FONDAMENTI DI TELECOMUNICAZIONI A

[Appunti Di Esercizi, Parte 2]

A CURA DI ALESSANDRO PAGHI

PROFESSORE: Giuliano Benelli (http://www3.diism.unisi.it/people/person.php?id=18)

LINK AL CORSO ANNO 2014/2015:
http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=54889&aa=2014

FREQUENTAZIONE: Consigliata.

17/06/2011

Es. 1

$y(t) = 4\cdot\cos(2\pi f_1 t) + 8\cdot\cos(2\pi f_2 t) + 8\cdot\cos(2\pi f_3 t) + 4\cdot\cos(2\pi f_4 t)$

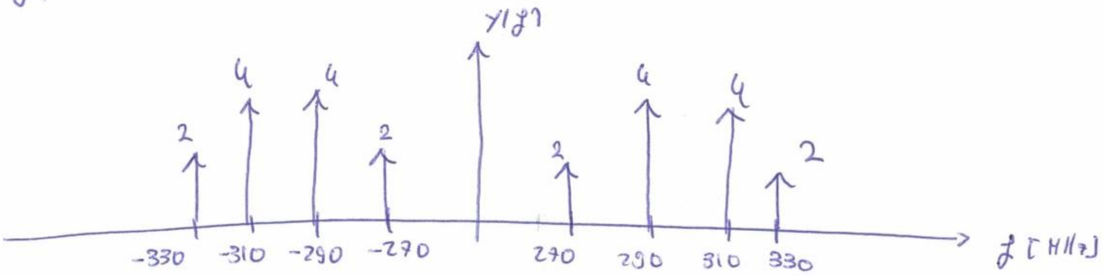
$f_1 = 270\ \text{MHz}$

$f_2 = 290\ \text{MHz}$

$f_3 = 310\ \text{MHz}$

$f_4 = 330\ \text{MHz}$

? $Y(f)$



? $y(t)$ DSB calcolare $s(t)$ e $m(t)$

$s(t) = V_0\cdot\cos(2\pi f_0 t)$, $f_0 = 300\ \text{MHz}$, $V_0 = 1\ \text{V}$

$m(t) = 16\cdot\cos(2\pi\, 10\ \text{M}t) + 8\cdot\cos(2\pi\, 30\ \text{M}t)$

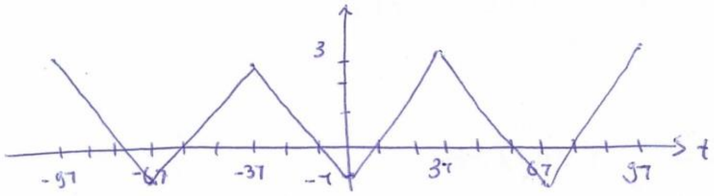
? $y(t)$ SB calcolare $s(t)$ e $m(t)$, $B_m = 70\ \text{MHz}$

$s(t) = V_0\cdot\cos(2\pi f_0 t)$, $V_0 = 1\text{V}$, $f_0 = 330 - 70 = 260\ \text{MHz}$

$m(t) = 4\cdot\cos(2\pi\, 10\ \text{M}t) + 8\cdot\cos(2\pi\, 30\ \text{M}t) + 8\cdot\cos(2\pi\, 50\ \text{M}t) + 4\cdot\cos(2\pi\, 70\ \text{M}t)$

20/06/2012

ES. 1

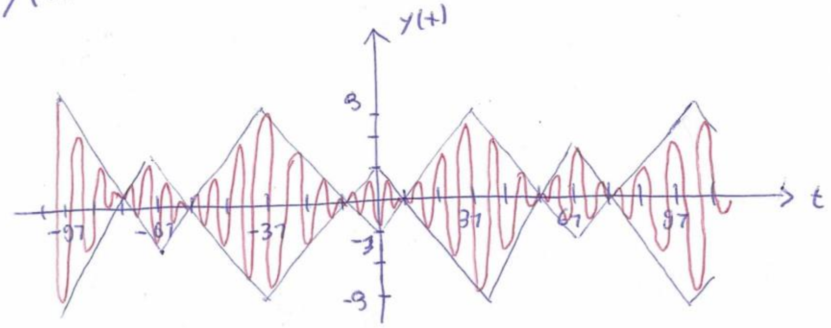


DSB

$H(f) = \text{rect}\left(\frac{f-f_0}{f_c}\right) + \text{rect}\left(\frac{f+f_0}{f_c}\right)$, $f_0 = 150$ MHz , $V_0 = 3$ V , $\tau = 10^{-4}$ sec

$f_c = 10$ KHz

? $y(t)$



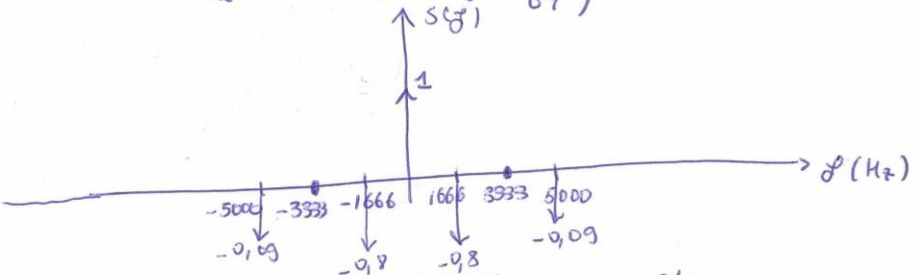
? $z(t)$

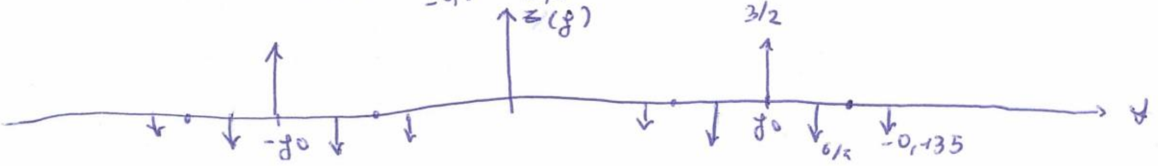
$s_\tau(t) = -4\,\text{tri}\left(\frac{t}{3\tau}\right)$

$$s_p(t) = \left[\sum_{m=-\infty}^{+\infty} -4\,\text{tri}\left(\frac{t-6m\tau}{\tau}\right)\right] + 3$$

$$S(f) = 3\delta(f) + \frac{1}{\cancel{2}\cancel{6\tau}} \sum_{m=-\infty}^{+\infty} -\overset{2}{\cancel{4}} \cdot \cancel{3\tau} \cdot \text{sinc}^2\left(\cancel{3\tau} \cdot \frac{m}{6\cancel{\tau}}\right) \cdot \delta\left(f - \frac{m}{6\tau}\right)$$

$$= 3\delta(f) - 2 \sum_{m=-\infty}^{+\infty} \text{sinc}^2\left(\frac{m}{2}\right) \cdot \delta\left(f - \frac{m}{6\tau}\right)$$





$z(t) = -0,27 \cdot \cos(2\pi (f_0 - 5000) t) - \frac{12}{5} \cdot \cos(2\pi (f_0 - 1666) t) + 3 \cos(2\pi f_0 t)$
$- \frac{12}{5} \cos(2\pi (f_0 + 1666) t) - 0,27 \cos(2\pi (f_0 + 5000) t)$

? SNR_u

$SNR_u = SNR_i = \frac{P_y}{N_0 B_{mod}} = \frac{10,33}{10^{-8} \cdot 5 \cdot 10^3} = 206658$

25/06/2013

ES. 1

$m_1(t) = \cos(2\pi f_1 t)$, $f_1 = 20\,kHz$

$m_2(t) = 2\cos(2\pi f_2 t)$, $f_2 = 60\,kHz$

$s(t) = m_1^2(t) \cdot m_2^2(t)$

AM)

$c(t) = V_0 \cdot \cos(2\pi f_0 t)$, $f_0 = 30\,MHz$, $V_0 = 1V$, $k = 0,2$

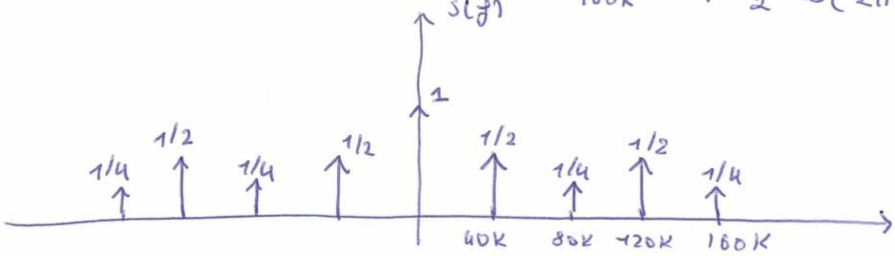
? $y(f)$

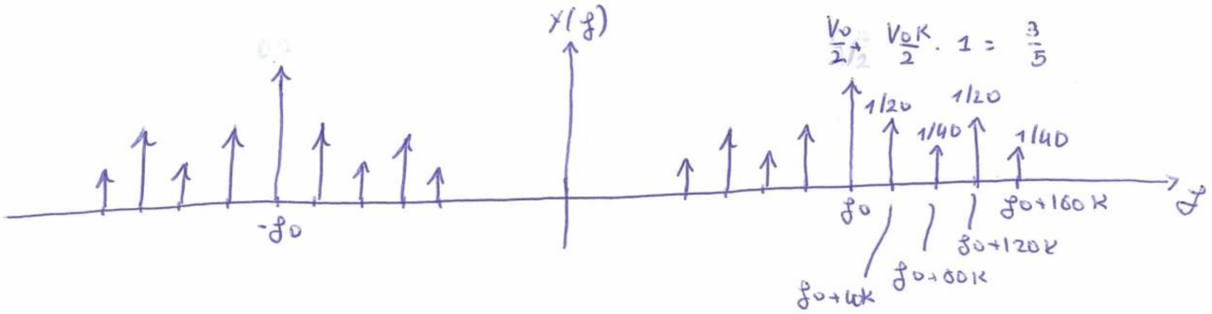
$$s(t) = 4\cdot\cos^2(2\pi f_1 t)\cos^2(2\pi f_2 t) = \frac{4}{4}\left(1+\cos(2\pi 2 f_1 t)\right)\left(1+\cos(2\pi 2 f_2 t)\right)$$

$$= 1 + \cos(2\pi 2 f_2 t) + \cos(2\pi 2 f_1 t) + \cos(2\pi 2 f_1 t)\cdot\cos(2\pi 2 f_2 t)$$

$$= 1 + \cos(2\pi 2 f_2 t) + \cos(2\pi 2 f_1 t) + \frac{1}{2}\left[\cos(2\pi(2f_1 + 2f_2)t) + \cos(2\pi(2f_1 - 2f_2)t)\right]$$

$$= 1 + \underset{120K}{\cos(2\pi 2 f_2 t)} + \underset{40K}{\cos(2\pi 2 f_1 t)} + \frac{1}{2}\underset{160K}{\cos(2\pi(2f_1 + 2f_2)t)} + \frac{1}{2}\underset{80K}{\cos(2\pi(2f_1 - 2f_2)t)}$$





? P_y

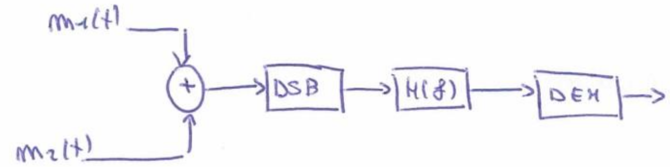
$$P_y = 8\cdot\left(\frac{1}{40}\right)^2 + 8\cdot\left(\frac{1}{20}\right)^2 + 2\cdot\left(\frac{3}{5}\right)^2 = 0{,}745 \text{ Watt}$$

19/06/2013

ES. 1

$$m_1(t) = \sum_{m=-\infty}^{+\infty} tri\left(\frac{t-2mT}{T}\right)$$

$$m_2(t) = \sum_{m=-\infty}^{+\infty} rect\left(\frac{t-4mT}{T}\right)$$



$$H(f) = rect\left(\frac{f-f_0}{f_c}\right) + rect\left(\frac{f+f_0}{f_c}\right)$$

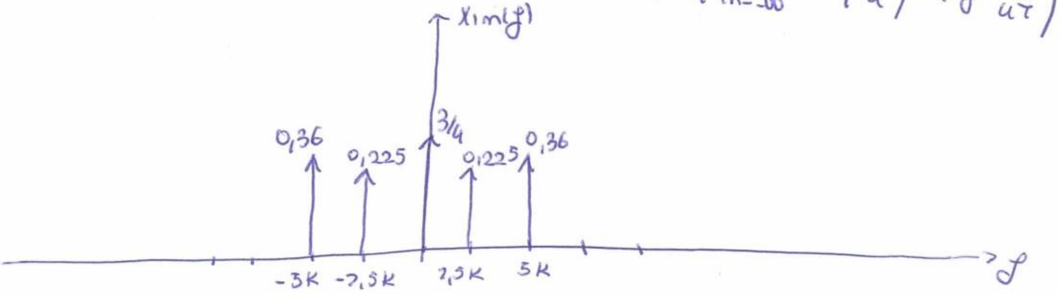
$f_0 = 10$ MHz , $V_0 = 2V$, $T = 10^{-4}$ sec , $f_c = 12$ kHz.

? $S_y(f)$

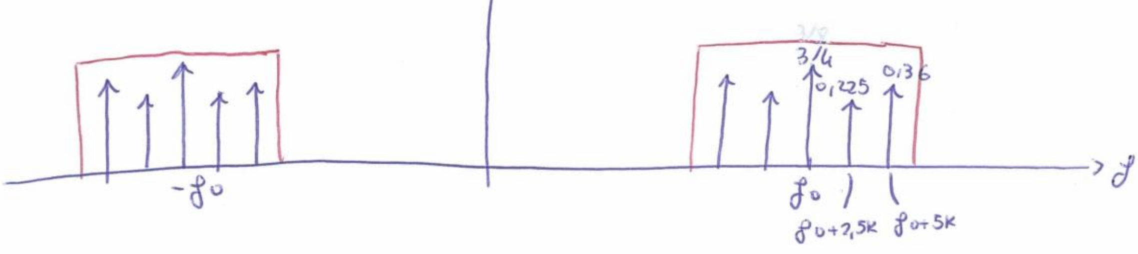
$$m_1(f) = \frac{1}{2T} \cdot \sum_{m=-\infty}^{+\infty} T \cdot sinc^2\left(T \cdot \frac{m}{2T}\right) \cdot \delta\left(f - \frac{m}{2T}\right)$$ Passo: 5K Hz

$$m_2(f) = \frac{1}{4T} \cdot \sum_{m=-\infty}^{+\infty} T \cdot sinc\left(T \cdot \frac{m}{4T}\right) \cdot \delta\left(f - \frac{m}{4T}\right)$$ Passo: 2,5 kHz

$$X_m(f) = m_1(f) + m_2(f) = \frac{1}{2} \sum_{m=-\infty}^{+\infty} sinc^2\left(\frac{m}{2}\right) \delta\left(f - \frac{m}{2T}\right) + \frac{1}{4} \sum_{m=-\infty}^{+\infty} sinc\left(\frac{m}{4}\right) \delta\left(f - \frac{m}{4T}\right)$$



$$0{,}36 = \left(\frac{2}{\pi}\right)^2 \cdot \frac{1}{2} + \frac{2}{\pi} \cdot \frac{1}{4}$$



? $y(t)$

$$y(t) = 0{,}72 \cdot \cos(2\pi(f_0 - 5K)t) + 0{,}45 \cdot \cos(2\pi(f_0 - 2{,}5K)t) +$$

$$+ \frac{3}{2} \cos(2\pi f_0 t) + 0{,}45 \cdot \cos(2\pi(f_0 + 2{,}5K)t) + 0{,}72 \cos\left(2\pi(f_0 + 5K)t\right).$$

? SNR_u

$SNR_i = SNR_u = \frac{P_y}{N_0 \cdot B_u}$ com $N_0 = 10^{-8}$

$$P_y = 4 \cdot 0{,}36^2 + 4 \cdot 0{,}225^2 + 2 \cdot 3/4^2 = 2{,}97 \text{ Watt}$$

$$SNR_u = \frac{2{,}97}{10^{-8} \cdot 5 \cdot 10^3} = 59400.$$

Es. 3

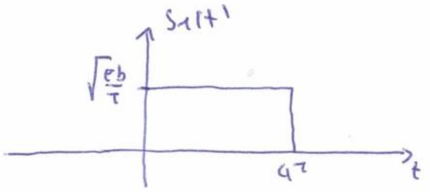
M=3 segnali equiprobabili:

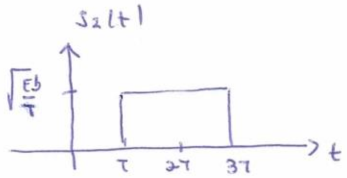
$$s_1(t) = \sqrt{\frac{E_b}{T}} \cdot \mathrm{rect}\left(\frac{t-2T}{4T}\right)$$

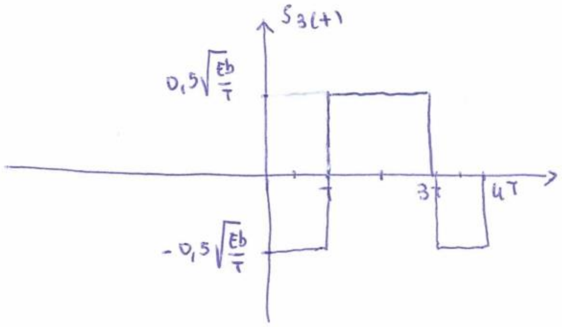
$$s_2(t) = \sqrt{\frac{E_b}{T}} \cdot \mathrm{rect}\left(\frac{t-2T}{2T}\right)$$

$$s_3(t) = -0,5\sqrt{\frac{E_b}{T}} \cdot \mathrm{rect}\left(\frac{t-0,5T}{T}\right) + 0,5\sqrt{\frac{E_b}{T}} \cdot \mathrm{rect}\left(\frac{t-2T}{2T}\right) - 0,5\sqrt{\frac{E_b}{T}} \cdot \mathrm{rect}\left(\frac{t-3,5T}{T}\right)$$

? Rappresentazione geometrica







$$s_3(t) = -0,5\, s_1(t) + s_2(t)$$

$$\psi_1(t) = \frac{s_1(t)}{\sqrt{E_{s_1}}} \quad , \quad E_{s_1} = \int_{-\infty}^{+\infty} |s_1(t)|^2 dt = \int_{0}^{4T} \frac{E_b}{T}\, dt = 4E_b$$

$$= \frac{s_1(t)}{\sqrt{4E_b}}$$

$$\psi_2(t) = \frac{v_2(t)}{\sqrt{E_{v_2}}} \quad , \quad v_2(t) = s_2(t) - \langle s_2(t), \psi_1(t) \rangle \psi_1(t)$$

$$\langle s_2(t), \psi_1(t) \rangle = \int_{-\infty}^{+\infty} s_2(t) \cdot \frac{s_1(t)}{\sqrt{4E_b}}\, dt = \frac{1}{\sqrt{4E_b}} \cdot \int_{T}^{3T} \frac{E_b}{T} \cdot dt = \frac{E_b}{T\sqrt{4E_b}} \cdot \left[t \right]_{T}^{3T}$$

$$= \frac{E_b}{\cancel{T}\sqrt{4E_b}} \cdot 2\cancel{T} = \frac{2E_b}{\sqrt{4E_b}} = \sqrt{E_b}$$

$$v_2(t) = s_2(t) - \sqrt{\cancel{E_b}} \cdot \frac{s_1(t)}{\sqrt{4} \cdot \sqrt{\cancel{E_b}}} = s_2(t) - \frac{1}{2} \cdot s_1(t)$$

$$E_{v_2} = \int_{-\infty}^{+\infty} \left| s_2(t) - \frac{1}{2} s_1(t) \right|^2 dt = \int_{T}^{3T} \frac{1}{4} \frac{E_b}{T} \, dt + \int_{0}^{T} \frac{1}{{}_2\cancel{4}} \frac{E_b}{T} \cdot \cancel{2} \, dt = \frac{1}{{}_2\cancel{4}} \frac{E_b}{\cancel{T}} \cdot \cancel{2T} + \frac{1}{2} \frac{E_b}{\cancel{T}} \cdot \cancel{T}$$

$$= E_b$$

$$\varphi_2(t) = \frac{v_2(t)}{\sqrt{E_b}}$$

$$\underline{s}_1 = (2\sqrt{E_b}, 0)$$

$$\underline{s}_2 = (\sqrt{E_b}, \sqrt{E_b})$$

$$\underline{s}_3 = (0, \sqrt{E_b})$$

25/06/2014

Es. 1

$m_1(t) = 2\cdot\cos(2\pi f_1 t) + \cos(2\pi f_2 t)$, $f_1 = 10$ kHz , $f_2 = 20$ kHz

$m_2(t) = 1 - 2\left(\sum_{m=-\infty}^{+\infty} rect\left(\frac{t-10mT}{5T}\right)\right)$, $T = 10^{-5}$ sec.

