Hilbert non modificando il modulo non modifica l'energia e nemmeno la funzione di autocorrelazione.

TEORIA DEI SEGNALI (ANALOGICO <=> DIGITALE)

Teorema di Shannon

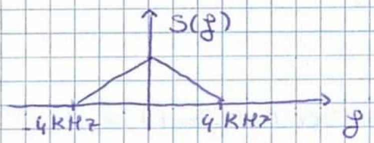
$s(t)$ con $S(f) \neq 0$ in $(-B, B)$ segnale analogico

$s(t)$ può essere ricostruito esattamente conoscendo i suoi campioni agli istanti $t_n = nT$, n intero se $T \leq \frac{1}{2B}$

$s(t_n = nT)$: campioni del segnale $s(t)$.

Campionare significa moltiplicare $s(t)$ per una serie di $\delta(t)$ poste a distanza T.

Segnale Telefonico

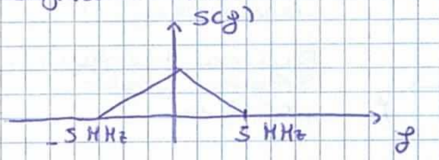


$B = 4.000\ Hz$

$T \leq \frac{1}{8.000}\ s$

n_c = # campioni da prendere in 1 sec $\geq 8.000\ c/s$

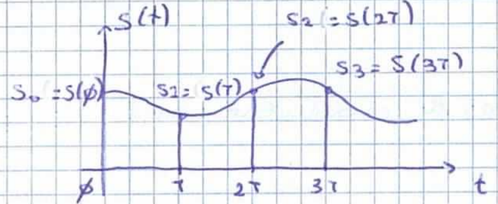
Segnale Video



$B = 5\ MHz$

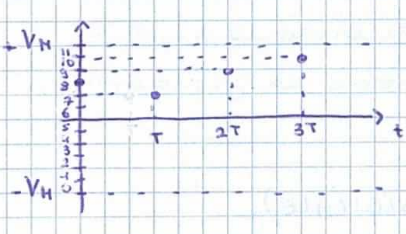
$T \leq \frac{1}{10 \cdot 10^6} = \frac{1}{10^7}$

$n_c = 10^7\ c/s$



$S_n = S(nt)$ -> numeri reali

V_m = min ; V_M = Max



QUANTIZZAZIONE

- Divido l'intervallo $-V_m \div V_m$ in parti, di ampiezza Δ.
- Poi trasferisco il numero di intervallo.

1) Campionamento $s(t) \rightarrow S_n = S(nT)$, $T \leq \frac{1}{2B}$

2) Quantizzazione S_n # reali -> # interi

3) Codifica (dei campioni) # interi -> binario

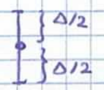
M: # di intervalli di quantizzazione

$S_0 \rightarrow 8 \rightarrow 1000$

$S_1 \rightarrow 6 \rightarrow 0110$

Quindi: 1000 ; 0110 ; ... è la sequenza che trasmetto.

nella ricostruzione se arriva 8:

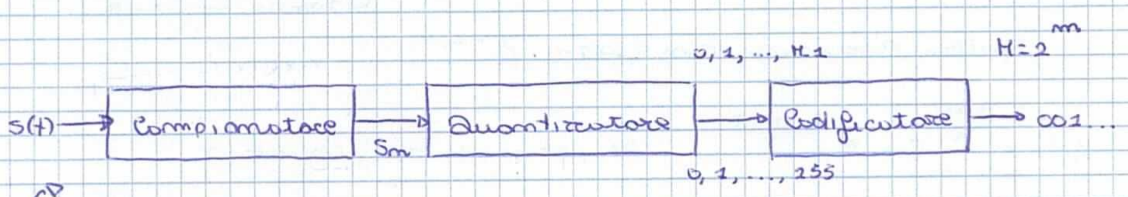


lo metto a metà dell'intervallo 8.

ma posso commettere un errore max di $\Delta/2$.

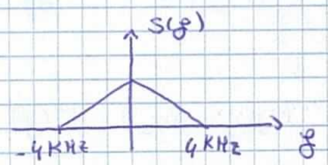
Quindi nella ricostruzione commetto errori di approssimazione.

Generalmente M è una potenza del 2.



Convertitore Analogico / Digitale (A/D)

Segnale Telefonico



1 Fase

$B = 4\ KHz$

$T = \frac{1}{8000}\ s$

$n_c = 8000\ c/s$

2 Fase

$M = 256 = 2^8$

3 Fase

Ogni campione lo rappresento con 8 bit, $m = 8$.

$n_b = n\ bit/s = n_c \cdot m = 64.000\ bit/s = 64\ Kb/s$

Devo avere una banda di 64 KHz senza distorsioni.

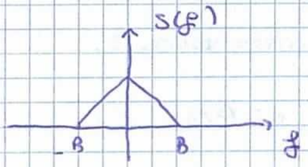
Quindi se il segnale telefonico analogico consuma una banda di 4 KHz, quello telefonico digitale consuma una banda di 64 KHz.

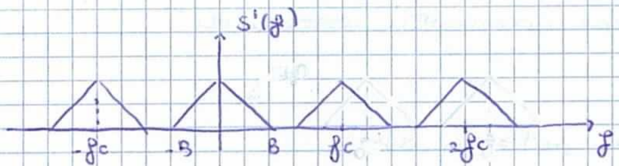
$$n_b = n_c \cdot m = 2B \cdot m$$

$s(t) \rightarrow S(f) \neq \emptyset$ in $(-B, B)$

Dimostrazione del Teorema di Shannon.

(ESAME)





Spettro periodico con periodo f_c

$f_c \geq 2B$ (spettri separati)

Fondamentale!

$$g(t) = \sum_{m=-\infty}^{+\infty} c_m \, e^{\frac{j2\pi m t}{T}} \quad \text{se periodico di periodo } T.$$

$$c_m = \frac{1}{T}\int_{-T/2}^{T/2} g(t)\, e^{-\frac{j2\pi m t}{T}}\, dt$$

$$s'(t) \rightarrow S'(f) = \sum_{m=-\infty}^{+\infty} c_m \, e^{\frac{j2\pi m f}{f_c}} \quad , \quad c_m = \frac{1}{f_c}\int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} S'(f)\, e^{-\frac{j2\pi m f}{f_c}}\, df.$$

ma in $(-f_c/2, f_c/2)$: $S'(f) = S(f)$

$$s(t) = \int_{-\infty}^{+\infty} S(f)\, e^{j2\pi f t}\, df = \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} S(f)\, e^{j2\pi f t}\, df$$

$$s_m = s\Big(\underbrace{\frac{m}{f_c}}_{t}\Big) = \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} S(f)\, e^{j2\pi f \frac{m}{f_c}}\, df$$

$$T = \frac{m}{f_c} = \frac{m}{2B} \quad , \quad f_c = 2B$$

$$s_m = \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} S(f)\, e^{j2\pi f \frac{m}{f_c}}\, df \quad , \quad c_m = \frac{1}{f_c}\int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} S(f)\, e^{-j2\pi f \frac{m}{f_c}}\, df$$

$$c_m = \frac{1}{f_c}\, s\left(\frac{-m}{f_c}\right) = \frac{1}{f_c}\, s_{-m}$$

Quindi: $s_m \Rightarrow c_m \Rightarrow S'(f) \Rightarrow S(f)$

$$s(t) = \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} S(f)\, e^{j2\pi f t}\, df = \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} S'(f)\, e^{j2\pi f t}\, df = \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} \sum_{m=-\infty}^{+\infty} c_m\, e^{\frac{j2\pi m f}{f_c}} \cdot e^{j2\pi \cdot f t}\, df$$

$$= \sum_{m=-\infty}^{+\infty} c_m \int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} e^{j2\pi f \left(t + \frac{m}{f_c}\right)}\, df \quad , \quad c_m = \frac{1}{f_c}\, s\left(-\frac{m}{f_c}\right)$$

$$= \frac{1}{f_c}\sum_{m=-\infty}^{+\infty} s\left(-\frac{m}{f_c}\right)\int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} e^{j2\pi f\left(t+\frac{m}{f_c}\right)}\,df \qquad , n = -m$$

$$= \frac{1}{f_c}\sum_{m=-\infty}^{+\infty} s\left(\frac{m}{f_c}\right)\int_{-\frac{f_c}{2}}^{\frac{f_c}{2}} e^{j2\pi f\left(t-\frac{m}{f_c}\right)}\,df$$

$$= \frac{1}{f_c}\sum_{m=-\infty}^{+\infty} s\left(\frac{m}{f_c}\right)\frac{1}{j2\pi\left(t-\frac{m}{f_c}\right)}\cdot\left[e^{j\cancel{2}\pi\frac{f_c}{\cancel{2}}\left(t-\frac{m}{f_c}\right)} - e^{-j\cancel{2}\pi\frac{f_c}{\cancel{2}}\left(t-\frac{m}{f_c}\right)}\right]$$

$$= \frac{1}{f_c}\sum_{m=-\infty}^{+\infty} s\left(\frac{m}{f_c}\right)\frac{1}{\pi\left(t-\frac{m}{f_c}\right)}\cdot \text{sen}\left[\pi f_c\left(t-\frac{m}{f_c}\right)\right]$$

$$= \sum_{m=-\infty}^{+\infty} s\left(\frac{m}{f_c}\right)\cdot \text{sinc}\left[f_c\left(t-\frac{m}{f_c}\right)\right] = s(t)$$

Dimostrato il Teorema di Shannon!

Esercizi Esame 17/03/2015

Numeri Complessi

$x^2 = -1$

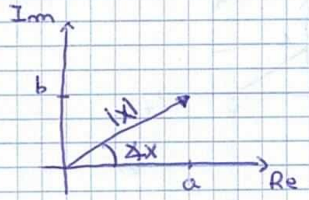
$i = \sqrt{-1}$

$x = a + ib \qquad , x \in \mathbb{C}$

$x^* = a - ib$ coniugato complesso di x.