$H(f) = rect\left(\frac{f}{f_c}\right)$, $f_c = 50$ kHz

AM)

$c(t) = V_0 \cdot \cos(2\pi f_0 t)$, $f_0 = 30$ MHz , $V_0 = 1V$, $K = 1/4$

? $X_{im}(f)$, $y_{AM}(t)$

$m_1(t) + m_2(t) = 2\cos(2\pi f_1 t) + \cos(2\pi f_2 t) + 1 - 2\sum_{m=-\infty}^{+\infty} rect\left(\frac{t-10mT}{5T}\right) = x_{im}(t)$

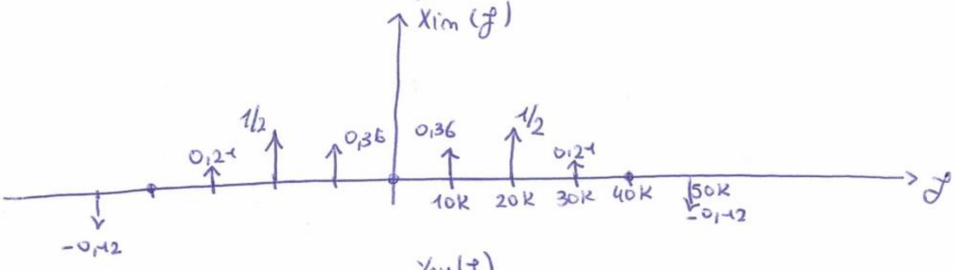
$X_{im}($

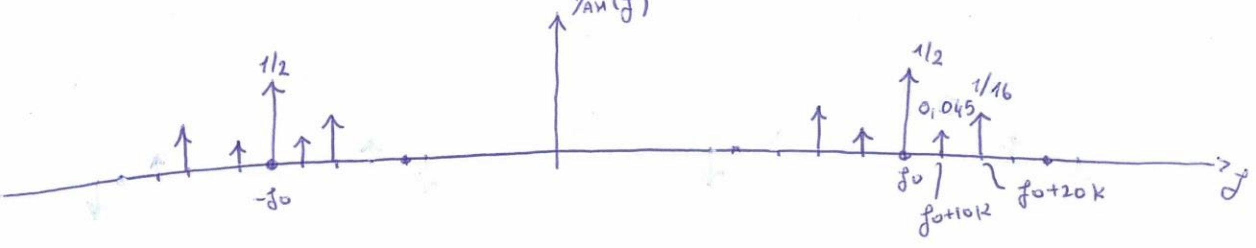
$y_{AM}(t) = V_0(1 + K x_{im}(t))\cos(2\pi f_0 t)$

$= (1 + \frac{1}{4}$

$X_{im}(f) = \delta(f-f_1) + \delta(f+f_1) + \frac{1}{2}(f-f_2) + \frac{1}{2}\delta(f+f_2) + \delta(f)$

$-\cancel{2}\left\{\frac{1}{\cancel{2}}\cdot\sum_{m=-\infty}^{+\infty} sinc\left(\frac{m}{2}\right)\cdot\delta\left(f-\frac{m}{10T}\right)\right\}$





$y_{AM}(t) = \frac{1}{8}\cos(2\pi(f_0-20K)t) + 0,09\cos(2\pi(f_0-10K)t) + 1/2\cdot\cos(2\pi f_0 t) +$

$+ 0,09\cdot\cos(2\pi(f_0+10K)t) + 1/8\cos(2\pi(f_0+20K)t)$.

? SNR_i , SNR_u

$$P_y = 4 \cdot \left(\frac{1}{16}\right)^2 + 4 \cdot \left(0{,}045\right)^2 + 2 \cdot \left(\frac{1}{2}\right)^2 = 0{,}52 \text{ Watt}$$

$$SNR_i = \frac{P_y}{N_0 B_u} = 2600$$

$$F_m = \frac{K^2 P_u}{1 + K^2 P_u} = 0{,}045 \qquad , P_u = 2 \cdot \left(\frac{1}{2}\right)^2 + 2 \cdot \left(0{,}36\right)^2 = 0{,}76 \text{ Watt}$$

$$SNR_u = SNR_i \cdot F_m = 117{,}9 \; .$$

ES. 2

$s(t) = 10 \cdot \cos(2\pi f_m t)$, $f_m = 50$ KHz

FM) $K_f = 10^4 \; \frac{Hz}{W}$

$m(t) = 4 \cdot \cos(2\pi f_0 t)$, $f_0 = 8$ MHz

Canale AWGN con $N_0 = 10^{-8} \; \frac{W}{Hz}$

? Segnale FM e Banda di Carson

$$y_{FM}(t) = V_0 \cdot \cos\left(2\pi f_0 t + 2\pi K_f \int_{\emptyset}^{t} s(\tau)\, d\tau\right)$$

$$= 4 \cdot \cos\left(2\pi f_0 t + 2\pi \cdot 10^4 \int_{\emptyset}^{t} 10 \cdot \cos(2\pi f_m \tau)\, d\tau\right)$$

$$= 4 \cdot \cos\left(2\pi f_0 t + 2\pi 10^5 \cdot \frac{1}{2\pi f_m} \cdot \operatorname{sen}(2\pi f_m t)\right)$$

$$m = \frac{K_f \cdot 10}{f_m} = 2$$

$$B_{98\%} = 2(m+1) \cdot B_m = 300 \text{ KHz}$$

? Ricevitore funziona

$$SNR_i|_{dB} \geq 13 + 10 \log_{10}(m+1) = 17,7 \text{ dB}$$

$$SNR_i = \frac{V_0^2}{2 N_0 B_{nast}} = 16.000 \qquad SNR_i|_{dB} = 42,04 \text{ dB}$$

? SNR_u

$$F_m = \frac{3}{2} m^2 = 6 \qquad \Rightarrow SNR_u|_{dB} = SNR_i|_{dB} + F_m|_{dB} = 49,8 \text{ dB}$$

? $P_{[79.00 \div 8200]\text{KHz}}$

$$\% P = 0,224^2 + 2 \cdot 0,577^2 + 2 \cdot 0,353^2 + 0,129^2 + 0,034^2 = 0,98$$

Es. 3

$E_b = 10^{-8}$

$N_0 = 10^{-9} = \frac{E_b}{10}$

$M = 4$

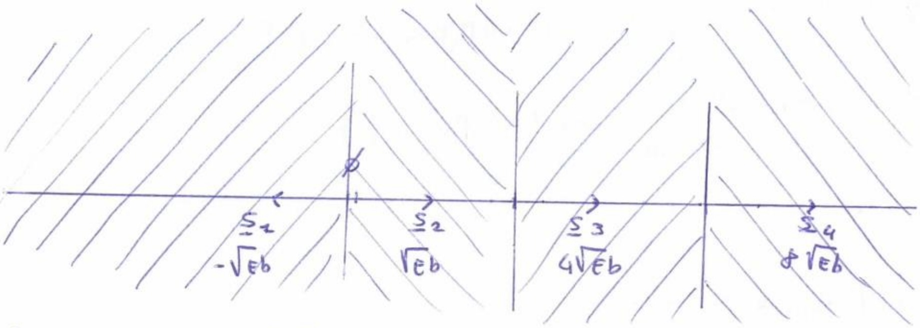
$s_1 = -\sqrt{E_b}$ $\qquad P(s_1) = 1/16$

$s_2 = \sqrt{E_b}$ $\qquad P(s_2) = 1/2$

$s_3 = 4\sqrt{E_b}$ $\qquad P(s_3) = 3/8$

$s_4 = 8\sqrt{E_b}$ $\qquad P(s_4) = 1/16$

? MAP.



$$I_{21} = \left\{ \underline{r}(s_2 - s_1) \gtrless \frac{N_0}{2} \ln\left(\frac{P(s_1)}{P(s_2)}\right) + \frac{E_2 - E_1}{2} \right\}$$

$$= \left\{ \underline{r}\, 2\sqrt{E_b} \gtrless \frac{E_b}{2 \cdot 10} \ln\left(\frac{1/16}{1/2}\right) + \cancel{\frac{\sqrt{E_b}^2 - \sqrt{E_b}^2}{2}} \right\} = \left\{ \underline{r} \gtrless \frac{-0{,}104\, E_b}{2\sqrt{E_b}} \right\}$$

$$= \left\{ r \gtrless -0{,}052\sqrt{E_b} \right\}$$

$$I_{32} = \left\{ \underline{r}(s_3 - s_2) \gtrless \frac{N_0}{2} \ln\left(\frac{P(s_2)}{P(s_3)}\right) + \frac{E_3 - E_2}{2} \right\}$$

$$= \left\{ \underline{r}\,(3\sqrt{E_b}) \gtrless \underbrace{\frac{E_b}{20} \underbrace{\ln\left(\frac{1/2}{3/8}\right)}_{0{,}28}}_{0{,}015 E_b} + \frac{16E_b - E_b}{2} \right\} = \left\{ \underline{r} \gtrless 2{,}505\sqrt{E_b} \right\}$$

$$I_{43} = \left\{ \underline{r}\, 4\sqrt{E_b} \gtrless \frac{E_b}{20} \ln\left(\frac{3/8}{1/16}\right) + \frac{64E_b - 16E_b}{2} \right\} = \left\{ \underline{r} \gtrless 6{,}0225\sqrt{E_b} \right\}$$

$$Pe = \sum_{i=1}^{4} Pe/s_i \cdot P(s_i)$$

$$Pe/\underline{s}_1 = Pr\left\{ s_1 + m \geq -0{,}032\sqrt{Eb} \right\} = Pr\left\{ m \geq 0{,}948\sqrt{Eb} \right\} = Q\left(0{,}948\sqrt{Eb} \cdot \sqrt{\frac{2}{N_0}} \right)$$

$$= Q\left(0{,}948 \cdot \sqrt{2} \cdot \sqrt{\frac{Eb}{N_0}} \right) = Q\left(0{,}423 \right)$$

$$Pe/\underline{s}_2 = Pr\left\{ s_2 + m \leq -0{,}052\sqrt{Eb} \right\} + Pr\left\{ s_2 + m \geq 2{,}505\sqrt{Eb} \right)$$

$$= Pr\left\{ m \leq -1{,}052\sqrt{Eb} \right\} + Pr\left\{ m \geq 3{,}505\sqrt{Eb} \right\}$$

$$= Q\left(-1{,}052\sqrt{Eb}\sqrt{\frac{2}{N_0}} \right) + Q\left(3{,}505\sqrt{Eb} \cdot \sqrt{\frac{2}{N_0}} \right) = Q\left(-0{,}471 \right) + Q\left(1{,}57 \right)$$

$$Pe/\underline{s}_3 = Pr\left\{ s_3 + m \leq 2{,}505\sqrt{Eb} \right\} + Pr\left\{ s_3 + m \geq 6{,}0225\sqrt{Eb} \right\}$$

$$= Pr\left\{ m \leq -1{,}495\sqrt{Eb} \right\} + Pr\left\{ m \geq 2{,}0225\sqrt{Eb} \right\}$$

$$= Q\left(-1{,}495\sqrt{Eb} \cdot \sqrt{\frac{2}{N_0}} \right) + Q\left(2{,}0225\sqrt{Eb} \cdot \sqrt{\frac{2}{N_0}} \right) = Q\left(0{,}669 \right) + Q\left(0{,}905 \right)$$

$$Pe/\underline{s}_4 = Pr\left\{ s_4 + m \leq 6{,}0225\sqrt{Eb} \right\} = Pr\left\{ m \leq -1{,}9775\sqrt{Eb} \right\}$$

$$= Q\left(-1{,}9775\sqrt{Eb} \cdot \sqrt{\frac{2}{N_0}} \right) = Q\left(0{,}884 \right)$$

$$Pe = Q(0{,}423) \cdot \frac{1}{16} + \left\{ Q(-0{,}471) + Q(1{,}57) \right\} \cdot \frac{1}{2} + \left\{ Q(0{,}669) + Q(0{,}905) \right\} \cdot \frac{3}{8} +$$

$$+ Q(0{,}384) \cdot \frac{1}{16} .$$

(*) ERRORE DI CALCOLO

17/06/2014

ES. 1

$m_1(t) = 2\cos(2\pi f_1 t)$, $f_1 = 10\,kHz$

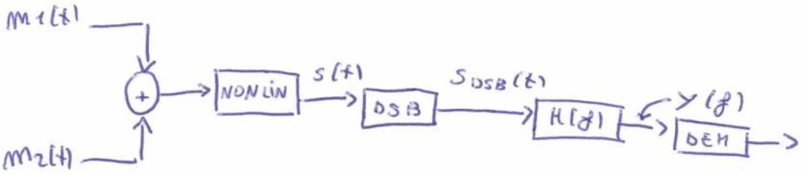
$m_2(t) = 3\cos(2\pi f_2 t)$, $f_2 = 15\,kHz$

$s(t) = x^2_{in}(t) + x_{in}(t)$

DSB)

$c(t) = V_0 \cdot \cos(2\pi f_0 t)$, $f_0 = 120\,MHz$, $V_0 = 3V$

$H(f) = rect\left(\frac{f-f_0}{f_c}\right) + rect\left(\frac{f+f_0}{f_c}\right)$, $f_c = 32\,kHz$



? $y(f)$

$x_{in}(t) = m_1(t) + m_2(t)$

$s(t) = (m_1(t) + m_2(t))^2 + m_1(t) + m_2(t)$

$= 4\cos^2(2\pi f_1 t) + 9\cos^2(2\pi f_2 t) + 12\cos(2\pi f_1 t)\cos(2\pi f_2 t) + 2\cos(2\pi f_1 t) +$

$+ 3\cos(2\pi f_2 t)$

$= 2\left[1 + \cos(4\pi f_1 t)\right] + \frac{9}{2}\left[1 + \cos(4\pi f_2 t)\right] + 6\left[\cos(2\pi(f_1 - f_2)t) + \cos(2\pi(f_1 + f_2)t)\right]$

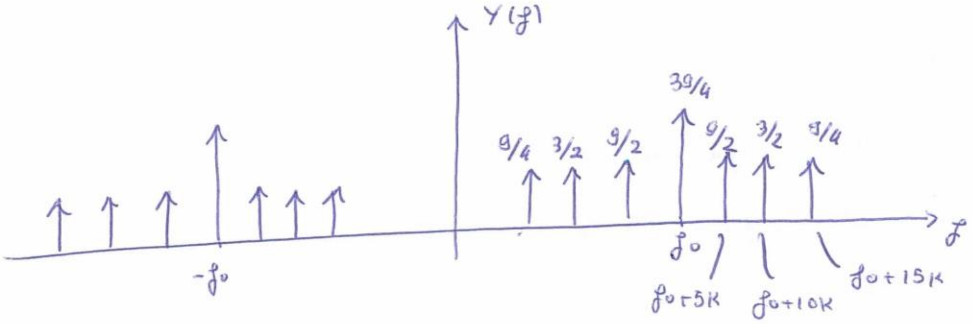
$+ 2\cos(2\pi f_1 t) + 3\cos(2\pi f_2 t) =$

$2 + \frac{9}{2} + 2\cos(2\pi 2 f_1 t) + \frac{9}{2} + \frac{9}{2}\cos(2\pi 2 f_2 t) + 6\cos(2\pi(f_1 - f_2)t) +$

$+ 6\cos(2\pi(f_1 + f_2)t) + 2\cos(2\pi f_1 t) + 3\cos(2\pi f_2 t)$

$= \frac{13}{2} + 6\cos(2\pi 5k t) + 2\cos(2\pi 10 k t) + 3\cos(2\pi 15k t) + 2\cos(2\pi 20k t) +$

$+ 6\cos(2\pi 25k t) + \frac{9}{2}\cos(2\pi 30 k t)$



? y(f)

$$y(f) = \frac{9}{2} \cdot \cos(2\pi(f_0-15K)t) + 3 \cdot \cos(2\pi(f_0-10K)t) + 9 \cos(2\pi(f_0-5K)t) +$$
$$+ \frac{39}{2} \cdot \cos(2\pi f_0 t) + 9 \cdot \cos(2\pi(f_0+5K)t) + 3 \cdot \cos(2\pi(f_0+10K)t) +$$
$$+ \frac{9}{2} \cdot \cos(2\pi(f_0+15K)t) .$$

? SNR_u , $N_0 = 10^{-8}$ W/Hz

$$SNR_u = SNR_i = \frac{P_y}{N_0 \cdot B}$$

$$P_y = 4 \cdot \left(\frac{9}{4}\right)^2 + 4 \cdot \left(\frac{3}{2}\right)^2 + 4 \cdot \left(\frac{9}{2}\right)^2 + 2 \cdot \left(\frac{39}{4}\right)^2 = 300,375 \text{ Watt}$$

$B = 15$ KHz

=> $SNR_i = 2002500$, $SNR_i|_{dB} = 63$ dB .

? SNR_u se $y(t) = y_{SSB}(t)$ com $B = f_0$

$$SNR_{u\,SSB} = \frac{P_y}{N_0 \cdot B} = 938671,875 \quad , \quad SNR_{u\,SSB}|_{dB} = 59,72 \text{ dB}$$

ES. 2

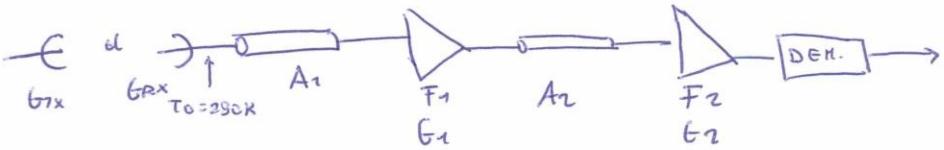
$s_1(t) = 2\cdot\cos(2\pi f_m t)$

$f_m = 50$ kHz

$s_2(t) = 8\cdot\cos(2\pi f_m t)$

FM)

$c(t) = 5\cdot\cos(2\pi f_0 t)$, $f_0 = 10$ GHz , $k_f = 5\cdot 10^4$



$G_{TX} = G_{RX} = 40$ dB

$d = 400$ Km

$A_1 = F_2 = G_2 = 10$ dB

$F_1 = G_1 = A_2 = 13$ dB $\Rightarrow 20$

? Effetto Soglia

$SNR_i|_{dB} \doteq 13 + 10\log_{10}(m+1)$, $m = \frac{k_f\cdot V_m}{f_m}$

$SNR_{i1}|_{dB} \doteq 17{,}77$ dB

$m_1 = 2$

$SNR_{i2}|_{dB} \doteq 22{,}54$ dB

$m_2 = 8$

$SNR_i|_{dB} = P_y|_{dB} + G_{TX}|_{dB} + G_{RX}|_{dB} - A_{fs}|_{dB} - 10\log_{10} k T_{SIST}\cdot B_H$

$P_y|_{dB} = \frac{V_0^2}{2} = \frac{25}{2}$ Watt

$P_y|_{dB} = 10{,}97$ dB

$A_{fs}|_{dB} = 32{,}4 + 20\log_{10} f_0|_{MHz} + 20\log_{10} d|_{km} = 152{,}4$ dB

$T_{SIST} = T_0 + T_{eq1} + T_{eq2}\cdot\frac{1}{G_1} + T_{eq3}\cdot\frac{1}{G_1G_2} + T_{eq}\cdot\frac{1}{G_1G_2G_3}$

$= T_0 + T_0(A_1-1) + T_0(F_1-1)A_1 + T_0(A_2-1)\cdot A_1\frac{1}{G_1} + T_0(F_2-1)\cdot\frac{A_1A_2}{G_1}$

$= \cancel{T_0} + T_0\cancel{A_1} - \cancel{T_0} + T_0F_1A_1 - T_0\cancel{A_1} + T_0\cancel{A_2A_1}\frac{1}{G_1} - T_0\frac{A_1}{G_1} + T_0F_2\frac{A_1\cancel{A_2}}{\cancel{G_1}} - \cancel{T_0\frac{A_1A_2}{G_1}}$

$= T_0\left(F_1A_1 - \frac{A_1}{G_1} + F_2A_1\right) = 86855$

$SNR_i|_{dB} = 10{,}79$ dB

NO!

?SNRu per segnale più efficente

$SNR_u\big|_{dB} = SNR_i\big|_{dB} + F_m\big|_{dB} = 11,56 \text{ dB}$ $F_m = \frac{3}{2} m^2 = 6$

? $B_{98\%}$

$B_{98\%} = 2 \cdot (m+1) \cdot B_{mod} = 300 \text{ KHz}$

? $B_{80\%}$

$B_{80\%} = 2 \cdot N \cdot B_{mod}$

$0,8 \leq \dots$

ES.3

M=4 segnali equiprobabili.

$$s_1(t) = \begin{cases} \sqrt{Eb} \cdot t & 0 \le t \le 1 \\ 0 & \text{altrimenti} \end{cases}$$

$$s_2(t) = \begin{cases} \sqrt{Eb} & 0 \le t \le 1 \\ 0 & \text{altrimenti} \end{cases}$$

$$s_3(t) = \begin{cases} -\sqrt{Eb} \cdot \frac{t}{2} & 0 \le t \le 1 \\ 0 & \text{altrimenti} \end{cases}$$

$$s_4(t) = \begin{cases} -3\sqrt{Eb} & 0 \le t \le 1 \\ 0 & \text{altrimenti} \end{cases}$$

? Rappresentazione di Gram-Schmidt

$$\psi_1(t) = \frac{s_1(t)}{\sqrt{E_{s1}}}$$

$$E_{s1} = \int_{-\infty}^{+\infty} |s_1(t)|^2 dt = \int_0^1 Eb \cdot t^2 \, dt = Eb \left[\frac{t^3}{3} \right]_0^1 = \frac{Eb}{3}$$

$$\psi_1(t) = \frac{s_1(t)}{\sqrt{\frac{Eb}{3}}}$$

$$\psi_2(t) = \frac{v_2(t)}{\sqrt{E_{v_2}}} \quad , \; v_2(t) = s_2(t) - \langle s_2(t), \psi_1(t) \rangle \psi_1(t)$$

$$\langle s_2(t), \psi_1(t) \rangle = \int_{-\infty}^{+\infty} s_2(t) \cdot \frac{s_1(t)}{\sqrt{Eb/3}} dt = \frac{1}{\sqrt{\frac{Eb}{3}}} \int_0^1 \cancel{\sqrt{Eb}} \cdot \cancel{\sqrt{Eb}} \cdot t \, dt =$$

$$= \frac{Eb}{\sqrt{\frac{Eb}{3}}} \cdot \left[\frac{t^2}{2} \right]_0^1 = \frac{Eb}{\sqrt{\frac{Eb}{3}}} \cdot \frac{1}{2} = \frac{\sqrt{3Eb}}{2}$$

$$v_2(t) = s_2(t) - \frac{\sqrt{3Eb}}{2} \cdot \frac{s_1(t)}{\sqrt{\frac{Eb}{3}}} = s_2(t) - \frac{3}{2} s_1(t)$$

$$E_{v_2} = \int_{-\infty}^{+\infty} |v_2(t)|^2 dt = \int_0^1 \left(\sqrt{Eb} - \frac{3}{2}\sqrt{Eb} \cdot t\right)^2 dt = \int_0^1 Eb + \frac{9}{4} Eb \cdot t^2 - 3\,Eb \cdot t \; dt$$

$$= \int_0^1 Eb \, dt + \int_0^1 \frac{9}{4} Eb \cdot t^2 \, dt - \int_0^1 3Eb \cdot t \, dt = \frac{1}{4} Eb$$

$\psi_2(t) = \frac{v_2(t)}{\sqrt{\frac{Eb}{4}}}$

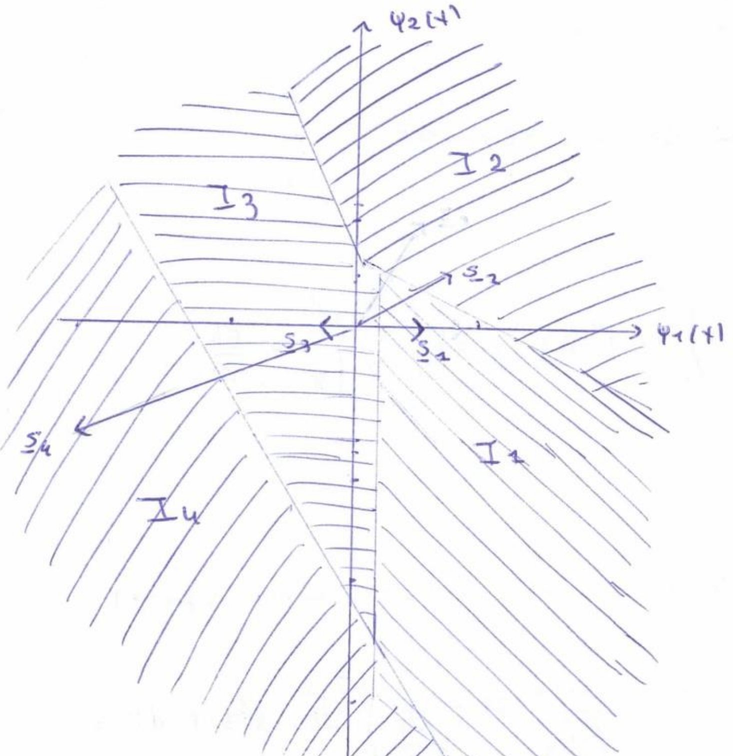
$\underline{S}_1 = \left(\sqrt{\frac{Eb}{3}} , 0 \right)$

$\underline{S}_2 = \left(\sqrt{\frac{3}{4} Eb} , \sqrt{\frac{1}{4} Eb} \right)$

$\underline{S}_3 = \left(-\frac{1}{2} \sqrt{\frac{Eb}{3}} , 0 \right)$

$\underline{S}_4 = \left(-3 \cdot \sqrt{\frac{3}{4} Eb} , -3 \sqrt{\frac{1}{4} Eb} \right)$

? Regioni di decisione



? U.B.