$x = |x| \cdot e^{j\measuredangle x} \qquad , |x| = \sqrt{a^2+b^2}$

$\measuredangle x = \text{arctg}\,\frac{b}{a}$



$$\begin{cases} a = |x|\cdot \cos\measuredangle x \\ b = |x|\cdot \text{sen}\measuredangle x \end{cases}$$

$x_1 = a + ib$

$x_2 = c + id$

$x_1 + x_2 = (a+c) + i(b+d)$

$x_1 - x_2 = (a-c) + i(b-d)$

$x_1 \cdot x_2 = (a+ib)(c+id) = ac + i(bc+ad) - bd$

$= (ac - bd) + i(bc + ad)$

$\frac{ai}{b} = \frac{ai}{b}\cdot\frac{i}{i} = -\frac{a}{ib} \Rightarrow i = -\frac{1}{i}$

Formule di Eulero

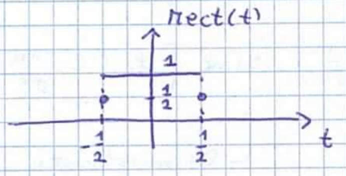
$$e^{i\varphi} = \cos\varphi + i \operatorname{sen}\varphi$$

$$e^{-i\varphi} = \cos\varphi - i \operatorname{sen}\varphi$$

$$\cos\varphi = \frac{e^{i\varphi} + e^{-i\varphi}}{2} \quad ; \quad \operatorname{sen}\varphi = \frac{e^{i\varphi} - e^{-i\varphi}}{2i}$$

Segnali Fondamentali

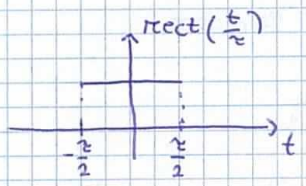
$$\text{rect}(t) = \begin{cases} 1 & \text{se } |t| < 1/2 \\ 1/2 & \text{se } |t| = 1/2 \\ \emptyset & \text{altrimenti} \end{cases}$$



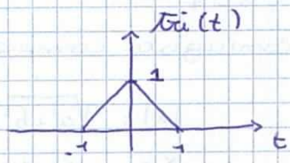
Il segnale sarà però:

$$\text{rect}(t) = \begin{cases} 1 & \text{se } |t| \leq 1/2 \\ \emptyset & \text{altrimenti} \end{cases}$$

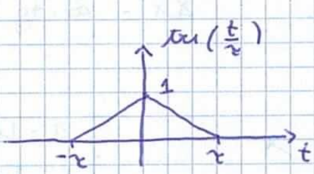
$$\text{rect}\left(\frac{t}{\tau}\right) = \begin{cases} 1 & \text{se } |t| \leq \tau/2 \\ \emptyset & \text{se altrimenti} \end{cases}$$



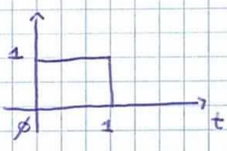
$$\text{tri}(t) = \begin{cases} 1 - |t| & \text{se } |t| < 1 \\ \emptyset & \text{altrimenti} \end{cases}$$



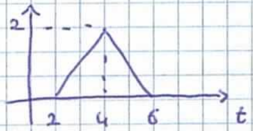
$$\text{tri}\left(\frac{t}{\tau}\right) = \begin{cases} 1 - |t| & \text{se } |t| < \tau \\ \emptyset & \text{altrimenti} \end{cases}$$



Esempio:

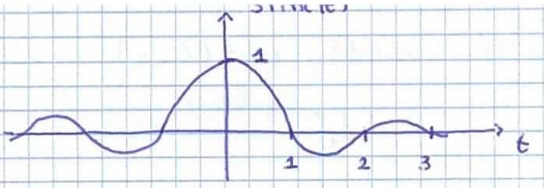


$$\text{rect}\left(t - \frac{1}{2}\right)$$

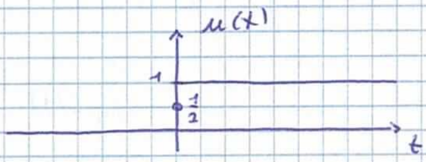


$$2 \cdot \text{tri}\left(\frac{t-4}{2}\right)$$

$$\mathrm{sinc}(t) = \begin{cases} \dfrac{\sin \pi t}{\pi t} & \text{se } t \neq 0 \\ 1 & \text{se } t = 0 \end{cases}$$

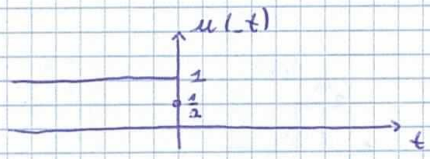


$$u(t) = \begin{cases} 1 & \text{se } t > 0 \\ 1/2 & \text{se } t = 0 \\ 0 & \text{altrimenti} \end{cases}$$