$\frac{Eb}{N_0} = 13 \text{ dB}$

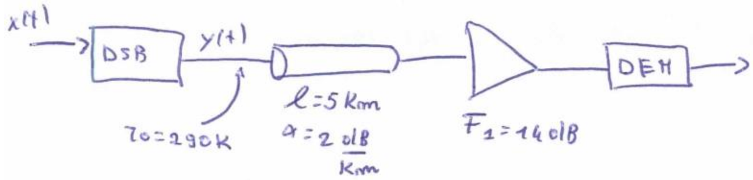
24/06/2008

ES. 1

$$x(t) = \sum_{m=-\infty}^{+\infty} g(t-mT) \qquad , \quad g(t) = \begin{cases} \frac{2|t|}{T} & |t| < \frac{T}{2} \\ \emptyset & \text{altrove} \end{cases}$$

$V_0 = 1V \quad , \; f_0 \gg \frac{1}{T} \quad , \; T = 1 \text{ msec}$

DSB



? P_y

$$g(t) = \text{rect}\left(\frac{t}{T}\right) - \text{tri}\left(\frac{t}{\frac{T}{2}}\right)$$

$$x(t) = \sum_{m=-\infty}^{+\infty} \text{rect}\left(\frac{t-mT}{T}\right) - \text{tri}\left(\frac{t-mT}{\frac{T}{2}}\right)$$

$$g(f) = T\,\text{sinc}(Tf) - \frac{T}{2}\,\text{sinc}^2\left(\frac{T}{2}f\right)$$

$$X(f) = \frac{1}{\cancel{T}} \sum_{m=-\infty}^{+\infty} \left[\cancel{T}\,\text{sinc}\left(\cancel{T}\cdot\frac{m}{\cancel{T}}\right) - \frac{\cancel{T}}{2}\,\text{sinc}^2\left(\frac{\cancel{T}}{2}\cdot\frac{m}{\cancel{T}}\right)\right]\cdot\delta\left(f-\frac{m}{T}\right)$$

$$= \sum_{m=-\infty}^{+\infty} \left[\text{sinc}(m) - \frac{1}{2}\,\text{sinc}^2(m/2)\right]\cdot\delta\left(f-\frac{m}{T}\right)$$

$$P_y = \frac{V_0^2}{2}\cdot\overline{x^2(t)}$$

$$\overline{x^2(t)} = \frac{1}{T}\int_{-\frac{T}{2}}^{\frac{T}{2}} \left(\frac{2|t|}{T}\right)^2 dt = 2\cdot\frac{1}{T}\left[\int_{\emptyset}^{\frac{T}{2}} \left(\frac{2t}{T}\right)^2 dt\right] = \frac{2}{T}\int_{\emptyset}^{\frac{T}{2}} \frac{4t^2}{T^2}\,dt = \frac{2}{T}\left[\frac{4t^3}{3T^2}\right]_{\emptyset}^{\frac{T}{2}}$$

$$= \frac{2}{\cancel{T}}\cdot\frac{4\cancel{T^3}}{24\cancel{T^2}} = \frac{1}{3}$$

$$P_y = \frac{1}{2}\cdot\frac{1}{3} = \frac{1}{6}\text{ Watt}$$

? T_{eq}

$T_{eq} = T_{eq1} + T_{eq2} \cdot \frac{1}{G_1} = T_0 (A_1 - 1) + T_0 (F_1 - 1) \cdot A_1$

$= - T_0 + T_0 F_1 A_1 = 72558\ K$

? SNR_u

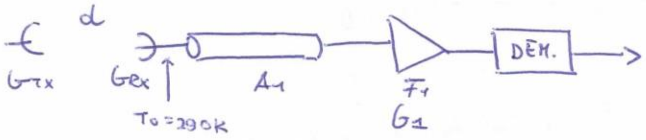
$B_u = \frac{1}{T} = 1\ KHz$

$SNR_i \big|_{dB} = SNR_u \big|_{dB} = P_y \big|_{dB} - 10 \log_{10} K T_{SIST} \cdot B_u = 142,192\ dB$

12/2008

ES. 1

$y_{DSB}(t) = 8(\cos(2\pi f_1 t) + \cos(2\pi f_2 t)) + 2(\cos(2\pi f_3 t) + \cos(2\pi f_4 t))$



$G_{TX} = G_{RX} = 10$ dB $d = 60$ Km

$A_1 = 10$ dB

$F_1 = G_1 = 10$ dB

$f_1 = 800$ MHz

$f_2 = 804$ MHz

$f_3 = 799$ MHz

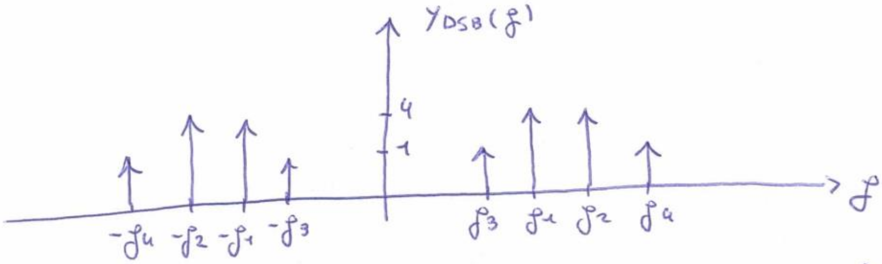
$f_4 = 805$ MHz

? $Y_{DSB}(f)$, P_y

$Y_{DSB}(f) = 4\delta(f-f_1) + 4\delta(f+f_1) + 4\delta(f-f_2) + 4\delta(f+f_2) +$
$+ \delta(f-f_3) + \delta(f+f_3) + \delta(f-f_4) + \delta(f+f_4)$

$P_y = 4 \cdot 4^2 + 4 \cdot 1^2 = 68$ Watt $P_y|_{dB} = 18{,}32$ dB

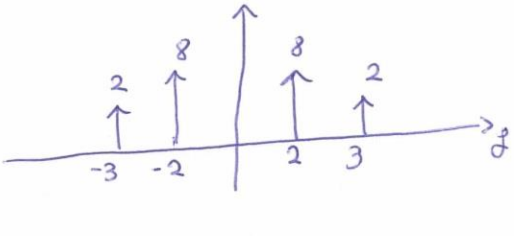
? modulante $m(t)$, portante $s(t)$



$f_0 = 802$ MHz , $s(t) = V_0 \cdot \cos(2\pi f_0 t)$

$m(t) = 16 \cdot \cos(2\pi 2M t) + 4\cos(2\pi 3M t)$

$V_0 = 1$ V.



$?SNR_u$

$SNR_i = SNR_u = P_{Tx}|_{dB} + G_{Tx}|_{dB} + G_{Rx}|_{dB} - A_{fs}|_{dB} - 10\log_{10} KT_{sist} B_H$

$A_{fs}|_{dB} = 32,4 + 20\log_{10} f_0|_{MHz} + 20\log_{10} d_{km} = 126 \text{ dB}$

$T_{sist} = T_0 + T_{eq} = T_0 + T_{eq1} + T_{eq2} \cdot \frac{1}{G_1} = T_0 + T_0(A_1 - 1) + T_0(F_1 - 1) \cdot A_2$

$= \cancel{T_0} + \cancel{T_0A_1} - \cancel{T_0} + T_0F_1A_1 - \cancel{T_0A_1} = T_0F_1A_1 = 29000 \text{ K}$

$SNR_u|_{dB} = SNR_i|_{dB} = 31,53 \text{ dB}$

Es: 2

$s(t) = \cos(2\pi \, 1K t)$

$c(t) = V_0 \cdot \cos(2\pi f_0 t)$, $V_0 = 1 mV$, $f_0 = 10 MHz$

$T_{SIST} = 10^6 K$

$K = 4\pi k \frac{Hz}{V}$

? Ricevitore funziona

$K_f = \frac{K}{2\pi} = \frac{{}^{2}\cancel{4\pi} \cdot 10^3}{\cancel{2\pi}} = 2 \cdot 10^3$

$SNR_i|_{dB} \geq 13 + 10 \log_{10}(m+1)$, $m = \frac{K_f \cdot V_m}{f_m} = 2$

$\geq 13 + 10 \log_{10}(3) = 17,7 \, dB$

$SNR_i = \frac{P_y}{K T_{SIST} \cdot B} = \frac{V_0^2 = (1 \cdot 10^{-3})^2}{2 \cdot 1,38 \cdot 10^{-23} \cdot 10^6 \cdot 10^3} = 36231884,06$

$SNR_i|_{dB} = 75,6 \, dB$

? $SNR_u|_{dB}$

$F_m = \frac{3}{2} \cdot m^2 = 6$

$SNR_u|_{dB} = SNR_i|_{dB} + 10 \log_{10}(6) = 83,4 \, dB$

? B_{carson}

$B_{98\%} = 2 \cdot (m+1) \cdot B_M = 6 \, KHz$

? $B_{80\%}$

$P \; 0,8 \leq 0,224^2 + 2 \cdot 0,577^2 + 2 \cdot 0,353^2 = 0,96$

$N = 2$

$B_{80\%} = 2 \cdot N \cdot B_M = 4 \, KHz$

ES. 3

$G_{TX} = G_{RX} = 10\,dB$ $\quad M = 5$

$T_{SIST} = 300\,K$

$T_{SIMB} = 3\,msec$

$\underline{S}_1 = (\sqrt{Eb}, 0)$

$\underline{S}_2 = (-\sqrt{Eb}, 0)$

$\underline{S}_3 = (0, \sqrt{Eb})$

$\underline{S}_4 = \left(-\frac{\sqrt{2Eb}}{2}, -\frac{\sqrt{2Eb}}{2}\right)$

$\underline{S}_5 = \left(\frac{\sqrt{2Eb}}{2}, -\frac{\sqrt{Eb}}{2}\right)$

?
- P_{TX}

$\frac{Eb}{N_0} = 10\,dB$

$$P_{TX}|_{dB} = \frac{Eb}{N_0}\Big|_{dB} - G_{TX}|_{dB} - G_{RX}|_{dB} + \overset{\to 0}{\cancel{A_{gs}}}|_{dB} + 10\log_{10} K T_{SIST} + 10\log_{10}\frac{\overset{=3}{\lceil \log_2 M \rceil}}{T_{SIMB}}$$

$$= -183,82\,dB$$

?
- stima probabilità di errore

D_{ij}^2	S_1	S_2	S_3	S_4	S_5
S_1	X	4Eb	2Eb	3,16Eb	0,34 Eb
S_2		X	2Eb	0,34 Eb	3,16 Eb
S_3			X	2,75Eb	2,75Eb
S_4				X	2Eb
S_5					X

$$U.B = \frac{2q}{k} \cdot Q\left(\sqrt{\frac{D_{ij\,MIN}^2}{2N_0}}\right)$$

$$= \frac{2\cdot 2}{5} \cdot Q\left(\sqrt{\frac{0,34}{2}\frac{Eb}{N_0}}\right)$$

$$= \frac{4}{5} \cdot Q(1,7)$$

$$= \frac{4}{5} \cdot \frac{1}{\sqrt{2\pi \cdot 1,7}} \cdot e^{-\frac{1,7^2}{2}} = 0,1$$

20/07/2009

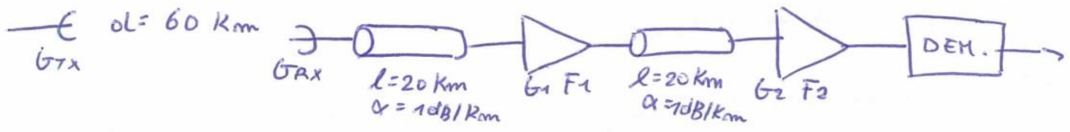
ES. 1

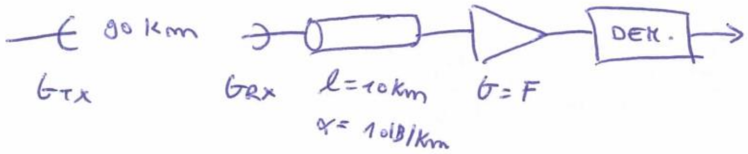
DSB

$s(t) = 8\cdot\cos(2\pi f_1 t) + 6\cdot\cos(2\pi f_2 t)$, $f_1 = 10\,kHz$

$f_2 = 30\,kHz$

$d_{INIZ} = 100$ Km





$A_1|_{dB} = 100$ dB

$A_1 = 100$

$A_2 = 10$ dB

1) $G_{TX} = G_{RX} = 10$ dB

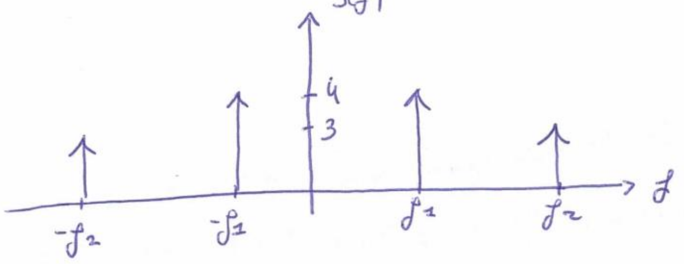
$G_1 = G_2 = F_1 = F_2 = 10$ dB

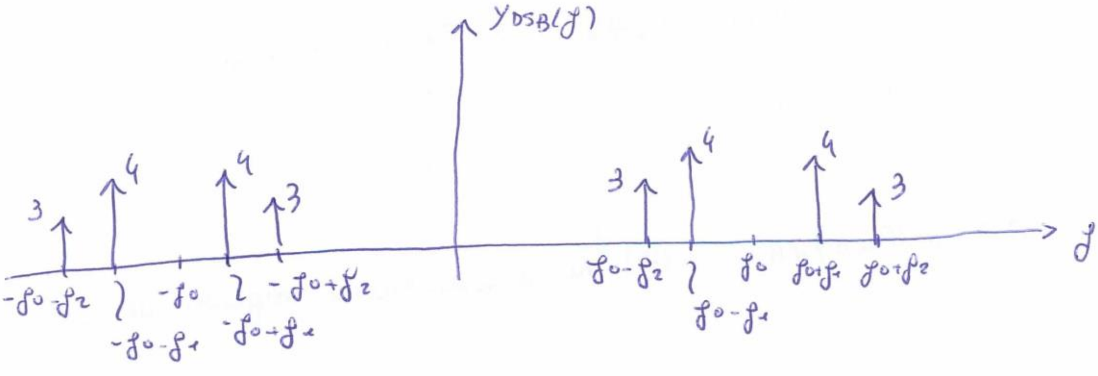
2) $G_{TX} = G_{RX} = 10$ dB

$G = F = 10$ dB

$c(t) = 2\cdot\cos(2\pi f_0 t)$, $f_0 = 1\,GHz$

? $Y_{DSB}(f)$





$P_{y_{DSB}}(t) = 4 \cdot 4^2 + 3^2 \cdot 4 = 100$ Watt , $P_y|_{dB} = 20$ dB

? SNR_u nei casi ① e ②

$T_i = T_0 = 290$ K

①

$$SNR_u|_{dB} = P_y|_{dB} + G_{TX} + G_{RX} - A_{fs} - 10\log_{10} K T_{SIST} B_{MODULANTE}$$

$$A_{fs} = 32{,}4 + 20\log_{10} f_{MHz} + 20\log_{10} D_{km}$$

$$= 32{,}4 + 20\log_{10} 1000 + 20\log_{10} 60 = 127{,}96 \text{ dB}$$

$$T_{SIST} = T_0 + T_{eq} = T_0 + T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1G_2} + \frac{T_{eq4}}{G_1G_2G_3}$$

$$= T_0 + T_0(A-1) + T_0(F-1)\cdot A + T_0(A-1)A\cdot\frac{1}{G} + T_0(F-1)\cdot\frac{A^2}{G}$$

$$= \cancel{T_0} + \cancel{T_0A} - \cancel{T_0} + T_0FA - \cancel{T_0A} + T_0\cancel{A^2}\frac{1}{G} - T_0A\frac{1}{G} + T_0FA^2\cdot\frac{1}{G} - \cancel{T_0}\frac{A^2}{G}$$

$$= T_0\left(FA - \frac{A}{G} + F\frac{A^2}{\cancel{G}}\right) = 3187100 \text{ K.}$$

$SNR_u|_{dB} = 30{,}8$ dB

②

$$A_{fs} = 32{,}4 + 20\log_{10} f_{MHz} + 20\log_{10} D_{km} = 131{,}48$$

$$T_{SIST} = T_0 + T_{eq} = T_0 + T_{eq1} + \frac{T_{eq2}}{G_1} = T_0 + T_0(A_1-1) + T_0(F_1-1)\cdot A_1$$

$$= \cancel{T_0} + \cancel{T_0A_1} - \cancel{T_0} + T_0F_1A_1 - \cancel{T_0A_1} = T_0F_1A_1 = 290.00 \text{ K}$$

$SNR_u|_{dB} = 47{,}72$ dB

$SNR_{u2}|_{dB} > SNR_{u1}|_{dB}$ ② ha prestazioni migliori di ①

Es. 2

FM

$y(t) = 5 \cdot \cos\left(2\pi f_0 t + 8 \operatorname{sen}\left(2\pi\, 10000\, t\right)\right)$, $f_0 = 2\,MHz$

trasmesso su canale AWGN con $\frac{N_0}{2} = 10^{-10} \frac{W}{Hz}$.

? Ricevitore funziona $\Rightarrow m = 8.$

$SNR_i|_{dB} \geq 13 + 10 \log_{10}(m+1) = 22,54\ dB$

$SNR_i|_{dB} = \frac{V_0^2}{\underset{\underset{10 \cdot 10^3}{\shortparallel}}{2N_0B}} = 625 \cdot 10^4$ $SNR_i|_{dB} = 67,96\ dB > 22,54\ dB$

<u>OK</u>

? SNR_u

$SNR_u = \underset{\underset{\text{non in dB}}{\uparrow}}{SNR_i} \cdot F_m = SNR_i \cdot \underset{\underset{96}{\shortparallel}}{\overset{\overset{F_m\ \cos()\ \text{in FM}}{\swarrow}}{\frac{3}{2} m^2}} = 6 \cdot 10^8$

$SNR_u|_{dB} = 87,8\ dB.$

? Banda per ottenere 80% del segnale ($B_{80\%}$)

$P_{80\%} = 0,172 + 2 \cdot 0,235^2 + 2 \cdot 0,113^2 + 2 \cdot 0,291^2 + 2 \cdot 0,105^2 + 2 \cdot 0,186^2 +$

$+ 2 \cdot 0,338^2 + 2 \cdot 0,321^2 = 0,86.$ $\Rightarrow N = 7$

$B_{80\%} = 2 \cdot N \cdot B_{\text{modulante}} = 2 \cdot 7 \cdot 10 \cdot 10^3 = 140\ KHz.$

? %Potenza fra $(1980 \div 2040)$ KHz .

Centrato in $f_0 = 2\,MHz$, Passo di 10 KHz.

$P_{\%B} =$ $\%P = 0,172^2 + 2 \cdot 0,235^2 + 2 \cdot 0,113^2 + 0,291^2 + 0,105^2 = 0,26$

<u>26%</u>

ES. 3

$M = 5$ Segnali equiprobabili

$S_1 = (-4\sqrt{Eb})$

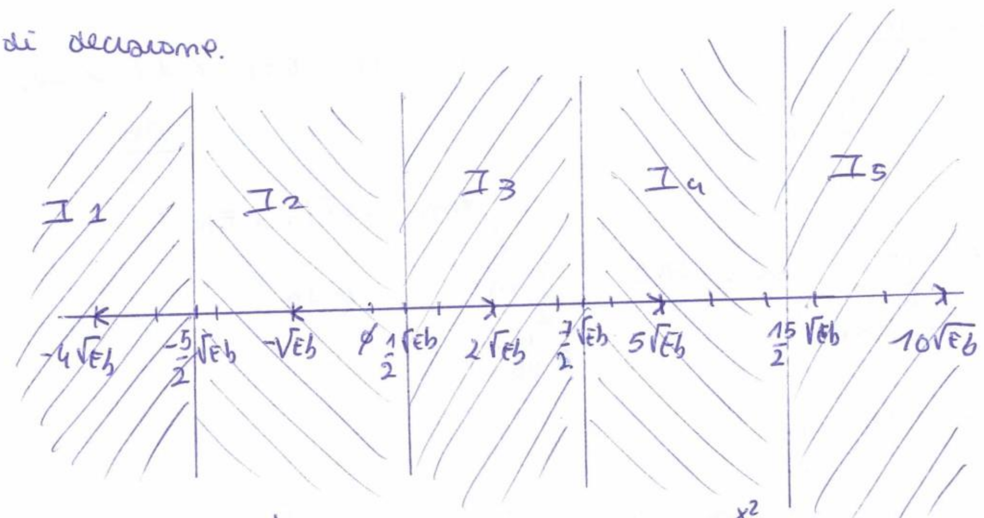
$S_2 = (-\sqrt{Eb})$

$S_3 = (2\sqrt{Eb})$

$S_4 = (5\sqrt{Eb})$

$S_5 = (10\sqrt{Eb})$

? Regioni di decisione.



? P_e con $\left.\frac{Eb}{N0}\right|_{dB} = 13\,dB \Rightarrow \frac{Eb}{N0} = 20$, $Q(x) = \frac{1}{\sqrt{2\pi}\,x} \cdot e^{-\frac{x^2}{2}}$

$$P_e = \frac{1}{M}\sum_{i=1}^{M} P_{e/s_i} = \frac{1}{5}\sum_{i=1}^{5} P_{e/s_i}$$

$$P_{e/\underline{s}_1} = P\{\underline{s}_1 + m \geq -2,5\sqrt{Eb}\} = P\{m \geq 1,5\sqrt{Eb}\} = Q\left\{\sqrt{\frac{9Eb}{2N_0}}\right\}$$

$$P_{e/\underline{s}_2} = P\{\underline{s}_2 + m \geq 0,5\sqrt{Eb}\} + P\{\underline{s}_2 + m \leq -2,5\sqrt{Eb}\} = Q\left\{\sqrt{\frac{9Eb}{2N_0}}\right\} + Q\left\{\sqrt{\frac{9Eb}{2N_0}}\right\}$$

$$P_{e/\underline{s}_3} = P\left\{\underline{s}_3 + m \geq 3,5\sqrt{Eb}\right) + P\left\{\underline{s}_3 + m \leq 0,5\sqrt{Eb}\right\} = Q\left(\sqrt{\frac{9Eb}{2N_0}}\right) + Q\left(\sqrt{\frac{9Eb}{2N_0}}\right)$$

$$P_{e/\underline{s}_4} = Q\left(\sqrt{\frac{9Eb}{2N_0}}\right) + Q\left(\sqrt{\frac{25Eb}{2N_0}}\right) \; ; \; P_{e/\underline{s}_5} = Q\left(\sqrt{\frac{25Eb}{2N_0}}\right)$$

$$P_e = \frac{1}{5}\left(6 \cdot Q\left(\sqrt{\frac{9Eb}{2N_0}}\right) + 2Q\left(\sqrt{\frac{25Eb}{2N_0}}\right)\right) = 1,44 \cdot 10^{-21}.$$

07/07/2009

ES. 1

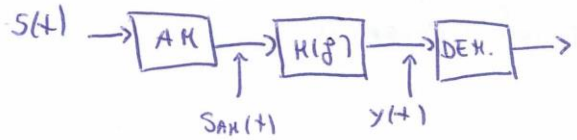
$$s(t) = \left(\sum_{m=-\infty}^{+\infty} \text{tri}\left(\frac{t-2mT}{T}\right) \right) - \frac{1}{2}$$

AM) $K_{AM} = 2$.

$$H(f) = \text{rect}\left(\frac{f-f_0}{f_c}\right) + \text{rect}\left(\frac{f+f_0}{f_c}\right)$$

$f_0 = 1\ \text{MHz}$, $V_0 = 1\text{V}$, $T = 10^{-4}$ sec, $f_c = 20\ \text{kHz}$.

? Segnale in ingresso al ricevitore



$$S_\tau(t) = \text{tri}\left(\frac{t}{\tau}\right) \;\text{->}\; S_\tau(f) = \tau\,\text{sinc}^2(\tau f)$$

$$S(f) = \frac{1}{2\cancel{\tau}} \cdot \sum_{m=-\infty}^{+\infty} \cancel{\tau} \cdot \text{sinc}^2\left(\cancel{\tau} \cdot \frac{m}{2\cancel{\tau}}\right) \cdot \delta\left(f - \frac{m}{2\tau}\right) - \frac{1}{2}\delta(f)$$

$$= \frac{1}{2} \cdot \sum_{m=-\infty}^{+\infty} \text{sinc}^2\left(\frac{m}{2}\right) \cdot \delta\left(f - \frac{m}{2\tau}\right) - \frac{1}{2}\delta(f).$$

$$s_{AM}(t) = V_0 (1 + K s(t)] \cdot \cos(2\pi f_0 t)$$

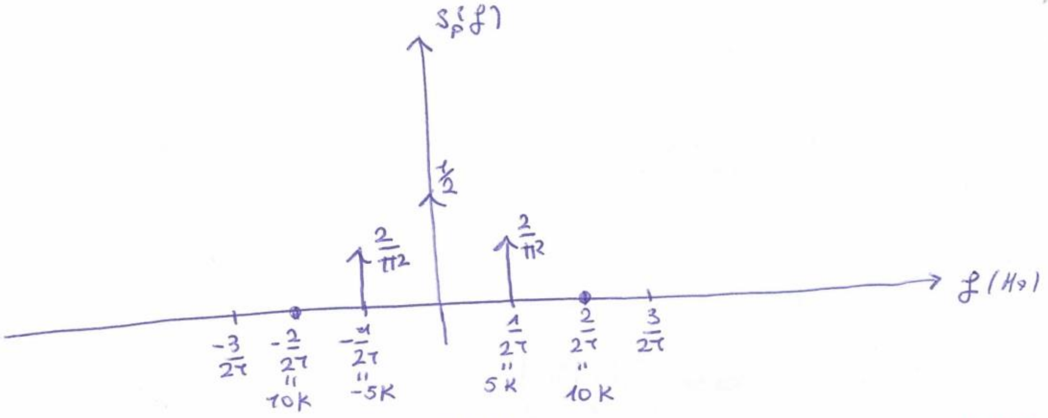
$$= 1 \cdot (1 + 2 \cdot s(t)) \cdot \cos(2\pi f_0 t) = \cos(2\pi f_0 t) + 2 s(t) \cdot \cos(2\pi f_0 t).$$

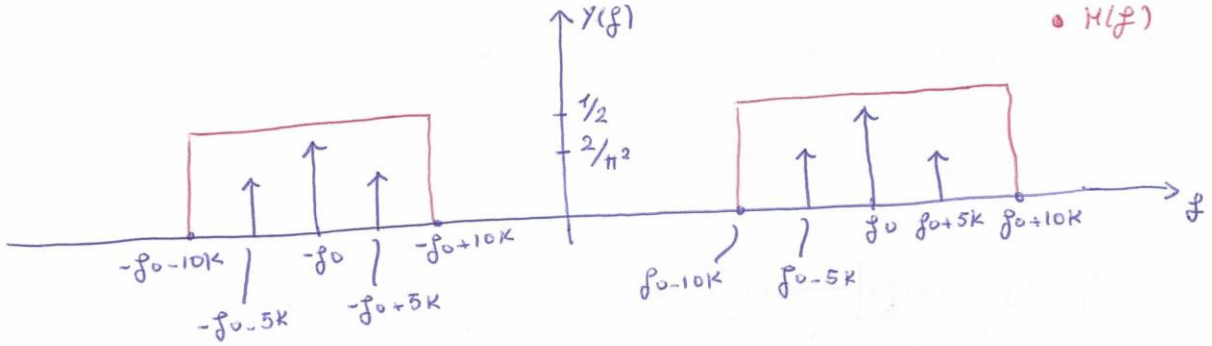
$$S_{AM}(f) = \frac{1}{2}\left\{\delta(f+f_0) + \delta(f-f_0)\right\} + 2 \cdot \frac{1}{2}\left\{S(f+f_0) + S(f-f_0)\right\}.$$

$$= \cancel{\frac{1}{2}\left\{\delta(f+f_0) + \delta(f-f_0)\right\}} + \left\{\frac{1}{2} \cdot \sum_{m=-\infty}^{+\infty} \text{sinc}^2\left(\frac{m}{2}\right) \delta\left(f + f_0 - \frac{m}{2\tau}\right) - \cancel{\frac{1}{2}\delta(f+f_0)} + \right.$$

$$\left. + \frac{1}{2} \sum_{m=-\infty}^{+\infty} \text{sinc}^2\left(\frac{m}{2}\right) \delta\left(f - f_0 - \frac{m}{2\tau}\right) - \cancel{\frac{1}{2}\delta(f-f_0)} \right\}$$

$$= \frac{1}{2} \sum_{m=-\infty}^{+\infty} \text{sinc}^2\left(\frac{m}{2}\right) \delta\left(f + f_0 - \frac{m}{2\tau}\right) + \frac{1}{2} \sum_{m=-\infty}^{+\infty} \text{sinc}^2\left(\frac{m}{2}\right) \delta\left(f - f_0 - \frac{m}{2\tau}\right).$$





con $f_0 = 1$ MHz.

$$Y(f) = \frac{1}{2}\left\{\delta(f+f_0) + \delta(f-f_0)\right\} + \frac{2}{\pi^2}\left\{\delta(f+f_0+5K) + \delta(f+f_0-5K) + \right.$$

$$\left. + \delta(f-f_0+5K) + \delta(f-f_0-5K)\right\}.$$

$$y(t) = \cos(2\pi f_0 t) + \frac{4}{\pi^2}\left\{\cos(2\pi(f_0-5K)t) + \cos(2\pi(f_0+5K)t)\right\}$$

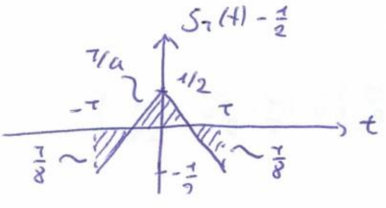
? Potenza media di tale segnale

$P_Y = 4\cdot\left(\frac{2}{\pi^2}\right)^2 + 2\cdot\left(\frac{1}{2}\right)^2 = 0{,}664$ Watt con δ .

$= \frac{1}{2} + 2\cdot\left(\frac{4}{\pi^2}\right)^2\Big/2 = 0{,}664$ Watt con cos().

? SNR_i, SNR_u con $B_{MODULANTE} = \overset{= 20\cdot 10^3\,Hz}{\frac{2}{\tau}}$, $N_0 = 10^{-10}\ \frac{W}{Hz}$

$SNR_i = \frac{P_Y}{N_0 \cdot B} = 332\,000$



OK media nulla .

$\overline{S(t)} = 0$, POSSO usare : $SNR_u = \dfrac{V_0^2 \, P_{MODULANTE} \cdot K_{AM}^2}{2 N_0 B}$.

$$P_M = \lim_{T \to \infty} \frac{1}{2T} \left[\int_{-T}^{0} \left(\frac{1}{2} + \frac{t}{T} \right)^2 dt + \int_{0}^{T} \left(\frac{1}{2} - \frac{t}{T} \right)^2 dt \right]$$

$$= \lim_{T \to \infty} \frac{1}{\cancel{2}T} \cancel{2} \cdot \int_{-T}^{0} \left(\frac{1}{2} + \frac{t}{T} \right)^2 dt = \lim_{T \to \infty} \frac{1}{T} \int_{-T}^{0} \frac{1}{4} + \frac{t^2}{T^2} + \frac{t}{T} \, dT$$

$$= \lim_{T \to \infty} \frac{1}{T} \left[\frac{t}{4} + \frac{t^3}{3T^2} + \frac{t^2}{2T} \right]_{-T}^{0} = \lim_{T \to \infty} \frac{1}{\cancel{T}} \left[- \left(- \frac{\cancel{T}}{4} - \frac{\cancel{T^3}}{3\cancel{T^2}} + \frac{\cancel{T^2}}{2\cancel{T}} \right) \right] = \frac{1}{12}$$

$$SNR_u = \frac{V_0^2 \cdot P_M \cdot K_{AM}^2}{2 N_0 B} = \frac{1^2 \cdot \frac{1}{12} \cdot 2^2}{2 \cdot 10^{-10} \cdot 20 \cdot 10^3} = 83333,3$$

ES. 2

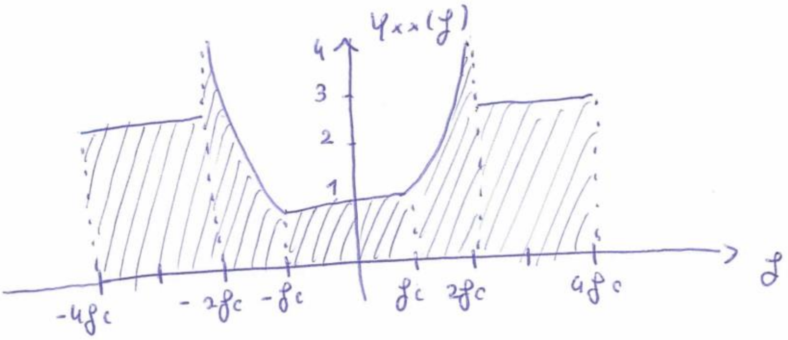
$\{x(t)\}$

$$\Psi_{x,x}(f) = \begin{cases} 1 & |f| \le f_c \\ \frac{f^2}{f_c^2} & f_c \le |f| \le 2f_c \\ 3 & 2f_c < |f| \le 4f_c \\ 0 & \text{altrimenti} \end{cases} \quad , f_c = 10 \text{ KHz}$$

FM) , $m=2$

$A = 80\,dB$, $\frac{N_0}{2} = 10^{-10}\ \frac{W}{Hz}$, $V_0 = 2V$, $K_f = 3$

? P_x , P_y .



$$P_x = \int_{-\infty}^{+\infty} \Psi_{x,x}(f)\ df = 2 \cdot \left[\underbrace{\int_{0}^{f_c} 1\ df}_{f_c} + \int_{f_c}^{2f_c} \frac{f^2}{f_c^2}\ df + \underbrace{\int_{2f_c}^{4f_c} 3\ df}_{6 f_c} \right]$$

$$= 2 \left[7f_c + \left[\frac{f^3}{3} \cdot \frac{1}{f_c^2} \right]_{f_c}^{2f_c} \right] = 14 f_c + 2 \left[\frac{(2f_c)^3}{3} \cdot \frac{1}{\cancel{f_c^2}} - \frac{f_c^3}{3} \cdot \frac{1}{\cancel{f_c^2}} \right]$$

$$= 14 f_c + 2 \left[\frac{7}{3} f_c \right] = \frac{56}{3} f_c = 186{,}7\ KW .$$

$$P_y = \frac{V_0^2}{2} = 2\,W$$

? Banda di Carlson

$B_{carlson} = 2(m+1) \cdot B = 2(2+1) \cdot 4 \cdot f_c = 240\ \text{KHz}$

? SNR_i , SNR_u

$$SNR_i = \frac{V_0^2}{2N_0B \cdot A}$$ con A non in dB $\Rightarrow A = 10^{\frac{80}{10}} = 10^8$

$$= \frac{\cancel{4}}{2 \cdot 2 \cdot 10^{-10} \cdot \cancel{4} \cdot 10^4 \cdot 10^8} = 2,5 \cdot 10^{-3}$$

$SNR_u = SNR_i \cdot F_m$

$$F_m = \frac{3 \cdot K_f^2 \cdot P_x}{B^2} = \frac{3 \cdot 3^2 \cdot 186,7 \cdot \cancel{10}^3}{16 \cdot 10\,\cancel{8}\,5} = 3,15 \cdot 10^{-3}$$

$SNR_u = 7,88 \cdot 10^{-6}$

ES. 3

$G_{TX} = G_{RX} = 10\ dB$

$A_{fs} = 152\ dB$, $T_{SIST} = 3 \cdot 10^{6}\ K$

$T_{SIMB} = 2\ msec$

$M = 4$

$$s_1 = \left(\frac{\sqrt{2E_b}}{2} , \sqrt{E_b} \right)$$

$$s_2 = \left(\frac{\sqrt{2E_b}}{2} , -\sqrt{E_b} \right)$$

$$s_3 = \left(-\frac{\sqrt{2E_b}}{2} , \frac{\sqrt{2E_b}}{2} \right)$$

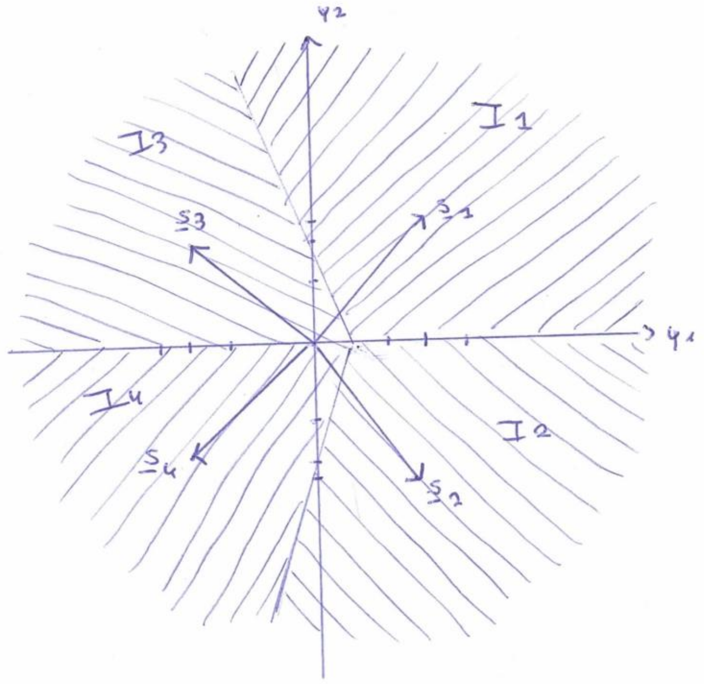
$$s_4 = \left(-\frac{\sqrt{2E_b}}{2} , -\frac{\sqrt{2E_b}}{2} \right)$$

? P_{TX} per avere $\frac{E_b}{N_0} = 13\ dB$

$$\frac{E_b}{N_0}\Big|_{dB} = P_{TX}|_{dB} + G_{TX}|_{dB} + G_{RX}|_{dB} - A_{fs}|_{dB} - 10\log_{10} K\, T_{SIST} - 10\log_{10} \frac{\lceil \log_2 M \rceil}{T_{SIMB}}$$

$P_{TX}|_{dB} = 11,17\ dB$

? Regioni di decisioni



?: Stima della probabilità di errore con il Bound Union

$$Q(x) = \frac{1}{\sqrt{2\pi} \cdot x} \cdot e^{-\frac{x^2}{2}}$$

$$D_{ij}^2 = (x_i - x_j)^2 + (y_i - x_j)^2$$

$D_{i,j}^2$	S_1	S_2	S_3	S_4
S_1	X	4Eb	2,08 Eb	4,92 Eb
S_2		X	4,92 Eb	2,08 Eb
S_3			X	2 Eb
S_4				X

$$BU = \frac{2}{M}4 \cdot Q\left(\sqrt{\frac{D_{ij\,MIN}^2}{2N_0}}\right)$$

$$= \frac{1}{2} \cdot Q\left(\sqrt{\frac{Eb}{N_0}}\right)$$

$$= \frac{1}{2} \cdot \frac{1}{\sqrt{2\pi \cdot 20}} \cdot e^{-\frac{20}{2}} = 2,025 \cdot 10^{-6}$$

$\sqrt{\frac{Eb}{N_0}}\Big|_{dB} = \sqrt{13} \Rightarrow \underline{\underline{\sqrt{20}}}$ non in dB

30/06/2009

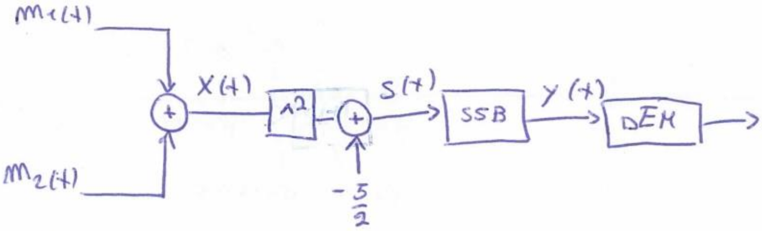
ES. 1

$m_1(t) = 2 \cdot \cos(2\pi f_1 t)$, $f_1 = 30$ KHz

$m_2(t) = \cos(2\pi f_2 t)$, $f_2 = 10$ KHz

$s(t) = x_1 m^2(t) - \frac{5}{2}$

SSB : $c(t) = V_0 \cdot \cos(2\pi f_0 t)$, $f_0 = 1$ MHz , $V_0 = 1$ V



$N_0 = 10^{-8} \ \frac{W}{Hz}$

? $Y(f)$

$x(t) = m_1(t) + m_2(t) = 2 \cdot \cos(2\pi f_1 t) + \cos(2\pi f_2 t)$

$s(t) = x(t)^2 - \frac{5}{2} = 4\cos^2(2\pi f_1 t) + \cos^2(2\pi f_2 t) + 4\cos(2\pi f_1 t)\cos(2\pi f_2 t) - \frac{5}{2}$

$= \frac{1}{\cancel{2}} \cdot \overset{2}{\cancel{4}} \left[1 + \cos(2\pi 2 f_1 t) \right] + \frac{1}{2} \left[1 + \cos(2\pi 2 f_2 t) \right] + \frac{1}{\cancel{2}} \cdot \overset{2}{\cancel{4}} \left[\cos(2\pi (f_1 - f_2) t) + \right.$

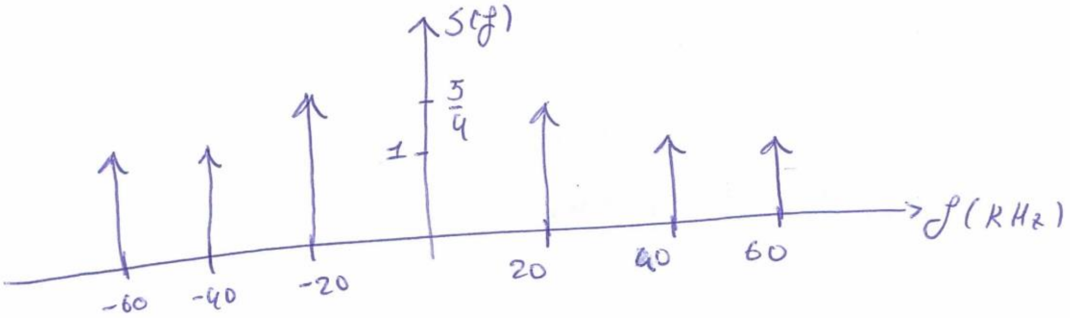
$\left. + \cos(2\pi (f_1 + f_2) t \right] - \frac{5}{2}$

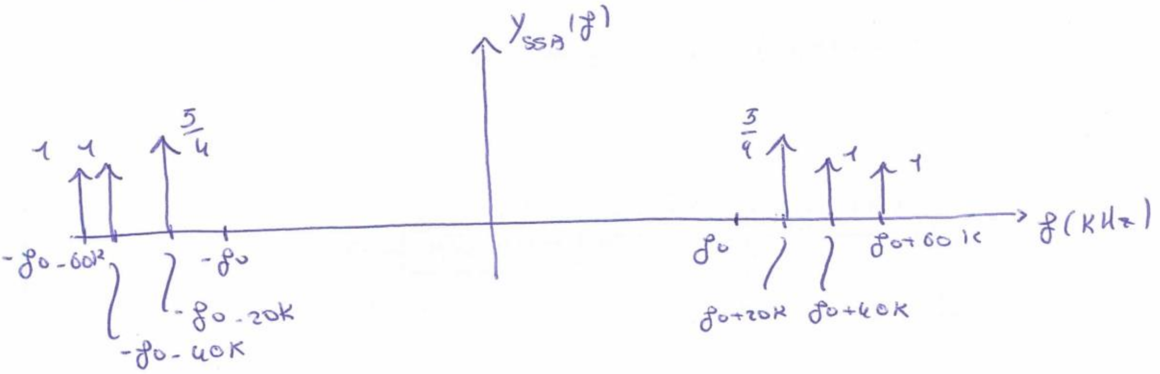
$= \cancel{2} + 2\cos(2\pi \overset{= 60k}{2 f_1} t) + \cancel{\frac{1}{2}} + \frac{1}{2}\cos(2\pi \overset{= 20k}{2 f_2} t) + 2\cos(2\pi \overset{= 20k}{(f_1 - f_2)} t) +$

$+ 2\cos(2\pi \overset{= 40k}{(f_1 + f_2)} t) - \cancel{\frac{5}{2}}$

$= \frac{5}{2}\cos(2\pi 20k t) + 2\cos(2\pi 40k t) + 2\cos(2\pi 60k t)$.