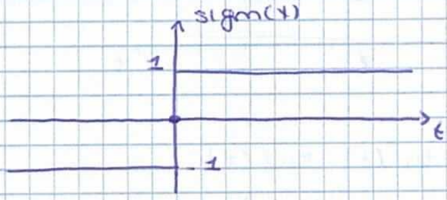


Il segnale reale:

$$u(t) = \begin{cases} 1 & \text{se } t \geq 0 \\ 0 & \text{altrimenti} \end{cases}$$



$$\mathrm{sign}(t) = \begin{cases} 1 & \text{se } t > 0 \\ 0 & \text{se } t = 0 \\ -1 & \text{se } t < 0 \end{cases}$$



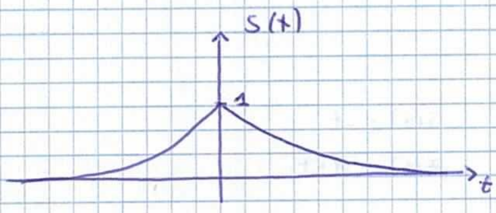
Sommando 2 gradini ottengo la funzione segno.

$$\mathrm{sign}(t) = u(t) - u(-t)$$

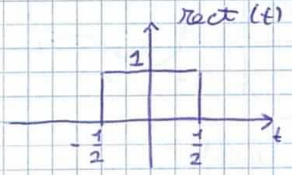
$$\mathrm{sign}(t) = \frac{t}{|t|}$$

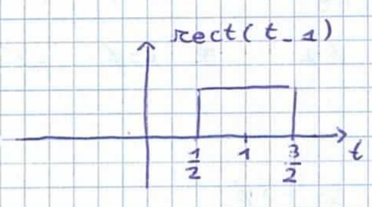
$$s(t) = e^{-|t|}$$

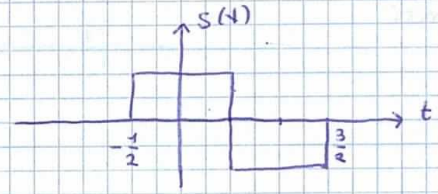
Da un segnale con $E_s = M < \infty$



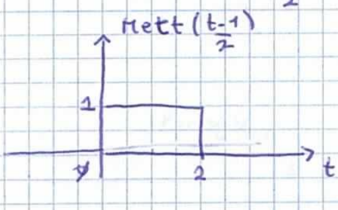
$s(t) = \text{rect}(t) - \text{rect}(t-1)$

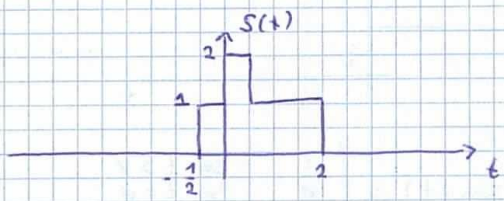




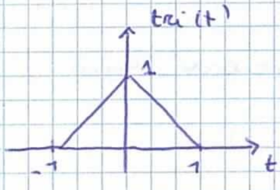


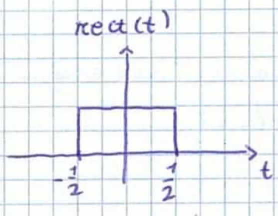
$s(t) = \text{rect}(t) + \text{rect}\left(\frac{t-1}{2}\right)$

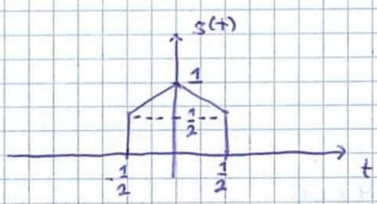




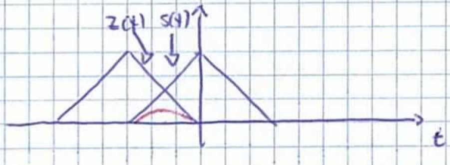
$s(t) = \text{tri}(t) \cdot \text{rect}(t)$







Esempio :