$S(f) = \frac{5}{2_1} \left\{ \delta(f - 20k) + \delta(f + 20k) \right\} + \frac{\cancel{2}}{\cancel{2}} \left\{ \delta(f - 40k) + \delta(f + 40k) + \delta(f - 60k) + \delta(f + 60k) \right\}$





? SNRu

$$SNR_u = \frac{V_0^2 P_M}{N_0 B} \quad , \quad P_M = 4 \cdot 1^2 + 2 \cdot \left(\frac{5}{4}\right)^2 = \frac{57}{8} W = 7,125\ W$$

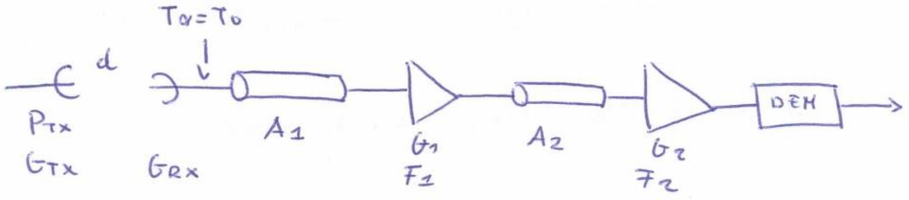
$$= \frac{1^2 \cdot 7,125}{10^{-8} \cdot 60 \cdot 10^3} = 11875$$

? SNRu DSB

$$SNR_u = \frac{V_0^2 P_M}{2 N_0 B} \quad , \quad P_M = 7,125\ W$$

$$= 5937,5$$

ES. 2



$s(t) = 2 \cdot \cos(2\pi f_m t)$, $f_m = 10$ KHz

FM: $c(t) = V_0 \cdot \cos(2\pi f_0 t)$, $f_0 = 1$ GHz

$\Delta f_{max} = 100$ KHz.

$P_{TX} = 10$ W

$T_0 = 290$ K

$G_{TX} = G_{RX} = 10$ dB

$d = 60$ Km

$A_1 = 10$ dB

$F_1 = G_1 = 10$ dB

$A_2 = 16$ dB

$G_2 = F_2 = 13$ dB.

? T_{eq}.

$$T_{eq} = T_{eq1} + T_{eq2} \cdot \frac{1}{G_1} + T_{eq3} \cdot \frac{1}{G_1 G_2} + T_{eq4} \frac{1}{G_1 G_2 G_3}$$

$$= T_0(A_1 - 1) + T_0(F_1 - 1) \cdot A_1 + T_0(A_2 - 1) \cdot A_1 \frac{1}{G_1} + T_0(F_2 - 1) \cdot A_1 \cdot \frac{1}{G_1} \cdot A_2$$

$$= \cancel{T_0} A_1 - T_0 + T_0 F_1 \overset{\cdot A_1}{\vee} - \cancel{T_0 A_1} + T_0 \cancel{A_1 A_2} \cdot \frac{1}{G_1} - T_0 \cancel{A_1} \cdot \frac{1}{\cancel{G_1}} + T_0 F_2 \cancel{A_1} \cdot \frac{1}{\cancel{G_1}} A_2$$

$$- \cancel{T_0 A_1 \cdot \frac{1}{G_1}} \cdot A_2$$

$$= T_0(-1 + F_1 A_1 - 1 + F_2 A_2) = 290 \cdot (-2 + 10 \cdot 10 + 20 \cdot 40) = 260420 \text{ K}$$

? controllare se il dimensionamento di tale sistema verifica la condizione sull'effetto soglia per la modulazione FM.

$$SNR_i|_{dB} \geq 13 + 10 \log_{10}(m+1) \Rightarrow \quad m = \frac{\Delta f_{max}}{f_m} = 5$$

$$= 13 + 10 \log_{10}(6) = 20{,}78 \text{ dB}$$

$$SNR_i|_{dB} = P_{TX}|_{dB} + G_{TX}|_{dB} + G_{RX}|_{dB} - A_{fs}|_{dB} - 10\log_{10} K T_{SIST} B_{\text{MODULANTE}}$$

$$A_{fs} = 32,4 + 20\log_{10} f_{MHz} + 20\log_{10} D_{km} = 127,96 \text{ dB}$$

$$SNR_i|_{dB} = 10 + 10 + 10 - 127,96 - 10\log_{10}\left(1,38\cdot 10^{-23}\cdot(290 + 260420)\cdot 20\cdot 10^{3}\right)$$

$$= 33,46 \text{ dB}$$

$33,46 > 20,78$ <u>OK</u>

? $B_{95\%}$

$$0,95 \leq 0,178^2 + 2\cdot 0,328^2 + 2\cdot 0,049^2 + 2\cdot 0,365^2 + 2\cdot 0,391^2 +$$
$$+ 2\cdot 0,261^2 = 0,96$$

$N = 5$

$$B_{95\%} = 2\cdot N\cdot B_{\text{modulante}} = 2\cdot 5\cdot 20\cdot 10^{3} = 200\cdot 10^{3} \text{ Hz}$$

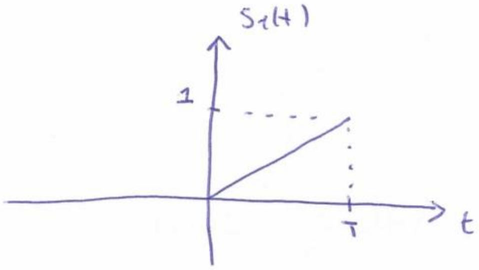
ES. 3

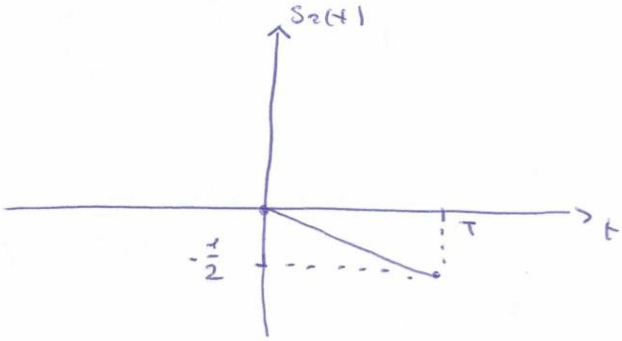
Sorgente equiprobabile, $M=3$.

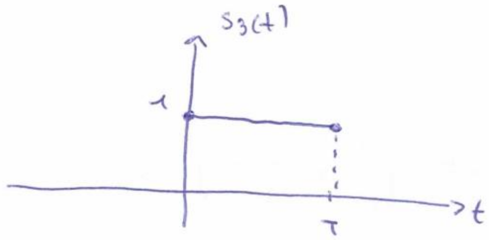
$$s_1(t) = \begin{cases} \frac{t}{T} & 0 \le t \le T \\ 0 & \text{altrimenti} \end{cases}$$

$$s_2(t) = \begin{cases} \frac{-t}{2T} & 0 \le t \le T \\ 0 & \text{altrimenti} \end{cases} \qquad s_3(t) = \begin{cases} 1 & 0 \le t \le T \\ 0 & \text{altrimenti} \end{cases}$$

? Rappresentazione di Gram-Schmidt







$$\psi_1(t) = \frac{s_1(t)}{\sqrt{E_{s_1}(t)}} \qquad E_{s_1}(t) = \int_{-\infty}^{+\infty} |s_1(t)|^2 dt = \int_0^T \frac{t^2}{T^2} dt = \left[\frac{t^3}{3} \cdot \frac{1}{T^2} \right]_0^T$$

$$= \frac{T^3}{\cancel{T^2}} \cdot \frac{1}{3} = \frac{T}{3}$$

$$\psi_1(t) = \frac{s_1(t)}{\sqrt{\frac{T}{3}}}$$

$$\psi_2(t) = \frac{v_2(t)}{\sqrt{E_{v_2}(t)}} \quad , \; v_2(t) = s_2(t) - \langle s_2(t), \psi_1(t) \rangle \psi_1(t)$$

$$\langle s_2(t), \psi_1(t) \rangle = \int_{-\infty}^{+\infty} s_2(t) \cdot \psi_1(t) \, dt = \int_0^T s_2(t) \cdot \frac{s_1(t)}{\sqrt{\frac{T}{3}}} dt = \frac{1}{\sqrt{\frac{T}{3}}} \int_0^T \frac{t}{T} \cdot \frac{-t}{2T} dt$$

$$= \frac{1}{\sqrt{\frac{T}{3}}} \int_0^T \frac{-t^2}{2T^2} dt = \frac{1}{\sqrt{\frac{T}{3}}} \cdot \left[-\frac{t^3}{3} \cdot \frac{1}{2T^2} \right]_0^T = \frac{1}{\sqrt{\frac{T}{3}}} \cdot \frac{-T^3}{6\cancel{T^2}} = \frac{1}{\sqrt{\frac{T}{3}}} \cdot \left(-\frac{T}{6} \right)$$

$$V_2(t) = S_2(t) - \frac{S_1(t)}{\sqrt{\frac{T}{3}}} \cdot \left(\frac{-T}{6\cdot\sqrt{\frac{T}{3}}}\right) = S_2(t) - S_1(t)\left(\frac{-T}{{}^{2}\cancel{6}\cdot\frac{T}{\cancel{3}}}\right) = S_2(t) + \frac{S_1(t)}{2}$$

$$= -\frac{1}{2}S_1(t) + \frac{1}{2}S_1(t) = 0 .$$

$$\varphi_3(t) = \frac{V_3(t)}{\sqrt{E_{V_3}(t)}}$$

$$V_3(t) = S_3(t) - \langle S_3(t), \varphi_1(t)\rangle\,\varphi_1(t)$$

$$\langle S_3(t), \varphi_1(t)\rangle = \int_{-\infty}^{+\infty} S_3(t)\cdot\varphi_1(t)\,dt = \int_0^T 1\cdot\varphi_1(t)\,dt = \int_0^T \frac{S_1(t)}{\sqrt{\frac{T}{3}}}\,dt$$

$$= \int_0^T \frac{1}{\sqrt{\frac{T}{3}}}\cdot\frac{t}{T}\,dt = \frac{1}{\sqrt{\frac{T}{3}}}\left[\frac{t^2}{2}\cdot\frac{1}{T}\right]_0^T = \frac{1}{\sqrt{\frac{T}{3}}}\cdot\frac{T^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{T}} = \frac{T}{2\sqrt{\frac{T}{3}}}$$

$$V_3(t) = S_3(t) - \frac{T}{2\sqrt{\frac{T}{3}}}\cdot\varphi_1(t) = S_3(t) - \frac{T}{2\sqrt{\frac{T}{3}}}\cdot\frac{S_1(t)}{\sqrt{\frac{T}{3}}} = S_3(t) - \frac{\cancel{T}}{2\,\frac{\cancel{T}}{3}}\cdot S_1(t)$$

$$= S_3(t) - \frac{3}{2}S_1(t)$$

$$E_{V_3}(t) = \int_{-\infty}^{+\infty} |V_3(t)|^2\,dt = \int_{-\infty}^{+\infty}\left(S_3(t) - \frac{3}{2}S_1(t)\right)^2 dt = \int_0^T \left(1 - \frac{3}{2}\frac{t}{T}\right)^2 dt$$

$$= \int_0^T 1 + \frac{9}{4}\cdot\frac{t^2}{T^2} - 3\frac{t}{T}\,dt = \left[t + \frac{9}{4T^2}\cdot\frac{t^3}{3} - \frac{3}{T}\cdot\frac{t^2}{2}\right]_0^T$$

$$= T + \frac{9}{4\cancel{T^2}}\cdot\frac{T^{\cancel{3}}}{3} - \frac{3}{\cancel{T}}\cdot\frac{T^{\cancel{2}}}{2} = T + \frac{\overset{3}{\cancel{9}}}{\underset{4}{\cancel{12}}}\cdot T - \frac{3}{2}T = \frac{1}{4}T .$$

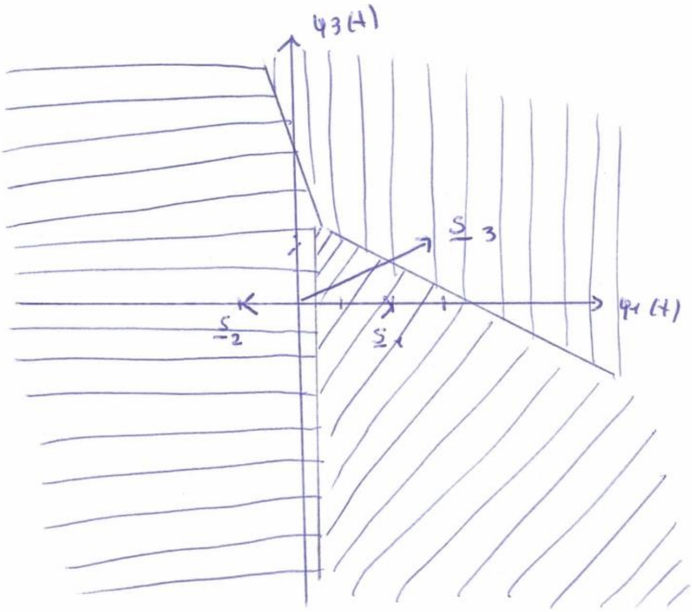
$$\varphi_3(t) = \frac{V_3(t)}{\sqrt{\frac{1}{4}T}}$$

$\underline{S_1}(t) = \left(\sqrt{\frac{T}{3}}\ ,\ 0\right)$

$\underline{S_2}(t) = \left(-\frac{1}{2}\sqrt{\frac{T}{3}}\ ,\ 0\right)$

$\underline{S_3}(t) = \left(\sqrt{\ }\ \frac{3}{2}\cdot\sqrt{\frac{T}{3}}\ ,\ \sqrt{\frac{T}{4}}\right) = \left(\frac{3}{2}\sqrt{\frac{T}{3}}\ ,\ \sqrt{\frac{3}{4}}\cdot\sqrt{\frac{T}{3}}\right)$

? Regioni di decisione



? Stima della probabilità con il Bound Union

$D_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2$

D_{ij}^2	S_1	S_2	S_3
S_1	X	$\frac{3}{4}T$	$\boxed{\frac{1}{3}T}$
S_2		X	$\frac{19}{12}T$
S_3			X

$$BU = \frac{2}{M}4 \cdot Q\left(\sqrt{\frac{D_{ij\,min}^2}{2N_0}}\right)$$

$$= \frac{2}{3} \cdot Q\left(\sqrt{\frac{1}{3} \cdot \frac{T}{2N_0}}\right)$$

/09/2009

ES. 1

Modulazione DSB

$$x(t) = \sum_{m=-\infty}^{+\infty} g(t - m2T) \qquad \text{con} \quad g(t) = \begin{cases} 2 & |t| \le \frac{T}{2} \\ \phi & \text{altrimenti} \end{cases}$$

$V_0 = 2V$, $f_0 \gg \frac{1}{T}$, con $T = 0{,}01$ msec

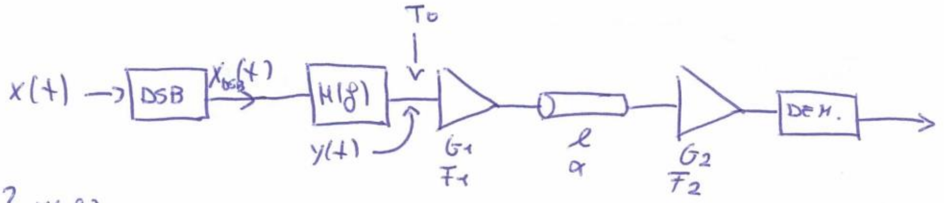
Canale ideale con $H(f) = rect\left(\frac{f - f_0}{\frac{2}{T}}\right) + rect\left(\frac{f + f_0}{\frac{2}{T}}\right)$

$G_1 = 10$ dB , $F_1 = 10$ dB $G_1 = F_1 =$

$\ell = 20$ Km , $\alpha = 1 \frac{dB}{Km}$ $A = \ell \cdot \alpha = 20$ dB

$G_2 = G_1$, $F_2 = F_1$

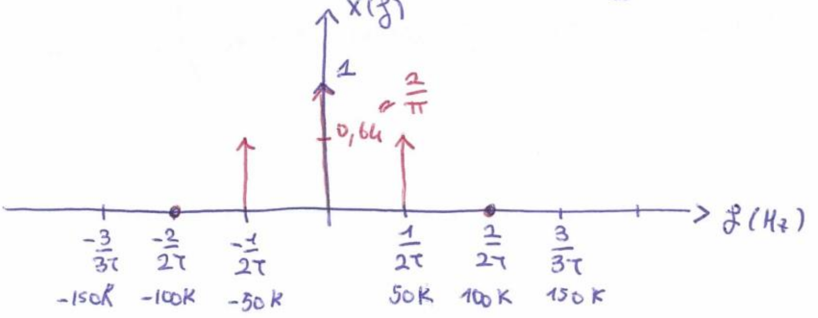
$T_i = T_0 = 290$ K

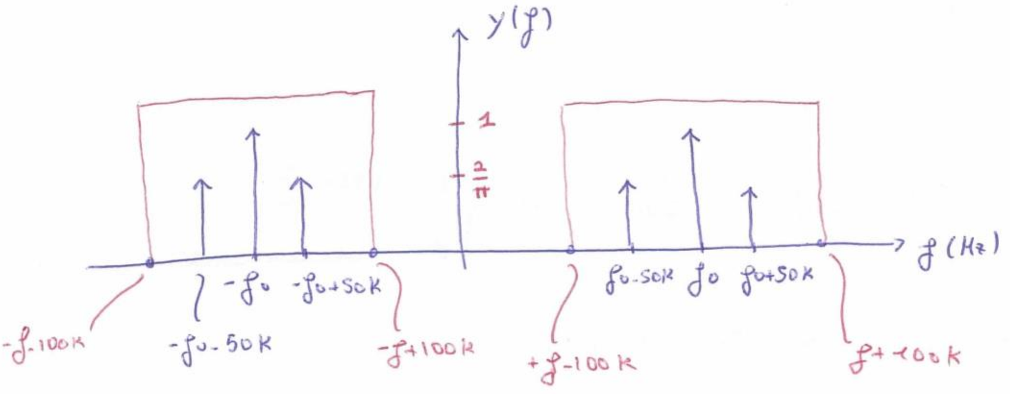


? $Y(f)$

$$x(t) = \sum_{m=-\infty}^{+\infty} 2 \cdot rect\left(\frac{t - m2T}{T}\right) \quad , \quad S_T = 2 rect\left(\frac{t}{\tau}\right) \to S_T(f) = 2\tau \, sinc(\tau \cdot f)$$

$$X(f) = \frac{1}{\cancel{2\tau}} \sum_{m=-\infty}^{+\infty} \cancel{2\tau} \, sinc\left(\cancel{\tau} \cdot \frac{m}{2\cancel{\tau}}\right) \cdot \delta\left(f - \frac{m}{2\tau}\right) = \sum_{m=-\infty}^{+\infty} sinc\left(\frac{m}{2}\right) \delta\left(f - \frac{m}{2\tau}\right).$$





? Potenza media del segnale utile all'ingresso dell'amplificatore.

$P_y = 2 \cdot 1^2 + 4 \cdot \left(\frac{2}{\pi}\right)^2 = 3,62$ Watt $\qquad P_y|_{dB} = 5,58$ dB

? SNR_u

$SNR_u = P_y|_{dB} - 10 \log_{10} K \cdot T_{SIST} \cdot B$ ~ modulante

?

$T_{SIST} = T_0 + T_{eq} = T_0 + T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2} = T_0 + T_0(F_1 - 1) + T_0(A_1 - 1) \cdot \frac{1}{G_1}$

$+ T_0(F_1 - 1) \cdot \frac{1}{G_1} \cdot A_1 = \cancel{T_0} + T_0 F_1 - \cancel{T_0} + \cancel{\frac{T_0 A_1}{G_1}} - \frac{T_0}{G_1} + \frac{T_0 \cancel{F_1} A_1}{\cancel{G_1}} - \cancel{\frac{T_0 A_1}{G_1}}$

$= T_0\left(F_1 - \frac{1}{G_1} + \overset{\simeq 100}{A_1}\right) = 31871$ K

$SNR_u = 5,58 - 10 \log_{10}\left(1,38 \cdot 10^{-23} \frac{J}{K} \cdot 31871 \cdot 50 \cdot 10^3\right) = 142,16$ dB

ES : 2

$\{x(t)\}$ caratterizzato da

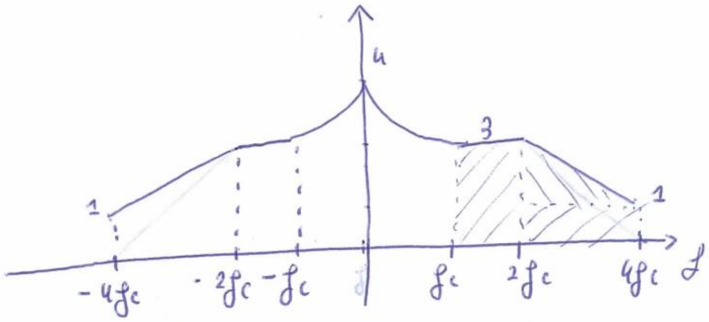
$$\Psi_{x,x}(f) = \begin{cases} 4 - \frac{f^2}{f_c^2} & |f| \leq f_c \\ 3 & f_c < |f| < 2f_c \\ 5 - \frac{f}{f_c} & 2f_c < |f| < 4f_c \\ 0 & \text{altrimenti} \end{cases} \quad , f_c = 10 \text{ kHz.}$$

Modulazione FM ($m = 1$)

$A = 60$ dB , $\frac{N_0}{2} = 10^{-10} \frac{W}{Hz}$

$V_0 = 2V$

? Potenza media di $x(t)$ e di $y(t)$.



$$P_x = \int_{-\infty}^{+\infty} \Psi_{x,x}(f)\, df = 2 \cdot \int_{0}^{f_c} 4 - \frac{f^2}{f_c^2}\, df + 2 \underbrace{\int_{f_c}^{2f_c} 3\, df}_{3 \cdot f_c} + 2 \underbrace{\int_{2f_c}^{4f_c} 5 - \frac{f}{f_c}\, df}_{2f_c \cdot 1 + \frac{2f_c \cdot \cancel{2}}{\cancel{2}} = 4f_c}$$

$$= 2 \cdot \left\{ 7f_c + \left[4f - \frac{f^3}{3f_c^2} \right]_0^{f_c} \right\} = 2 \left\{ 7f_c + \left[4f_c - \frac{f_c^{\cancel{3}}}{3\cancel{f_c^2}} \right] \right\} = 2 \left\{ 7f_c + 4f_c - \frac{f_c}{3} \right\} = \frac{64}{3} f_c$$

$= 21,3 f_c$ Watt

$$P_y = \frac{V_0^2}{2} = 2 .$$

? Banda di Carlson

$$B_{98\%} = 2\cdot(m+1)\cdot \underset{\text{Banda modulante}}{\underset{\wr}{B}} = 16\, f_c = 160 \text{ KHz}.$$

$$?\ SNR_i = \frac{V_0^2}{2N_0B\cdot A} = \frac{2^2}{2\cdot 2\cdot 10^{-10}\cdot 4\cdot 10^{4}\cdot 10^{\frac{60}{10}}} = 0,25$$

? F_m , $K_f = 2$

$$F_m = \frac{3\cdot K_f^2\cdot P_X \sim \text{modulante}}{\underset{\wr\ \text{modulante}}{B^2}} = 1,6\cdot 10^{-3}$$

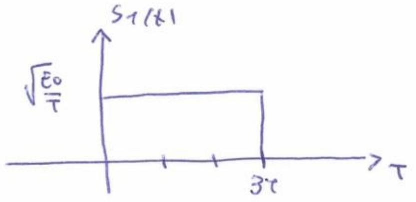
ES. 3

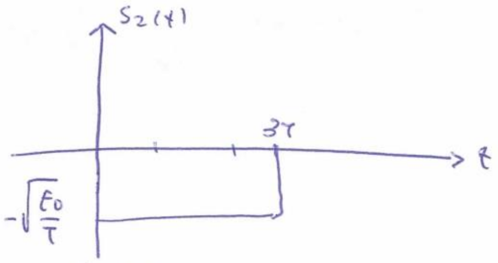
Simboli equiprobabili , $M=4$

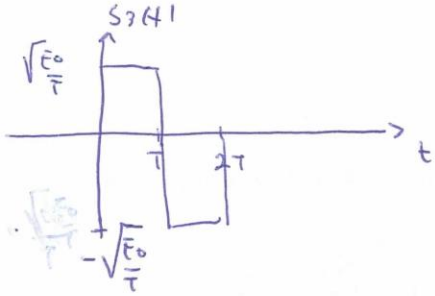
$$s_1(t) = \begin{cases} \sqrt{\frac{E_0}{T}} & 0 \le t \le 3T \\ 0 & \text{altrimenti} \end{cases} \qquad s_3(t) = \begin{cases} \sqrt{\frac{E_0}{T}} & 0 \le t \le T \\ -\sqrt{\frac{E_0}{T}} & T \le t \le 2T \\ 0 & \text{altrimenti} \end{cases}$$

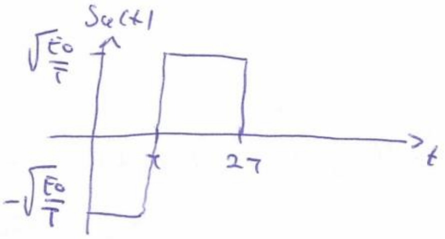
$$s_2(t) = \begin{cases} -\sqrt{\frac{E_0}{T}} & 0 \le t \le 3T \\ 0 & \text{altrimenti} \end{cases} \qquad s_4(t) = \begin{cases} -\sqrt{\frac{E_0}{T}} & 0 \le t \le T \\ \sqrt{\frac{E_0}{T}} & T \le t \le 2T \\ 0 & \text{altrimenti} \end{cases}$$

? Rappresentazione geometrica Gram - Schmidt









$$\varphi_1(t) = \frac{s_1(t)}{\sqrt{E_{s_1}(t)}} \quad , \; E_{s_1}(t) = \int_{-\infty}^{+\infty} |s_1(t)|^2 \, dt = \int_0^{3T} \frac{E_0}{T} \, dt = 3E_0$$

$$= \frac{s_1(t)}{\sqrt{3E_0}}$$

$$\varphi_2(t) = \frac{v_2(t)}{\sqrt{E_{v_2}(t)}} \quad , \; v_2(t) = s_2(t) - \langle s_2(t), \varphi_1(t) \rangle \, \varphi_1(t)$$

$$\langle s_2(t), \varphi_1(t) \rangle = \int_{-\infty}^{+\infty} s_2(t) \cdot \varphi_1(t) \, dt = \int_0^{3T} -\sqrt{\frac{E_0}{T}} \cdot \frac{\sqrt{\frac{E_0}{T}}}{\sqrt{3E_0}} \, dt = \int_0^{3T} -\frac{\frac{E_0}{T}}{\sqrt{3E_0}} \, dt = \int_0^{3T} -\frac{\sqrt{E_0}}{\sqrt{3}T} \, dt$$

$$= -\frac{\sqrt{E_0}}{\sqrt{3}} \cdot 3 = -\sqrt{3E_0}$$

$$v_2(t) = s_2(t) + \cancel{\sqrt{3E_0}} \cdot \frac{s_1(t)}{\cancel{\sqrt{3E_0}}} = s_2(t) + \frac{s_1(t)}{\sqrt{2E_0}} = 0$$

$\varphi_2(t)$ non in Base.

$$\varphi_3(t) = \frac{v_3(t)}{\sqrt{E_{v_3}(t)}}$$

$$v_3(t) = s_3(t) - \cancelto{0}{\langle s_3(t), \varphi_1(t)\rangle} \varphi_1(t)$$

$$\langle s_3(t), \varphi_1(t)\rangle = \int_{-\infty}^{+\infty} s_3(t)\,\varphi_1(t)\,dt = \int_0^{2T} s_3(t) \cdot \frac{s_1(t)}{\sqrt{3E_0}}\,dt = 0.$$

$$v_3(t) = s_3(t)$$

$$E_{v_3}(t) = E_{s_3}(t) = \int_{-\infty}^{+\infty} |s_3(t)|^2\,dt = \int_0^{2T} \frac{E_0}{T}\,dt = 2E_0$$

$$\varphi_3(t) = \frac{s_3(t)}{\sqrt{2E_0}}$$

$$\varphi_4(t) = \frac{v_4(t)}{\sqrt{E_{v_4}}(t)}$$

$$v_4(t) = s_4(t) = \cancelto{0}{\langle s_4(t), \varphi_1(t)\rangle} \varphi_1(t) - \langle s_4(t), \varphi_3(t)\rangle \varphi_3(t)$$

$$\langle s_4(t), \varphi_1(t)\rangle = \int_{-\infty}^{+\infty} s_4(t) \cdot \varphi_1(t)\,dt = \int_0^{T} -\sqrt{\frac{E_0}{T}} \cdot \frac{\sqrt{\frac{E_0}{T}}}{\sqrt{3E_0}}\,dt \int_T^{2T} \sqrt{\frac{E_0}{T}} \cdot \frac{\sqrt{\frac{E_0}{T}}}{\sqrt{3E_0}}\,dt$$

$$= 0$$

$$\langle s_4(t), \varphi_3(t)\rangle = \int_{-\infty}^{+\infty} s_4(t) \cdot \varphi_3(t)\,dt = \int_{-\infty}^{+\infty} s_4(t) \cdot \frac{s_3(t)}{\sqrt{2E_0}}\,dt = \int_0^{2T} -\frac{E_0}{T} \cdot dt \cdot \frac{1}{\sqrt{2E_0}}$$

$$= \frac{1}{\sqrt{2E_0}} \cdot \left(-2E_0\right) = -\sqrt{2E_0}$$

$$v_4(t) = s_4(t) + \cancel{\sqrt{2E_0}} \cdot \frac{s_3(t)}{\cancel{\sqrt{2E_0}}} = 0.$$

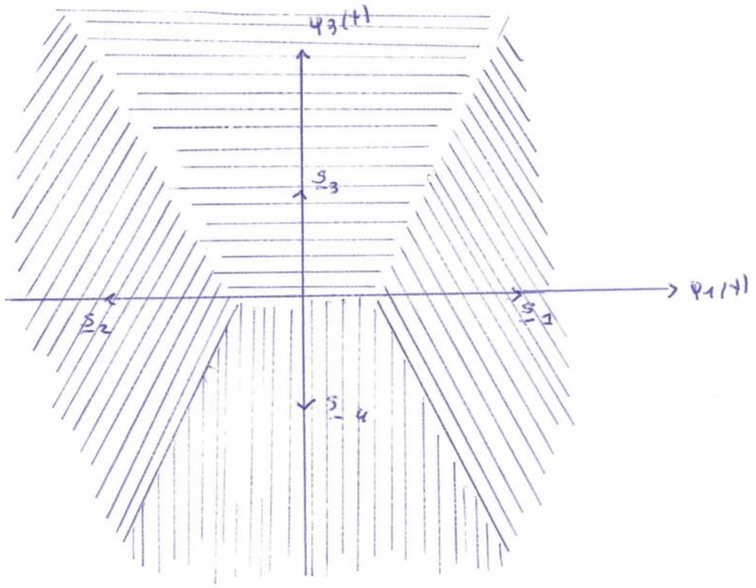
$\underline{s}_1 = (\sqrt{3E_0},\ 0)$

$\underline{s}_2 = (-\sqrt{3E_0},\ 0)$

$\underline{s}_3 = (0,\ \sqrt{2E_0})$

$\underline{s}_4 = (0,\ -\sqrt{2E_0})$

? Regioni di decisione con criterio MAP.



? Stima della probabilità di errore con il Bound Union con $\frac{E_b}{N_0} = 10\,dB$.

$D_{ij}^2 = (x_i - x_j)^2 + (y_i - y_j)^2$

D_{ij}^2	S_1	S_2	S_3	S_4
S_1	$12E_0$	$12E_b$	$\boxed{5E_b}$	$\boxed{5E_b}$
S_2		X	$\boxed{5E_b}$	$\boxed{5E_b}$
S_3			X	$8E_b$
S_4				X

$$BU = \frac{24}{M} \cdot Q\left(\sqrt{\frac{D_{ijMIN}^2}{2N_0}}\right)$$

$$= \frac{2\cdot M}{\cancel{M}} \cdot Q\left(\sqrt{\frac{5}{2}\frac{E_b}{N_0}}\right)$$

$$= 2\cdot Q(5) = 2\cdot\frac{1}{5\sqrt{2\pi}}\cdot e^{-\frac{25}{2}} = 5{,}94 \cdot 10^{-9}$$

29/09/2009

ES. 1

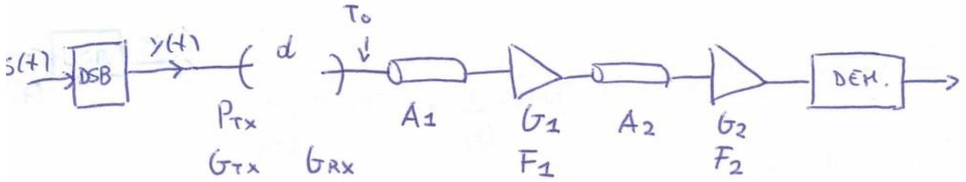
$s(t) = 8\cos(2\pi f_m t)$, $f_m = 25\ KHz$

modulazione DSB:

$f_0 = 3\ MHz$, $V_0 = 2V$

$c(t) = V_0 \cdot \cos(2\pi f_0 t)$ PORTANTE

$y(t) = V_0 \cdot s(t) \cdot \cos(2\pi f_0 t)$ S. MODULATO.



$P_{Tx} = 10\ W$

$T_a = T_0 = 290\ K$

$G_{Tx} = G_{Rx} = 13\ dB$

$d = 60\ Km$

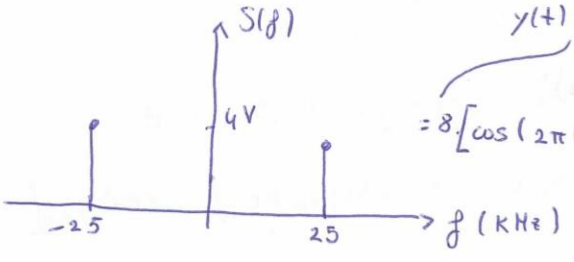
$A_1 = 10\ dB$

$G_1 = F_1 = 10\ dB$

$A_2 = 13\ dB$

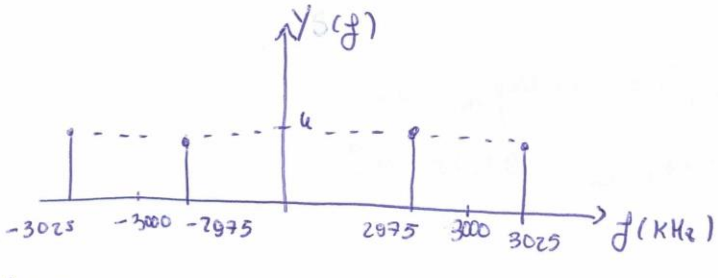
$G_2 = F_2 = 13\ dB$

? Spettro di ampiezza del segnale modulato DSB



$y(t) = 16 \cdot \cos(2\pi 25Kt)\cos(2\pi 3Mt)$

$= 8\left[\cos(2\pi(3M+25k)t) + \cos(2\pi(3M-25k)t)\right]$



$Y(f) = 4\left(\delta(f+3025K) + \delta(f+2975K) + \delta(f-2975K) + \delta(f-3025K)\right)$.

? Temperatura equivalente di Rumore complessiva del sistema in ricezione.

$$T_{eq} = T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1G_2} + \frac{T_{eq4}}{G_1G_2G_3}$$

$$= + T_0(A_1-1) + T_0(F_1-1) \cdot A_1 + T_0(A_2-1) \cdot A_1 \frac{1}{G_1} +$$

$$+ T_0 \cdot (F_2-1) \cdot A_1 \cdot \frac{1}{G_1} \cdot A_2$$

$$= 0 + T_0\cancel{A_1} - T_0 + T_0A_1F_1 - \cancel{T_0A_1} + T_0\cancel{A_2}A_1\frac{1}{G_1} - T_0A_1\frac{1}{G_1}$$

$$+ T_0F_2A_1\frac{1}{G_1}A_2 - T_0\cancel{A_1\frac{1}{G_1}} \cdot A_2$$

$$= T_0A_1F_1 - T_0\cancel{A_1} \cdot \frac{1}{\cancel{G_1}} + T_0F_2\cancel{A_1}\frac{1}{\cancel{G_1}}A_2 - T_0$$

$$= T_0(F_1A_1 - 1 - 1 + F_2A_2) = 290(10 \cdot 10 - 1 - 1 + 20 \cdot 20) = 144420 \text{ K}$$

? Rapporto Segnale Rumore SNR_u all'uscita del demodulatore DSB

Rumore AWGN con $K = 1,38 \cdot 10^{-23}$ J/K.

$$SNR_u = P_{TX} + G_{TX} + G_{RX} - A_{fs} - 10\log_{10} K \cdot T_{sist} \cdot B$$

$$= 10 + 13 + 13 - (32,4 + 20\log_{10} 3 + 20\log_{10} 60) - 10\log_{10}[K \cdot [(T_0 + T_{eq}) \cdot 25 \cdot 10^3]$$

con $A_{fs} = 32,4 + 20\log_{10} f_{MHz} + 20\log_{10} D_{km}$

$$= 10 + 13 + 13 - 77,50 + 133,02 = 91,52 \text{ dB}.$$

ES. 2

$s(t) = 4 \cdot \cos(2\pi f_m t)$, $f_m = 20\ KHz$

Modulato FM con $K_f = 10^4 \frac{Hz}{W}$, $m(t) = \cos(2\pi f_0 t)$, $f_0 = 2\ MHz$

trasmesso su un canale AWGN con densità spettrale di potenza media monolatera $N_0 = 10^{-10} \frac{W}{Hz}$

? Controllare se il ricevitore funziona correttamente

$$y(t) = \underset{1}{\overset{}{V_0}} \cdot \cos\left[2\pi \underset{2\,MHz}{f_0} t + 2\pi K_f \int_0^t s(t)\, dt\right]$$

$$m = \frac{K_f V_m}{f_m} = \frac{10^4 \cdot 4}{20 \cdot 10^3} = 2$$

$$SNR_i\big|_{dB} \geq 13 + 10 \log_{10}(m+1) = 17,77\ dB$$

$$SNR_i = \frac{V_0^2}{2 N_0 \underset{\text{Banda } s(t)}{B}} = \frac{1^2}{2 \cdot 10^{-10} \cdot 2 \cdot 10^4} = 0,25 \cdot 10^6 = 53,97\ dB$$

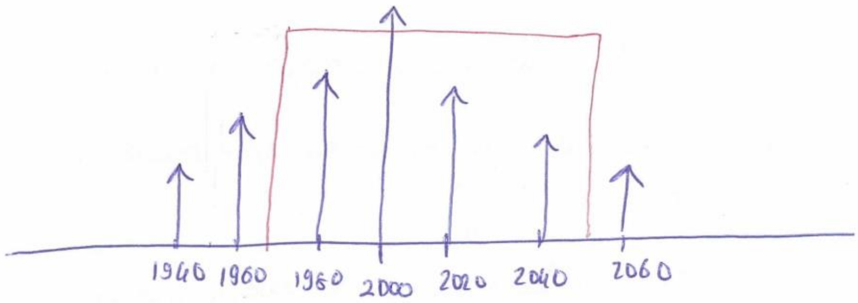
53,97 > 17,77 -> Ricevitore Funziona

? Rapporto SNR_u in uscita al demodulatore.

$SNR_u = SNR_i \cdot F_m$, con F_m = per coseno FM $= \frac{3}{2} m^2 = 6$

$= 1,5 \cdot 10^{+6} = 61,76\ dB$

? Calcolare la percentuale di potenza del segnale modulato contenuta tra 1970 kHz e 2050 kHz.



Considerato $n = 2$ uso $n = 0, 1, 2$

$\% P_y = 0,224^2 + 2 \cdot 0,577^2 + 0,353^2 = 0,84$

$\rightarrow$ 84 %.