$s(t) = -t$

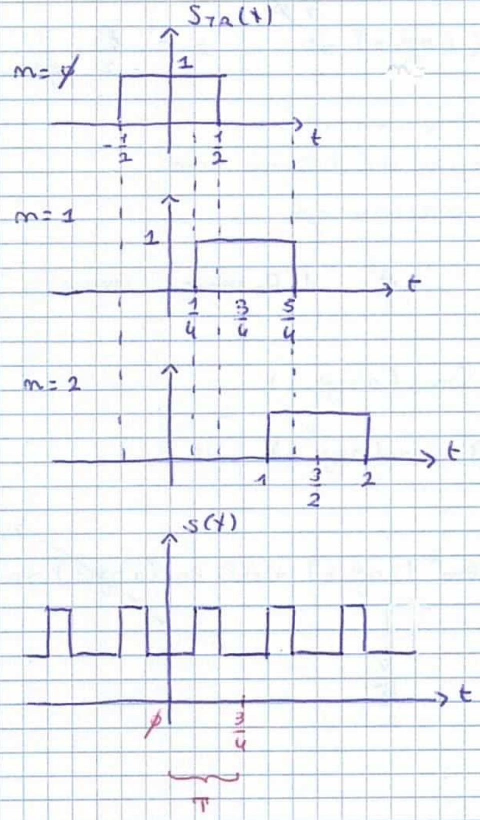
$z(t) = 1 + t$

$p(t) = s(t) \cdot z(t) = -t^2 - t$

Scrivere un segnale Periodico

$$s(t) = \sum_{n=-\infty}^{+\infty} rect\left(t - \frac{3}{4}n\right)$$

$S_{TR}(t)$: segnale con $n = \emptyset$ = $rect(t)$



Per caso: $s(t) = \sum_{n=-\infty}^{+\infty} (-1)^n \, rect\left(t - \frac{3}{4}n\right)$

$$s(t) = 1 + sign(1 - t)$$

$$s(t) = sinc(t) \cdot sign(t)$$

$s(t)$, $E_s = \int_{-\infty}^{+\infty} |s(t)|^2 \, dt$

$$P_s = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |s(t)|^2 \, dt$$

$E_s = M < \infty \Rightarrow P_s = \emptyset$. necessario per la Trasformata di Fourier.

Segnale Periodico, $E_s = +\infty$, $P_s = N < \infty$.

$$P_{media} = \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} |s(t)|^2 \, dt.$$

$s(t) = A \cdot \cos(2\pi f_0 t)$

$$P_s = f_0 \cdot \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} A^2 \cdot \underbrace{\cos^2(2\pi f_0 t)}_{\frac{1}{2}(1+\cos(2x))} \, dt = A^2 \cdot \frac{f_0}{2} \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} 1 + \cos(4\pi f_0 t) \, dt$$

$$= A^2 \cdot \frac{f_0}{2} \cdot \left[\int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} 1 \cdot dt + \underbrace{\cancel{\int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} \cos(4\pi f_0 t) \, dt} \to \emptyset}_{} \right] = A^2 \cdot \frac{\cancel{f_0}}{2} \cdot \frac{1}{\cancel{f_0}} = \frac{A^2}{2}.$$

Integrale del coseno nel suo periodo $= \emptyset$

Potenza cos = Potenza sen $= \frac{A^2}{2}$. (Potenza media).

$s(t) = A \cdot \cos(2\pi f_0 t) + B \cdot \operatorname{sen}(2\pi f_0 t)$

$$P_s = f_0 \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} \left[A \cdot \cos(2\pi f_0 t) + B \cdot \operatorname{sen}(2\pi f_0 t) \right]^2 dt$$

$$= f_0 \cdot \int_{-\frac{1}{2f_0}}^{\frac{1}{2f_0}} \underbrace{A^2 \cos^2(2\pi f_0 t)}_{\frac{A^2}{2}} + \underbrace{B^2 \cdot \operatorname{sen}^2(2\pi f_0 t)}_{\frac{B^2}{2}} + \cancel{2AB \cos(2\pi f_0 t) \operatorname{sen}(2\pi f_0 t)} \to \emptyset \; dt$$

$$= \frac{A^2}{2} + \frac{B^2}{2}.$$

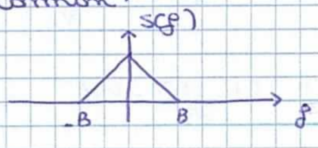
$s(t) = A \cdot \cos(2\pi f_1 t) + B \cdot \operatorname{sen}(2\pi f_2 t)$

$$P_s = \frac{A^2}{2} + \frac{B^2}{2}$$

19/03/2015

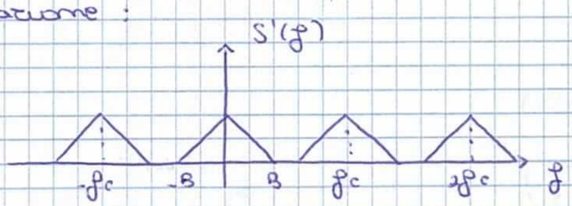
Teorema del campionamento (Shannon)

$s(t)$, $S(f) \neq \emptyset$ in $(-B, B)$



$T \leq \frac{1}{2B}$, $s_n = s(nT)$

Dimostrazione :

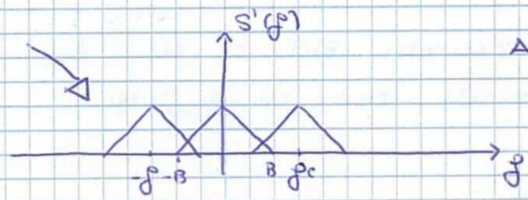


$f_c \geq 2B$

$$s(t) = \sum_{n=-\infty}^{+\infty} s\left(\frac{n}{f_c}\right) \cdot \frac{\operatorname{sen}\left[\pi f_c \left(t - \frac{n}{f_c}\right)\right]}{\pi f_c \left(t - \frac{n}{f_c}\right)}$$

Ciascun sinc ha ampiezza che dipende dal campione.