? Calcolare la banda necessaria a contenere almeno l'85 % della potenza del segnale

$B_{85\%} = 2 \cdot N \cdot \underset{f_m}{\underset{\shortparallel}{B}} = 2 \cdot 2 \cdot 20\text{K} = 80 \text{ kHz}$

$0,224^2 + 2 \cdot 0,577^2 + 2 \cdot 0,353^2 = 0,96.$

ES. 3

$G_{TX} = 15\ dB$ $\quad A_{fs} = 160\ dB$ $\quad T_{SIMB} = 3\ msec$

$G_{RX} = 15\ dB$ $\quad T_{SIST} = 4 \cdot 10^6\ K$

Modulazione con 5 segnali equiprobabili con la seguente notazione vettoriale :

$S_1 = (2\sqrt{E_b}, \phi)$

$S_2 = (\sqrt{E_b}, \sqrt{2E_b})$

$S_3 = (\sqrt{E_b}, -\sqrt{2E_b})$

$S_4 = (-\sqrt{E_b}, \sqrt{E_b})$

$S_5 = (-\sqrt{E_b}, -\sqrt{E_b})$

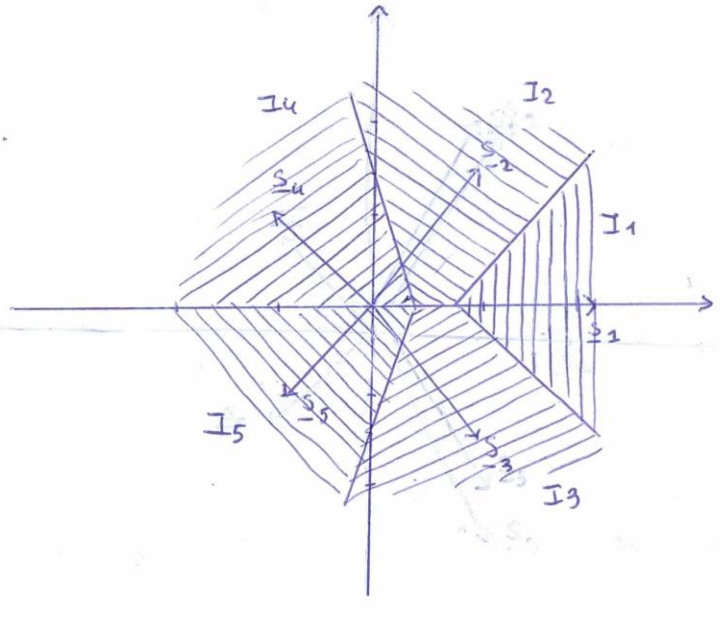
? $\frac{E_b}{N_0}$ se $P_{TX} = 20\ dB$.

$$\left.\frac{E_b}{N_0}\right|_{dB} = P_{TX}|_{dB} + G_{TX}|_{dB} + G_{RX}|_{dB} - A_{fs}|_{dB} - 10\log_{10} K T_{SIST} - 10\log_{10}\frac{\lceil \lg_2 M \rceil}{T_{SIMB}}$$

$$= 20 + 15 + 15 - 160 - 10\log_{10}(1{,}38 \cdot 10^{-23} \cdot 4 \cdot 10^{6}) - 10\log_{10}\frac{\lceil \lg_2 5 \rceil}{3 \cdot 10^{-3}}$$

$$= 22{,}58\ dB$$

? Disegnare regioni di decisione.



? Fornire una stima della probabilità di errore con il Baund Union

D_{ij}^2	S_1	S_2	S_3	S_4	S_5
S_1	X	3Eb	3Eb	10Eb	10Eb
S_2		X	8Eb	4,17 Eb	9,8Eb
S_3			X	9,8 Eb	4,17 Eb
S_4				X	4 Eb
S_5					X

$$BU = \frac{2\;4}{M} \cdot Q\left(\sqrt{\frac{D_{ijMIN}^2}{2N_0}}\right) = \frac{2\cdot 2}{5} \cdot Q\left(\sqrt{\frac{3Eb}{2N_0}}\right)$$

05/07/2008

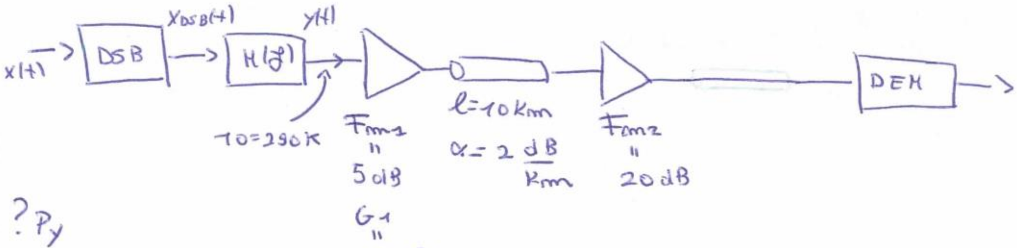
ES. 1

DSB)

$$x(t) = \sum_{m=-\infty}^{+\infty} g(t - mT) \quad , \quad g(t) = \begin{cases} 1 - \frac{2|t|}{T} & , |t| \leq \frac{T}{2} \\ \emptyset & , \text{altrove} \end{cases}$$

$V_0 = 3V$, $f_0 \gg \frac{1}{T}$, $T = 1\,\mu\text{sec}$

$$H(f) = \text{rect}\left(\frac{f - f_0}{\frac{4}{T}}\right) + \text{rect}\left(\frac{f + f_0}{\frac{4}{T}}\right)$$



? P_y

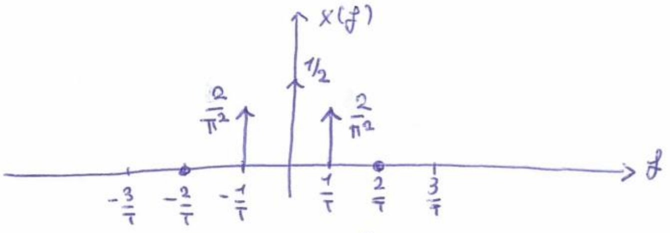
$F_1 = 3,16$
$F_2 = 100$
$G_1 = 31,62$
$A_1 = 100$

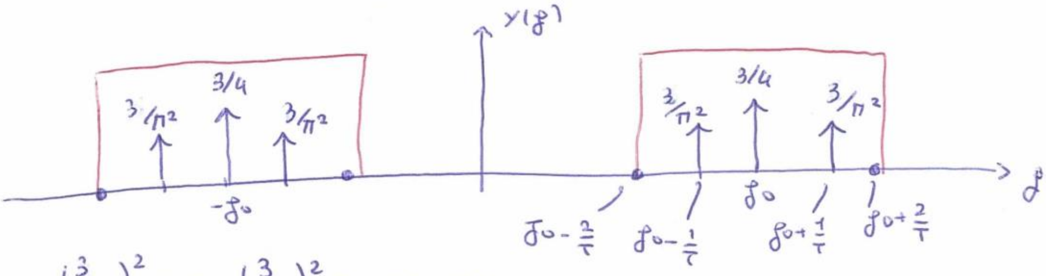
$$g(t) = \text{tri}\left(\frac{t}{\frac{T}{2}}\right)$$

$$x(t) = \sum_{m=-\infty}^{+\infty} \text{tri}\left(\frac{t - mT}{\frac{T}{2}}\right)$$

$$X(f) = \frac{1}{\cancel{T}} \sum_{m=-\infty}^{+\infty} \frac{\cancel{T}}{2} \text{sinc}^2\left(\frac{\cancel{T}}{2} \cdot \frac{m}{\cancel{T}}\right) \cdot \delta\left(f - \frac{m}{T}\right)$$

$$= \frac{1}{2} \sum_{m=-\infty}^{+\infty} \text{sinc}^2\left(\frac{m}{2}\right) \delta \cdot \left(f - \frac{m}{T}\right)$$





$$P_y = 4 \cdot \left(\frac{3}{\pi^2}\right)^2 + 2 \cdot \left(\frac{3}{4}\right)^2 = 1,49 \text{ Watt.}$$

? SNR_u

$B_x = \frac{1}{T} = 1 \cdot 10^{6}$ Hz $= 1$ MHz.

$$SNR_i|_{dB} = SNR_u|_{dB} = P_{Tx}|_{dB} + \cancelto{0}{G_{Tx}|_{dB}} + \cancelto{0}{G_{Rx}|_{dB}} - \cancelto{0}{A_{gx}|_{dB}} - 10\log_{10} K T_{SIST} B_x$$

$$T_{SIST} = T_0 + T_{eq} = T_0 + T_{eq1} + \frac{T_{eq2}}{G_1} + \frac{T_{eq3}}{G_1 G_2}$$

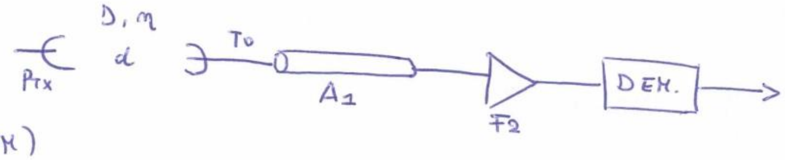
$$= T_0 + T_0(F_1 - 1) + T_0(A_1 - 1) \cdot \frac{1}{G_1} + T_0(F_2 - 1) \cdot \frac{1}{G_1} \cdot A_1$$

$$= \cancel{T_0} + T_0 F_1 - \cancel{T_0} + T_0 \cancel{A_1 \cdot \frac{1}{G_1}} - \frac{T_0}{G_1} + T_0 F_2 \cdot \frac{1}{G_1} \cdot A_1 - T_0 \cdot \cancel{\frac{1}{G_1}} \cdot A_1$$

$$= T_0\left(F_1 - \frac{1}{G_1} + F_2 \cdot \frac{1}{G_1} \cdot A_1\right) = 92621 \text{ K}$$

$SNR_u|_{dB} = 120,66$ dB

ES. 2



FM)

$s(t) = 2 \cdot \cos(2\pi 10^4 t)$

$c(t) = V_0 \cdot \cos(2\pi f_0 t)$, $f_0 = 1 \text{G Hz}$

$\Delta f_{max} = 20 \text{ kHz}$

$P_{Tx} = 10 \text{ W}$

$T_0 = 290 \text{ K}$

$D = 1 \text{ m}$, $\eta = 0{,}6$, $d = 100 \text{ Km}$

$A_1 = 10 \text{ dB}$, $F_2 = 10 \text{ dB}$

? m_{fm} , B_{Tx} Carlson

$$m = \frac{\Delta f_{max}}{f_m} = 2$$

$B_{Tx} = 2 \cdot (m+1) \cdot B_M = 6 \cdot 10^4 = 60 \text{ KHz}.$

? Teq

$$T_{eq} = T_{eq1} + T_{eq2} \cdot \frac{1}{G_1} = T_0 (A_1 - 1) + T_0 (F_1 - 1) \cdot A_1$$

$$= -T_0 + T_0 F_1 A_1 = 28710 \text{ K}$$

? sistema funziona

$SNR_i|dB \geq 13 + 10 \log_{10} (m+2) = 17{,}77 \text{ dB}$

$$G|dB = 20{,}4 + 10 \log_{10} \eta + 20 \log_{10} f|_{GHz} + 20 \log_{10} d|_m$$

$$= 18{,}18 \text{ dB} = G_{Tx}|dB = G_{Rx}|dB$$

$$A_{fs}|dB = 32{,}4 + 20 \log_{10} f|_{MHz} + 20 \log_{10} d|_{Km} = 132{,}4 \text{ dB}$$

$$SNR_i|dB = P_{Tx}|dB + G_{Tx}|dB + G_{Rx}|dB - A_{fs}|dB - 10 \log_{10} (K T_{SIST} \cdot B_{MOD})$$

$$= 57{,}94 \text{ dB}$$

OK

Es. 3

M=3 simboli equiprobabili

$$S_K(t) = \sqrt{\frac{6Eb}{T}} \cdot \cos\left(\frac{2\pi}{T}\cdot t + \frac{K\cdot\pi}{2}\right) \quad , \quad t\in[0,T]$$

$K = 0, 1, 2$;

? Rappresentazione di Gram - Schmidt

$$S_1(t) = \sqrt{\frac{6Eb}{T}} \cdot \cos\left(\frac{2\pi}{T}\cdot t\right)$$

$$S_2(t) = \sqrt{\frac{6Eb}{T}} \cdot \cos\left(\frac{2\pi}{T}\cdot t + \frac{\pi}{2}\right)$$

$$S_3(t) = \sqrt{\frac{6Eb}{T}} \cdot \cos\left(\frac{2\pi}{T}\cdot t + \pi\right)$$

$$\psi_1(t) = \frac{S_1(t)}{\sqrt{E_{S1}}}$$

$$E_{S1} = \int_{-\infty}^{+\infty} |S_1(t)|^2 dt = \int_{-\infty}^{+\infty} \frac{6Eb}{T}\cdot \cos^2\left(\frac{2\pi}{T}\cdot t\right) dt = \frac{6Eb}{T}\int_0^T \frac{1}{2}\left(1+\cos\left(\frac{4\pi}{T}\cdot t\right)\right) dt$$

$$= \frac{6Eb}{T}\left\{\int_0^T \frac{1}{2} dt + \overset{\to 0}{\cancel{\int_0^T \frac{1}{2}\cdot\cos\left(\frac{4\pi}{T}\cdot t\right) dt}}\right\} = \frac{3\cancel{6}Eb}{\cancel{T}}\cdot\frac{1}{\cancel{2}}\cancel{T} = 3Eb$$

$$\psi_1(t) = \frac{S_1(t)}{\sqrt{3Eb}}$$

$$\psi_2(t) = \frac{V_2(t)}{\sqrt{E_{V_2}}}$$

$$V_2(t) = S_2(t) - \langle S_2(t), \psi_1(t)\rangle \cdot \psi_1(t)$$

$$\langle S_2(t), \psi_1(t)\rangle = \int_{-\infty}^{+\infty} S_2(t)\cdot\psi_1(t)\, dt = \int_0^T \sqrt{\frac{6Eb}{T}}\cdot\cos\left(\frac{2\pi}{T}\cdot t + \frac{\pi}{2}\right)\cdot\sqrt{\frac{6Eb}{T}}\cdot\cos\left(\frac{2\pi}{T}\cdot t\right)$$

$$\cdot\frac{1}{\sqrt{3Eb}}\, dt = \frac{6Eb}{T}\cdot\frac{1}{\sqrt{3Eb}}\cdot\int_0^T \text{sen}\left(\frac{2\pi}{T}\cdot t\right)\cdot\cos\left(\frac{2\pi}{T}\cdot t\right) dt$$

$$= \frac{6Eb}{T\cdot\sqrt{3Eb}}\cdot\int_0^T \frac{1}{2}\left[\overset{\to 0}{\cancel{\text{sen}\left(\frac{4\pi t}{T}\right)}} + 0\right] dt = 0$$

$$V_2(t) = S_2(t)$$

$$\psi_2(t) = \frac{S_2(t)}{\sqrt{E_{S_2}}}$$

$$\sqrt{E_{S_2}} = \int_{-\infty}^{+\infty} |\underline{s}_2(t)|^2 dt = \int_0^T \frac{6E_b}{T} \cdot \operatorname{sen}^2\left(\frac{2\pi t}{T}\right) dt = \frac{6E_b}{T}\int_0^T \frac{1}{2}\left[\cos(0) - \cos\left(\frac{4\pi t}{T}\right)\right] dt$$

$$= \frac{6E_b}{T}\int_0^T \frac{1}{2}\, dt = 3E_b$$

$$\psi_2(t) = \frac{S_2(t)}{\sqrt{3E_b}}$$

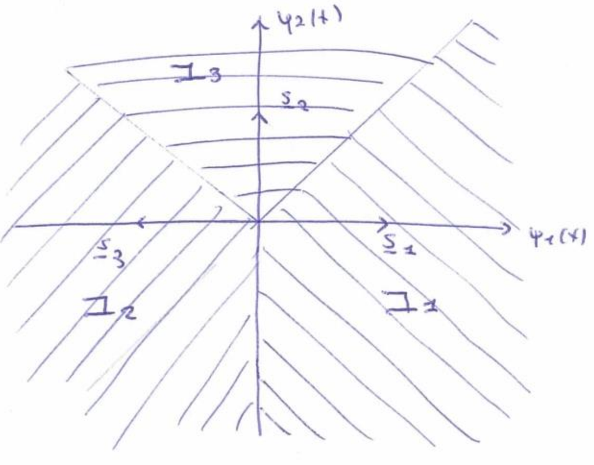
$S_3(t)$ antipodale a $S_1(t)$

$\underline{S}_1 = (\sqrt{3E_b}, 0)$

$\underline{S}_2 = (0, \sqrt{3E_b})$

$\underline{S}_3 = (-\sqrt{3E_b}, 0)$

? Regioni di decisione



? B.U.

$\frac{E_b}{N_0} = 10$ dB

D_{ij}^2	S_1	S_2	S_3
S_1	X	$\boxed{6E_b}$	$12E_b$
S_2		X	$\boxed{6E_b}$
S_3			X

$$B.U. = \frac{2q}{M} \cdot Q\left(\sqrt{\frac{D_{ij,MIN}^2}{2N_0}}\right) = \frac{4}{3}Q\left(\sqrt{\frac{3E_b}{N_0}}\right)$$

$$= \frac{4}{3}Q\left(\sqrt{30}\right) = \frac{4}{3} \cdot \frac{1}{\sqrt{2\pi \cdot 30}} \cdot e^{-\frac{30}{2}} = 2{,}97 \cdot 10^{-8}$$

17/07/2008

Es. 1

$m(t)=\cos(2\pi f_m t)$, $f_m = 10$ KHz

$s(t)=x_m^2(t)+\frac{1}{2}x_m(t)-\frac{1}{2}$

AM)

$c(t)=V_0\cos(2\pi f_0 t)$, $f_0 = 1$ MHz , $V_0 = 1$ V , $K = 1$

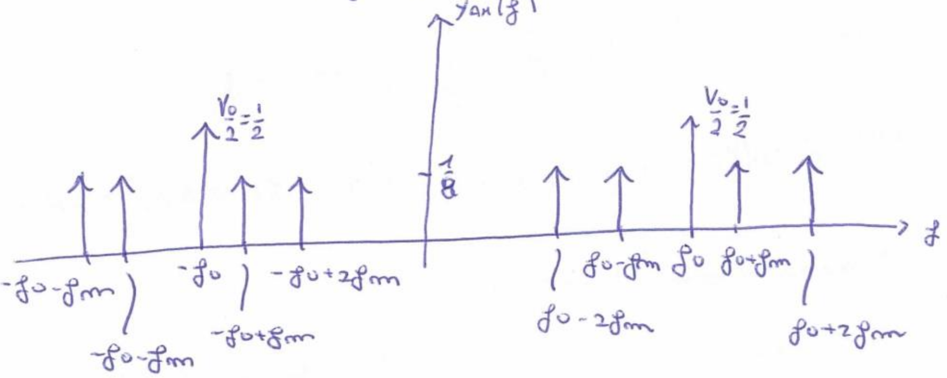
? $Y_{AM}(f)$

$$s(t)=m^2(t)+\frac{1}{2}m(t)-\frac{1}{2}=\cos^2(2\pi f_m t)+\frac{1}{2}\cos(2\pi f_m t)-\frac{1}{2}$$

$$=\frac{1}{2}(1+\cos(2\pi 2f_m t))+\frac{1}{2}\cos(2\pi f_m t)-\frac{1}{2}$$

$$=\frac{1}{2}\cos(2\pi 2f_m t)+\frac{1}{2}\cos(2\pi f_m t)$$

$y_{AM}(t)=V_0(1+Ks(t))\cdot\cos(2\pi f_0 t)$



? SNR_u , F_m

$P_s = 4\cdot\left(\frac{1}{4}\right)^2=\frac{1}{4}$ Watt

$N_0 = 10^{-8}\ \frac{W}{Hz}$

$$SNR_i = \frac{V_0^2(1+K^2P_s)}{2N_0B_s}=3125$$

$$SNR_u=\frac{V_0^2K^2P_s}{2N_0B_s}=625$$

$$F_m=\frac{SNR_u}{SNR_i}=\frac{1}{5}$$

ES. 2

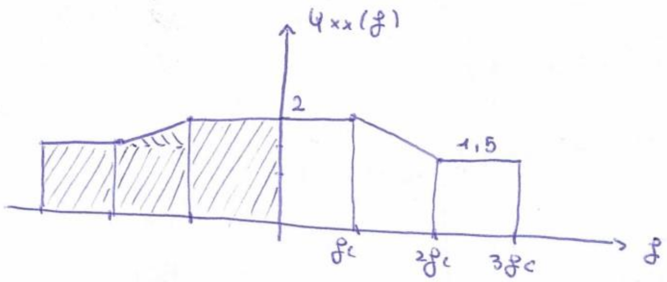
$\{x(t)\}$

$$\Psi_{x,x}(f) = \begin{cases} 2 & |f| \le f_c \\ 2,5 - \frac{|f|}{2f_c} & f_c \le |f| \le 2f_c \\ 1,5 & 2f_c \le |f| \le 3f_c \\ 0 & \text{altrimenti} \end{cases} \quad , f_c = 1 \text{ KHz}$$

FM) $m = 5$

$A_{fs} = 120$ dB , $\frac{N_0}{2} = 10^{-12}$

? P_x



$P_x = \int_{-\infty}^{+\infty} \Psi_{x,x}(f)\, df = 2 \cdot \left(f_c \cdot 2 + f_c \cdot 1,5 + f_c \cdot 1,5 + f_c \cdot 0,5/2 \right) = 10,5\, f_c .$

? Banda di Trasmissione (Carlson)

$B_{Tx} = 2(m+1)\, B_{modulante} = 2 \cdot (5+1) \cdot 3 \cdot 10^3 = 36$ KHz .

? SNR_u

$V_0 = 3V$, $K_f = 1$

$SNR_i|_{dB} = \frac{V_0^2}{2}\Big|_{dB} - A_{fs}|_{dB} - 10 \log_{10} N_0 B =$

$= 6,53 - 120 - 10 \log_{10} (2 \cdot 10^{-12} \cdot 3 f_c) = -31,25$ dB

$F_m = \frac{3K^2 P_x}{B_x^2} = 3,5 \cdot 10^{-3}$

$SNR_u|_{dB} = SNR_i|_{dB} + F_m|_{dB} = -55,81$ dB .