$f_c < 2B$



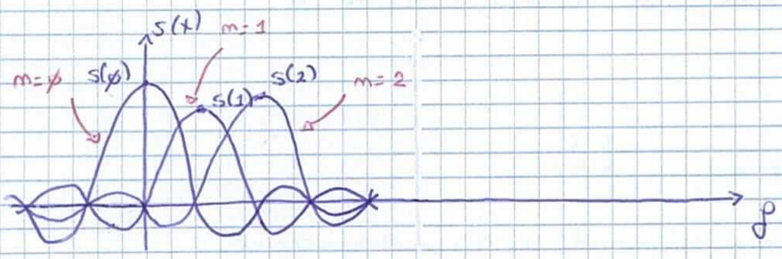
ALIASING

Viene introdotta una sovrapposizione di frequenze

=> Distorto!

f_c : frequenza di Nyquist.

$T = \frac{1}{f_c} \leq \frac{1}{2B}$



$f_c = 2B$

$n = 0$, $s(t) = \frac{\operatorname{sen}(\pi f_c t)}{\pi f_c t}$

$\pi f_c t = k\pi \Rightarrow t = \frac{k}{f_c} = \frac{k}{2B}$

$m = 1 \quad , \quad s(t) = \dfrac{\operatorname{sen}\left[\pi f_c \left(t - \frac{1}{f_c}\right)\right]}{\pi f_c \left(t - \frac{1}{f_c}\right)}$

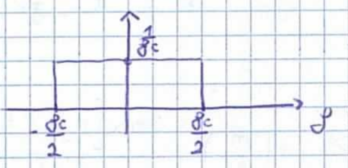
$$\cancel{\pi} f_c \left(t - \frac{1}{f_c}\right) = k\cancel{\pi} \quad \Rightarrow \quad t = \frac{k+1}{f_c}$$

Siccome:

$$\operatorname{rect}\left(\frac{t}{T}\right) \rightarrow T\operatorname{sinc}(fT)$$

$$T\operatorname{sinc}(tT) \rightarrow \operatorname{rect}\left(\frac{f}{T}\right)$$

$$\operatorname{sinc}(t f_c) \rightarrow \underbrace{\frac{1}{f_c}\operatorname{rect}\left(\frac{f}{f_c}\right)}_{\text{È un filtro passa basso ideale!}}$$

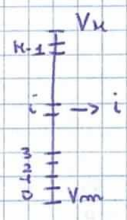


$$\operatorname{sinc}\left[\pi f_c \left(t - \frac{1}{f_c}\right)\right]$$

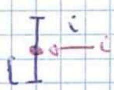
I campioni vengono quindi filtrati dai filtri passa basso e poi combinati tramite le fasi.

Passaggi Fondamentali:

1) Campionamento: $s(t) \rightarrow s_n$
2) Quantizzazione: num. reale s_n approssimato con un numero intero.



Intervalli di quantizzazione M.

nella ricostruzione: 

L'errore si chiama rumore di quantizzazione!

$M = 2^m$

3) Codifica: trasformazione dei campioni in binario

n_c : num. campioni al secondo $= \frac{1s}{T} = 2B$

$M = 2^m$ → Ogni campione viene trasformato in m bit

n_b = n bit/s = $2B \cdot m$

Segnale Telefonico:

$B = 4$ KHz

$M = 2^8 = 256$

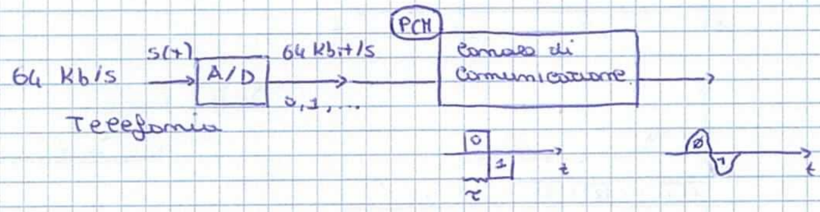
$n_b = 64$ Kb/s

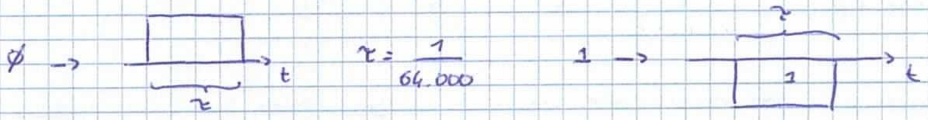
$B_t = 64$ KHz → Banda Trasmissione

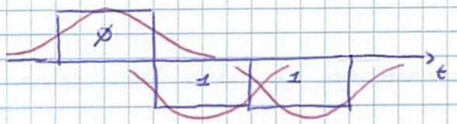
Posso scambiare i termini velocità ↔ Banda!