Es. 3

M=3 simboli equiprobabili

$$s_1(t) = \sqrt{\frac{6E_b}{T}} \cdot \cos\left(\frac{2\pi}{T} \cdot t\right) \quad , t \in [0, T]$$

$$s_2(t) = \sqrt{\frac{6E_b}{T}} \cdot \cos\left(\frac{2\pi}{T} \cdot t + \pi\right) \quad , t \in [0, T]$$

$$s_3(t) = 0 \quad , t \in [0, T]$$

? Rappresentazione di Gram-Schmidt

$$\varphi_1(t) = \frac{s_1(t)}{\sqrt{E_{s1}}} \qquad E_{s1} = \int_{-\infty}^{+\infty} |s_1(t)|^2 dt = \int_{-\infty}^{+\infty} \frac{6E_b}{T} \cdot \cos^2\left(\frac{2\pi}{T} \cdot t\right) dt$$

$$= \frac{6E_b}{T} \int_{-\infty}^{+\infty} \frac{1}{2}\left(1 + \cos\left(\frac{4\pi}{T} \cdot t\right)\right) dt = \frac{6E_b}{T} \left\{ \int_0^T \frac{1}{2} dt + \int_0^T \cos\left(\frac{4\pi}{T} \cdot t\right) dt \right\}$$

$$= \frac{6E_b}{T} \left\{ \left[\frac{1}{2} t\right]_0^T + \left[\frac{T}{4\pi} \cancelto{0}{\text{sen}\left(\frac{4\pi}{T} \cdot t\right)}\right]_0^T \right\} = 3E_b .$$

$$\varphi_1(t) = \frac{s_1(t)}{\sqrt{3E_b}}$$

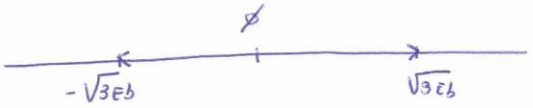
Considerato che $s_2(t) = -s_1(t)$ i segnali sono antipodali!

=>

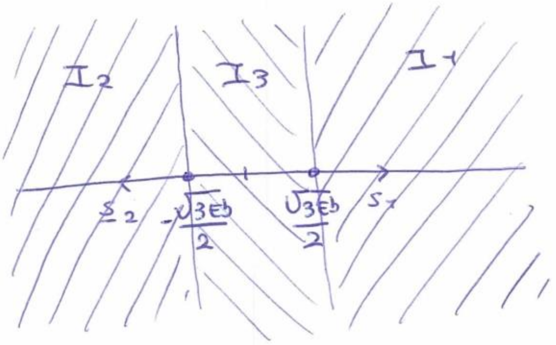
$$\underline{s}_1 = (\sqrt{3E_b})$$

$$\underline{s}_2 = (-\sqrt{3E_b})$$

$$\underline{s}_3 = (0)$$



? MAP



? Pe

$$P_e = \frac{1}{3} \sum_{m=1}^{3} P_{e/\underline{s}_i}$$

$$P_{e/\underline{s}_1} = P_e \left\{ m + s_1 \leq \frac{\sqrt{3E_b}}{2} \right\} = P\left(m \leq -\frac{1}{2}\sqrt{3E_b} \right) = Q\left(+\frac{\sqrt{3E_b}}{2} \cdot \sqrt{\frac{2}{N_0}} \right)$$

$$= Q\left(\sqrt{\frac{3}{2} \cdot \frac{E_b}{N_0}} \right)$$

$$P_{e/\underline{s}_2} = P_{e/\underline{s}_1} = Q\left(\sqrt{\frac{3}{2} \cdot \frac{E_b}{N_0}} \right)$$

$$P_{e/\underline{s}_3} = P_e \left\{ m + s_3 \leq -\frac{1}{2}\sqrt{3E_b} \right\} + P_e \left\{ s_3 + m \geq \frac{1}{2}\sqrt{3E_b} \right\}$$

$$= P\left(m \leq -\frac{1}{2}\sqrt{3E_b} \right) + P\left(m \geq \frac{1}{2}\sqrt{3E_b} \right) = 2 \cdot Q\left(\sqrt{\frac{3}{2} \cdot \frac{E_b}{N_0}} \right)$$

$$P_e = \frac{4}{3} \cdot Q\left(\sqrt{\frac{3}{2} \cdot \frac{E_b}{N_0}} \right) = \frac{4}{3} \cdot Q\left(\sqrt{15} \right) = \frac{4}{3} \cdot \frac{1}{\sqrt{2\pi 15}} \cdot e^{-\frac{15}{2}} = 7,59 \cdot 10^{-5}$$

--. 09/2008

ES. 1

AM)

$x(t) = \sum_{m=-\infty}^{+\infty} g(t - m3\tau)$, $g(t) = Tri_{\tau}(t)$

$V_0 = 2V$, $f_0 \gg \frac{1}{3\tau}$ con $\tau = 1 msec$

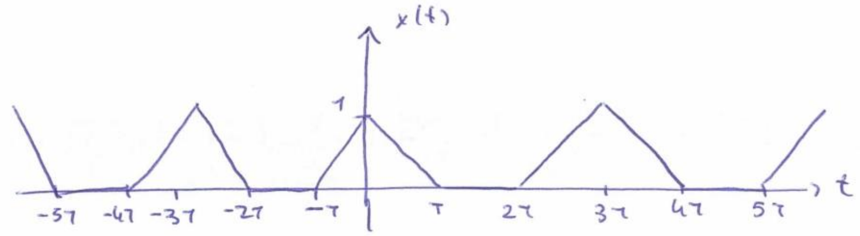
$K = \frac{1}{2}$

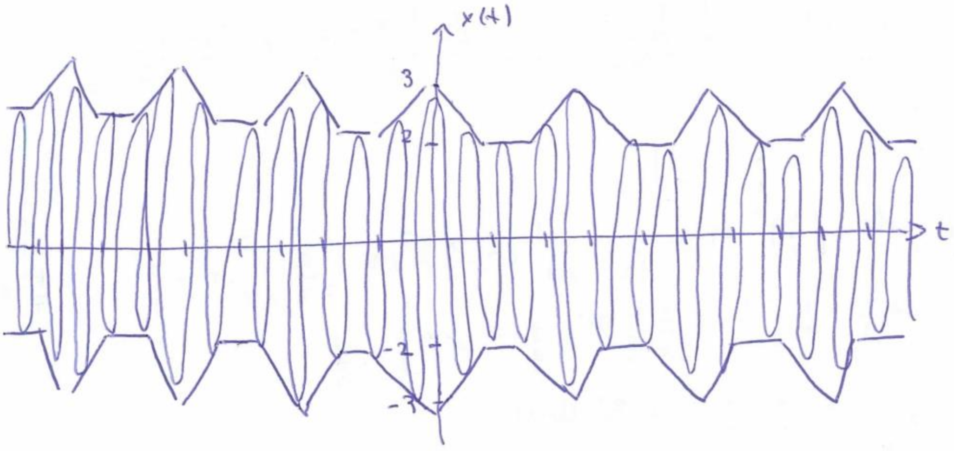
? $y_{AM}(t)$

$y_{AM}(t) = V_0(1 + K \cdot x(t)) \cdot \cos(2\pi f_0 t)$

$= 2 \cdot (1 + \frac{1}{2} \cdot x(t)) \cdot \cos(2\pi f_0 t)$

$x(t) = \sum_{m=-\infty}^{+\infty} Tri\left(\frac{t - m3\tau}{\tau}\right)$





? Calcolare $Y_{AM}(f)$

$$X(f) = \frac{1}{3\cancel{\tau}} \cdot \sum_{m=-\infty}^{+\infty} \cancel{\tau} \cdot \mathrm{sinc}^2\left(\cancel{\tau} \cdot \frac{m}{3\cancel{\tau}}\right) \cdot \delta\left(f - \frac{m}{3\tau}\right)$$

$$= \frac{1}{3} \sum_{m=-\infty}^{+\infty} \mathrm{sinc}^2\left(\frac{m}{3}\right) \cdot \delta\left(f - \frac{m}{3\tau}\right)$$

$$y_{AM}(t) = V_0 \cos(2\pi f_0 t) + V_0 K x(t) \cdot \cos(2\pi f_0 t)$$

$$= 2 \cos(2\pi f_0 t) + \cancel{2} \cdot \frac{\cancel{1}}{\cancel{2}} x(t) \cdot \cos(2\pi f_0 t)$$

$$Y_{AM}(f) = \delta(f - f_0) + \delta(f + f_0) + \frac{1}{2}\left(X(f - f_0) + X(f + f_0)\right).$$

? SNR_i, SNR_u

$T_{sist} = 500\,K$, $B = \frac{1}{3\tau}$

$$SNR_i = \frac{P_y}{K T_{sist} \cdot B_x}$$

P_y: potenza di un cos( ) $= \frac{(V_0(1 + K x(t))^2}{2} = \frac{V_0^2}{2}\left(1 + K^2 \overline{x^2(t)} + \overline{x(t)} \cdot 2K\right)$

con $\overline{x^2(t)} = P_x$

$$\overline{x}(t) = \frac{1}{3\tau} \int_{-\tau}^{\tau} \mathrm{tri}\left(\frac{t}{\tau}\right) dt = \frac{1}{3\tau} \cdot 2 \int_{0}^{\tau} 1 - \frac{t}{\tau}\, dt = \frac{1}{3\tau} \cdot 2 \left[t - \frac{t^{\cancel{2}}}{2} \cdot \frac{1}{\cancel{\tau}}\right]_{0}^{\tau}$$

$$= \frac{2}{3\tau}\left[\tau - \frac{\tau}{2}\right] = \frac{1}{3}$$

$$\overline{x^2(t)} = P_x = \frac{1}{3\tau} \int_{-\tau}^{\tau} \mathrm{tri}\left(\frac{t}{\tau}\right)^2 dt = \frac{1}{3\tau} \cdot 2 \int_{0}^{\tau} \left(1 - \frac{t}{\tau}\right)^2 dt = \frac{2}{3\tau} \int_{0}^{\tau} 1 + \frac{t^2}{\tau} - 2\frac{t}{\tau}\, dt$$

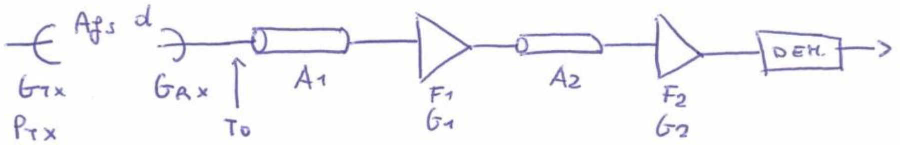
$$= \frac{2}{3\tau}\left[t + \frac{t^3}{3} \cdot \frac{1}{\tau^2} - \cancel{2}\frac{t^2}{\cancel{2}} \cdot \frac{1}{\tau}\right]_{0}^{\tau} = \frac{2}{3\cancel{\tau}}\left[\cancel{\tau} + \frac{1}{3}\tau \cancel{-\tau}\right] = \frac{2}{9}$$

$$P_y = \frac{\overset{2}{\cancel{4}}}{\cancel{2}} \cdot \left(1 + \frac{1}{_2\cancel{4}} \cdot \frac{\cancel{2}}{9} + \cancel{2} \cdot \frac{\cancel{1}}{\cancel{2}} \cdot \frac{1}{3}\right) = \frac{25}{9}\ \text{Watt}$$

$SNR_i = 1{,}2 \cdot 10^{+18}$

$$SNR_u = \frac{V_0^2 K^2 \overline{s^2(t)}}{2 N_0 B} = \frac{4 \cdot \frac{1}{4} \cdot \frac{2}{9}}{2 \cdot 1{,}38 \cdot 10^{-23} \cdot 500 \cdot \frac{1}{3} \cdot 10^3} = 4{,}8 \cdot 10^{16}.$$

2



$s(t) = \cos(2\pi f_m t)$, $f_m = 20$ kHz

$c(t) = V_0 \cdot \cos(2\pi f_0 t)$, $f_0 = 5$ GHz

$m = 5$

$P_{Tx} = 10$ W
$T_0 = 290$ K
$G_{Tx} = G_{Rx} = 16$ dB ; $d = 60$ Km
$A_1 = F_1 = G_1 = 10$ dB
$A_2 = 13$ dB
$F_2 = G_2 = 13$ dB

? T_{eq}

$$T_{eq} = T_{eq1} + T_{eq2} \cdot \frac{1}{G_1} + T_{eq3} \cdot \frac{1}{G_1 G_2} + T_{eq4} \frac{1}{G_1 G_2 G_3}$$

$$= T_0(A_1 - 1) + T_0(F_1 - 1) . A_1 + T_0 (A_2 - 1) A_1 \frac{1}{G_1} + T_0 (F_2 - 1) . \frac{A_1 A_2}{G_1}$$

$$= \cancel{T_0 A_1} - T_0 + T_0 F_1 A_1 - \cancel{T_0 A_1} + T_0 A_2 \cancel{A_1 \frac{1}{G_1}} - T_0 \cancel{A_1} \frac{1}{\cancel{G_1}} + T_0 F_2 \cancel{A_1} A_2 \frac{1}{\cancel{G_1}} - T_0 \cancel{A_1 A_2 \frac{1}{G_1}}$$

$$= T_0 (-1 + F_1 A_1 - 1 + F_2 A_2) = 144420 \text{ K}$$

? Ricevitore funziona

$SNR_i|dB \geq 13 + 10 \log_{10}(m+1) = 20,78$ dB

$SNR_i|dB = P_{Tx}|dB + G_{Tx}|dB + G_{Rx}|dB - A_{fs}|dB - 10 \log_{10}(K T_{sist} B_n)$

$A_{fs}|dB = 32,4 + 20 \log_{10} f_0|_{MHz} + 20 \log_{10} d|_{km} = 141,94$

$SNR_i|dB = 34,05$ dB

? SNR_u

$F_m = \frac{3}{2} m^2 = \frac{75}{2}$ => $SNR_u|dB = SNR_i|dB + 10 \log_{10}\left(\frac{75}{2}\right) = 49,79$ dB

ES. 3

$M = 5$ segnali equiprobabili

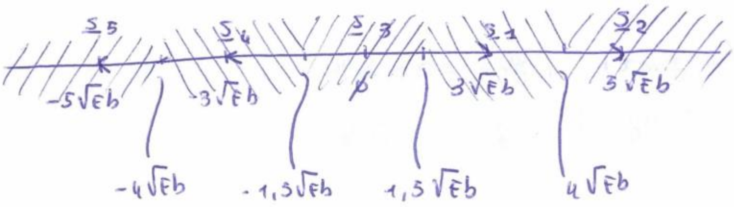
$s_1(t) = 3\sqrt{E_b}$

$s_2(t) = 5\sqrt{E_b}$

$s_3(t) = 0$

$s_4(t) = -3\sqrt{E_b}$

$s_5(t) = -5\sqrt{E_b}$



$$P_e = \frac{1}{5}\sum_{i=1}^{5} P_{e/\underline{s}_i}$$

$$P_{e/\underline{s}_5} = P\left\{ s_5 + \underline{m} \geq -4\sqrt{E_b} \right\} = P\left(\underline{m} \geq \sqrt{E_b} \right) = Q\left(\sqrt{E_b} \cdot \sqrt{\frac{2}{N_0}} \right) = Q\left(\sqrt{\frac{2E_b}{N_0}} \right)$$

$$P_{e/\underline{s}_4} = P\left\{ s_4 + m \leq -4\sqrt{E_b} \right\} + P\left\{ s_4 + m \geq -1,5\sqrt{E_b} \right\} = P\left(m \leq -\sqrt{E_b} \right) +$$
$$P\left(m \geq 1,5\sqrt{E_b} \right) = Q\left(+\sqrt{E_b} \cdot \sqrt{\frac{2}{N_0}} \right) + Q\left(1,5\sqrt{E_b} \cdot \sqrt{\frac{2}{N_0}} \right) = Q\left(\sqrt{\frac{2E_b}{N_0}} \right) + Q\left(\sqrt{\frac{9E_b}{2N_0}} \right)$$

$$P_{e/\underline{s}_3} = P\left\{ S_3 + m \leq -1,5\sqrt{E_b} \right\} + P\left\{ S_3 + m \geq 1,5\sqrt{E_b} \right\} + P\left(m \leq -1,5\sqrt{E_b} \right)$$
$$+ P\left(m \geq 1,5\sqrt{E_b} \right) = Q\left(+1,5\sqrt{E_b} \cdot \sqrt{\frac{2}{N_0}} \right) + Q\left(1,5\sqrt{E_b} \cdot \sqrt{\frac{2}{N_0}} \right))$$
$$= 2 \cdot Q\left(\sqrt{\frac{9E_b}{2N_0}} \right)$$

$$P_{e/\underline{s}_2} = P_{e/\underline{s}_4} \quad ; \quad P_{e/\underline{s}_1} = P_{e/\underline{s}_5}$$

$$P_e = \frac{1}{5}\left(4 \cdot Q\left(\sqrt{\frac{2E_b}{N_0}} \right) + 4 \cdot Q\left(\sqrt{\frac{9E_b}{2N_0}} \right) \right).$$

11/09/2008

Es. 1

$y(t) = (A + B s(t)) \cos(2\pi f_0 t) + C \hat{s}(t) \text{ sen}(2\pi f_0 t)$

$P_c = 1 \text{ Watt} = \frac{V_0^2}{2} \Rightarrow V_0 = \sqrt{2}$

? Parametri modulazioni

AM) $A = V_0$, $B = V_0 K$, $C = 0$

DSB) $A = 0$, $B = V_0$, $C = 0$

SSB) $A = 0$, $B = V_0$, $C = V_0$

? SNRi, SNRu per le tre modulazioni

$s(t) = 4 \cdot \cos(2\pi f_m t)$, $f_m = 10 \text{ MHz}$, $N_0 = 10^{-11} \frac{W}{Hz}$, $K = 0,5$.

AM) $P_M = \frac{4^2}{2} = 8 \text{ Watt}$

$$SNR_i = \frac{V_0^2(1 + K^2 P_M)}{2 N_0 B_M} = 30000$$

$$SNR_u = \frac{V_0^2 K^2 P_M}{2 N_0 B_M} = 20000$$

DSB)

$$SNR_u = SNR_i = \frac{V_0^2 P_M}{2 N_0 B_M} = 80000$$

SSB)

$$SNR_u = SNR_i = \frac{V_0^2 P_M}{N_0 B_M} = 160000$$

ES. 2

$m(t) = \cos(2\pi f_m t)$, $f_m = 15 \text{ KHz}$

$c(t) = V_0 \cdot \cos(2\pi f_0 t)$, $f_0 = 20 \text{ MHz}$, $V_0 = 3 V$

$K_f = 75 K \frac{Hz}{V}$

$N_0 = 10^{-14} \frac{W}{Hz}$, $A = 70 \text{ dB}$

? $B_{88\%}$

$$m = \frac{K_f \cdot V_m}{f_m} = 5$$

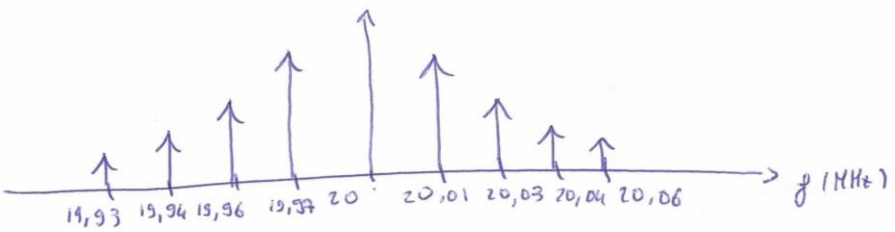
$$0,88 \leq 0,178^2 + 2 \cdot 0,328^2 + 2 \cdot 0,047^2 + 2 \cdot 0,365^2 + 2 \cdot 0,391^2 + 2 \cdot 0,261^2$$
$$= 0,96 .$$

$N = 5$

$B_{88\%} = 2N \cdot B_M = 150 \text{ KHz}$

? $P_{[19,94 \div 20,06] \text{ MHz}}$

$P_y = \frac{V_0^2}{2} = \frac{9}{2}$ Watt



$N = 4$

$\% P = 0,178^2 + 2 \cdot 0,328^2 + 2 \cdot 0,047^2 + 2 \cdot 0,365^2 + 2 \cdot 0,391^2 = 0,523$

$P = P_y \cdot \% P = 3,7$ Watt

? P_{Tx} affinchè il ricevitore funzioni

$SNR_i|dB \geq 13 + 10 \log_{10} (m+1) = 20,78 \text{ dB}$

$SNR_i|dB = P_{Tx}|dB - B N_0|dB - A|dB \Rightarrow P_{Tx}|dB = SNR_i|dB + N_0 B|dB + A|dB$
$= -7,46 \text{ dB}$.

? $SNR_u|dB$

$F_m = \frac{3}{2} m^2 = \frac{75}{2}$ $SNR_u|dB = SNR_i|dB + 10 \log_{10} \frac{75}{2} = 36,5 \text{ dB}$.

ES: 3

$G_{TX} = G_{RX} = 10$ dB

$A_{fs} = 180$ dB

$T_{SIST} = 200$ K

$T_{SIMB} = 3$ msec

$M = 5$ segnali equiprobabili

$s_1(t) = (\sqrt{Eb}, 0)$

$s_2(t) = (0, \sqrt{Eb})$

$s_3(t) = (-\sqrt{Eb}, 0)$

$s_4(t) = (\sqrt{Eb}, \sqrt{Eb})$

$s_5(t) = (-\sqrt{Eb}, \sqrt{Eb})$

? P_{TX} per avere $\frac{Eb}{N_0} = 5$ dB

$$P_{TX}|_{dB} = \frac{Eb}{N_0}\Big|_{dB} - G_{TX}|_{dB} - G_{RX}|_{dB} + A_{fs}|_{dB} + 10\log_{10} K T_{SIST} + 10 \log_{10} \frac{[\log_2 M]}{T_{SIMB}}$$

$$= -10,6 \text{ dB}$$

? Stima di P_e

D_{ij}^2	S_1	S_2	S_3	S_4	S_5
S_1	X	2Eb	4Eb	$\boxed{Eb}$	5Eb
S_2		X	2Eb	$\boxed{Eb}$	$\boxed{Eb}$
S_3			X	5Eb	$\boxed{Eb}$
S_4				X	4Eb
S_5					X

$$BU = \frac{2q}{M} \cdot Q\left(\sqrt{\frac{D_{ijMIN}^2}{2N_0}}\right)$$

$$= \frac{8}{5} \cdot Q\left(\sqrt{\frac{3}{2}}\right)$$

$$= \frac{8}{5} \cdot \frac{1}{\sqrt{2\pi} \cdot \sqrt{\frac{3}{2}}} \cdot e^{-3/4} = 0,246.$$