Quello del telefono è un modello standard.

Le leggi di quantizzazione non sono le stesse in tutto il mondo!







Non trasmetto il rettangolo perché ci vorrebbe banda infinita ma le loro approssimazioni.

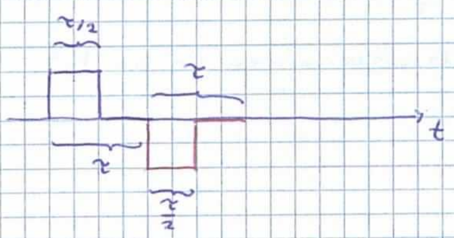
N.B: Le approssimazioni si possono sovrapporre ma devono comunque mantenere un certo segno per capire cosa trasmetto.

Il sistema ∅ : quadrato positivo

1 : quadrato negativo

si chiama NRZ (Non Return to Zero).

RZ (Return to Zero)

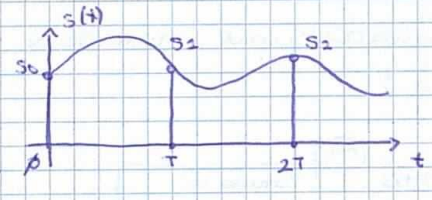


Questo sistema richiede banda doppia rispetto a quello NRZ.

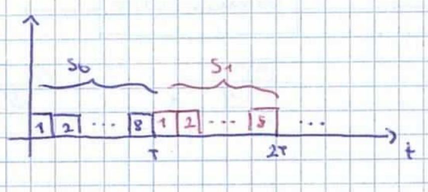
Il sistema NRZ è pericoloso per i ritardi in quanto se ho una stringa di 1 non ci sono discese e salite ma il segnale è costante. Non riesco bene a seguire la sequenza.

$T = \frac{1}{2B} = \frac{1}{8000} = 125\ \mu s$ nelle reti telefoniche.

$M = 2^8 = 256$



Ogni campione è 8 Bit



PCM = Pulse Code Modulation

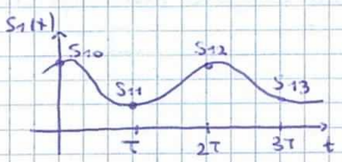
1 s -> 64.000 bit

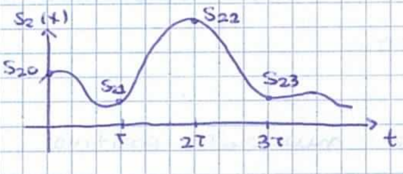
Grazie alla compressione riesco a ridurre il numero di bit da trasmettere => Riduco la Banda!

Ho due telefonate che voglio trasmettere su un solo canale

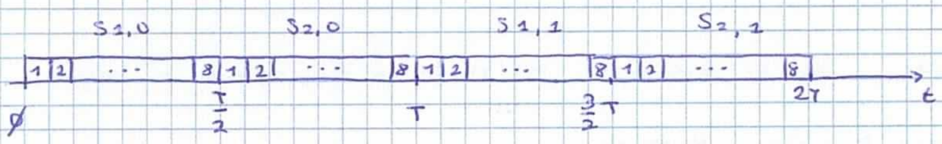
$s_1(t)$, $s_2(t)$

Campionate con $T = 125\ \mu s$ e $M = 2^8$ livelli di quantizzazione.





Adesso :



Entrambi i segnali vengono trasmessi sullo stesso canale!

=> MULTIPLEX 2 canali

$m_b = m\,bit/s = 128\ Kbit/s$

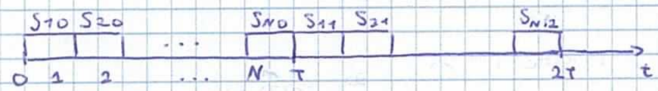
$B_t \doteq 128\ KHz$

La banda è stata Raddoppiata!

Adesso trasmetto N segnali

$s_1(t), s_2(t), \ldots, s_N(t)$

$T = \frac{1}{2B} = 125\ \mu s$



$S_{10} = 8\ bit$

...

$m_b = N \cdot 64\ Kb/s$

$B_t \doteq N \cdot 64\ KHz$

=> multiplex con N canali

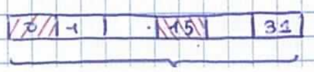
Parametri da definire:

$T(f_c)$, M, N

nel caso della telefonia

$N = 1 \rightarrow m_b = 64\ Kb/s$, $B_t = 64\ KHz$

$N = 32 \rightarrow m_b = 32 \cdot 64\ Kb/s = 2048\ Kb/s \cong 2Mb/s$ in Europa



Canale Ø e 15 non usati!

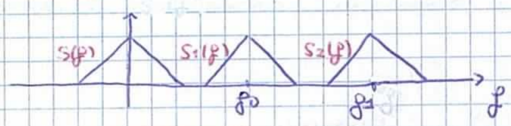
USA $\rightarrow N = 24$, $m_b \simeq 1,5$ Mb/s

Il multiplex visto si chiama: MULTIPEX a divisione di tempo.

TDM: Time Division Multiplex

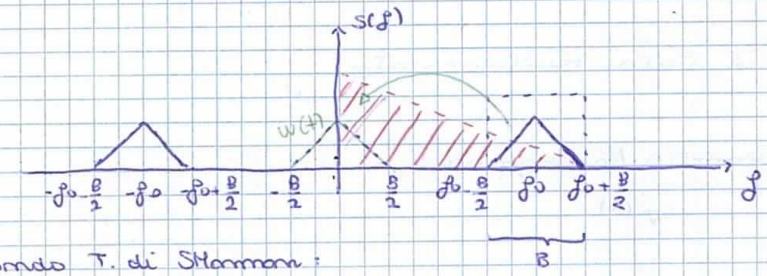
$s_1(t) \cdot \cos(2\pi f_0 t)$

$s_2(t) \cdot \cos(2\pi f_1 t)$



Multiplex a divisione di Frequenza

FDM: Frequency Division Multiplex.



applicando T. di Shannon:

$T = \frac{1}{2(f_0 + \frac{B}{2})}$ con questo T posso campionare anche il segnale rosso.

$f_0 = 1$ GHz , $T = \frac{1}{10^9}$

Teorema del campionamento in alta frequenza

$$s(t) = a(t) \cdot \cos[2\pi f_0 t - \theta(t)] = Re\{\underbrace{a(t) \cdot e^{j[2\pi f_0 t - \theta(t)]}}_{v(t)}\}$$

$s(t) \rightarrow S(f)$

$s(t)\, e^{j2\pi f_0 t} \rightarrow S(f - f_0)$

$$v(t) = \underbrace{a(t) \cdot e^{-j\theta(t)}}_{w(t)} \cdot e^{j2\pi f_0 t}$$

$$T = \frac{1}{2 \cdot \frac{B}{2}} = \frac{1}{B} \quad , \quad w(t) = \sum_{n=-\infty}^{+\infty} w\left(\frac{n}{f_c}\right) \cdot \frac{\sin\left[\pi f_c \left(t - \frac{n}{f_c}\right)\right]}{\pi f_c \left(t - \frac{n}{f_c}\right)} = \circledast$$

$w(t) = a(t) \cdot e^{-j\theta(t)}$

$w\left(\frac{n}{f_c}\right) = a\left(\frac{n}{f_c}\right) \cdot e^{-j\theta(n/f_c)}$

$$W(t) = \circledast = \sum_{m=-\infty}^{+\infty} a\left(\frac{m}{f_c}\right) e^{-j\theta\left(m/f_c\right)} \cdot \operatorname{sinc}\left[f_c\left(t - \frac{m}{f_c}\right)\right].$$

$$V(t) = \sum_{m=-\infty}^{+\infty} a\left(m/f_c\right) e^{-j\theta\left(m/f_c\right)} \cdot \operatorname{sinc}\left[f_c\left(t - \frac{m}{f_c}\right)\right] \cdot e^{j2\pi f_0 t}$$

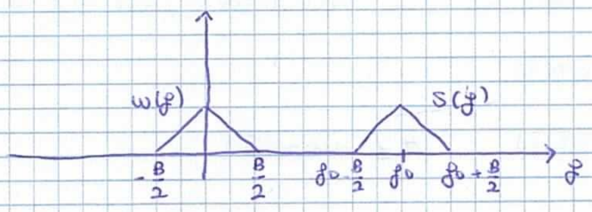
$$= \sum_{m=-\infty}^{+\infty} a\left(m/f_c\right) \operatorname{sinc}\left[f_c\left(t - \frac{m}{f_c}\right)\right] e^{j\left[2\pi f_0 t - \theta\left(\frac{m}{f_c}\right)\right]}.$$

$\cos = Re[e^{\cdots}]$

$$s(t) = \sum_{m=-\infty}^{+\infty} a\left(m/f_c\right) \operatorname{sinc}\left[f_c\left(t - \frac{m}{f_c}\right)\right] \cdot \cos\left[2\pi f_0 t - \theta\left(\frac{m}{f_c}\right)\right]$$

$f_c = B$, $T = \frac{1}{B} = \frac{1}{f_c}$, Campiono con questo periodo

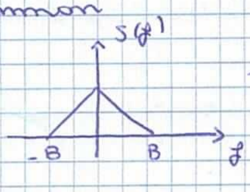
Disegno solo la parte positiva:



$$T = \frac{1}{2\left(f_0 + \frac{B}{2}\right)}$$

Invece di campionare a $T = \frac{1}{2\left(f_0 + \frac{B}{2}\right)}$, in alta frequenza posso campionare a $T = \frac{1}{B}$.

Shannon



$$T' = \frac{1}{2B}$$

Con il nuovo Teorema il numero di campioni da prendere è lo stesso avendo stessa banda in quanto campiono sia ampiezza che fase.