FONDAMENTI DI TELECOMUNICAZIONI A

[Appunti Di Esercizi, Parte 1]

A CURA DI ALESSANDRO PAGHI

PROFESSORE: Giuliano Benelli (http://www3.diism.unisi.it/people/person.php?id=18)

LINK AL CORSO ANNO 2014/2015:
http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=54889&aa=2014

FREQUENTAZIONE: Consigliata.

6/05/14 PROVA A

Es. 1

$$s(t) = \int_{-\infty}^{t} \left[4\,\mathrm{sinc}^2(2\tau) e^{j4\pi\tau} * \frac{d}{d\tau} 4\,\mathrm{sinc}(8\tau) \right] d\tau$$

$K(\tau) = p(\tau) * z(\tau)$

$p(\tau) = 4\,\mathrm{sinc}^2(2\tau) e^{j4\pi\tau}$

$z(\tau) = \frac{d}{d\tau} 4\,\mathrm{sinc}(8\tau)$

$s(t) = \int_{-\infty}^{t} K(\tau)\, d\tau \quad \longleftrightarrow \quad S(f) = \frac{1}{j2\pi f} \cdot K(f)$

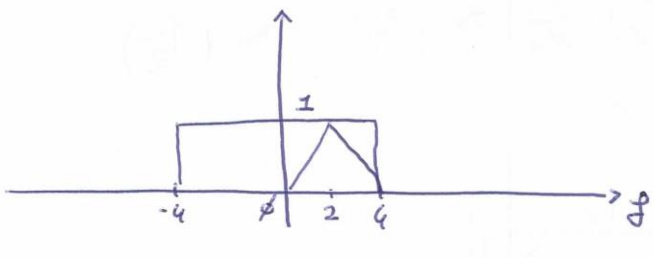
$K(f) = P(f) \cdot Z(f)$

$4\,\mathrm{sinc}^2(2\tau) \longleftrightarrow \frac{4}{2}\,\mathrm{tri}\left(\frac{f}{2}\right) = 2\,\mathrm{tri}\left(\frac{f}{2}\right)$

$P(f) = 2\,\mathrm{tri}\left(\frac{f-2}{2}\right)$

$Z(f) = j\cancel{2}\pi f \cdot \frac{\cancel{4}}{\cancel{8}}\,\mathrm{rect}\left(\frac{f}{8}\right) = j\pi f\,\mathrm{rect}\left(\frac{f}{8}\right)$

$S(f) = \frac{1}{j\cancel{2\pi f}} \cdot \cancel{2}\,\mathrm{tri}\left(\frac{f-2}{2}\right) \cdot \cancel{j\pi f} \cdot \mathrm{rect}\left(\frac{f}{8}\right) = \mathrm{tri}\left(\frac{f-2}{2}\right).$



6/05/14 PROVA B

ES. 1

$$s(t) = \frac{d}{dt}\left[4\,\mathrm{sinc}(8t) * \int_{-\infty}^{t} 4\,\mathrm{sinc}^2(2\tau)\, e^{j4\pi\tau}\, d\tau \right]$$

$$s(t) = \frac{d}{dt} k(t) \quad \longleftrightarrow \quad S(f) = j2\pi f \cdot K(f)$$

$$k(t) = p(t) * z(t) \quad \longleftrightarrow \quad K(f) = P(f) \cdot Z(f)$$

$$p(t) = 4\,\mathrm{sinc}(8t)$$

$$P(f) = \frac{4}{8}\,\mathrm{sinc}\left(\frac{f}{8}\right) = \frac{1}{2}\,\mathrm{rect}\left(\frac{f}{8}\right)$$

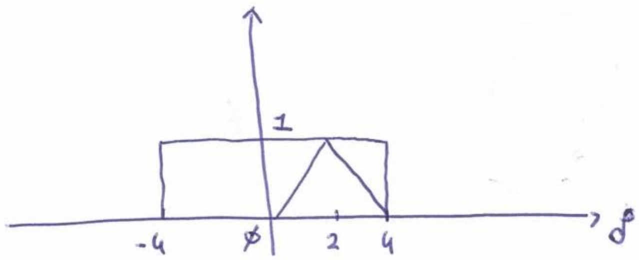
$$z(t) = \int_{-\infty}^{t} 4\,\mathrm{sinc}^2(2\tau)\, e^{j4\pi\tau}\, d\tau$$

$$4\,\mathrm{sinc}^2(2\tau) \longleftrightarrow \frac{4}{2}\,\mathrm{tri}\left(\frac{f}{2}\right) = 2\,\mathrm{tri}\left(\frac{f}{2}\right)$$

$$4\,\mathrm{sinc}^2(2\tau) \cdot e^{j4\pi\tau} \longleftrightarrow 2\,\mathrm{tri}\left(\frac{f-2}{2}\right)$$

$$Z(f) = \frac{1}{j\cancel{2}\pi f} \cdot \cancel{2}\,\mathrm{tri}\left(\frac{f-2}{2}\right) = \frac{1}{j\pi f} \cdot \mathrm{tri}\left(\frac{f-2}{2}\right)$$

$$S(f) = j\cancel{2\pi f} \cdot \frac{1}{\cancel{2}}\,\mathrm{rect}\left(\frac{f}{8}\right) \cdot \frac{1}{\cancel{j\pi f}} \cdot \mathrm{tri}\left(\frac{f-2}{2}\right) = \mathrm{tri}\left(\frac{f-2}{2}\right)$$



25/06/14

Es. 1

$$s(t) = 4\,\mathrm{sinc}^2(2t)\cos(6\pi t) * \frac{d}{dt}\left[6\,\mathrm{sinc}(4t)\,\mathrm{sen}(6\pi t)\right]$$

$$s(t) = p(t) * z(t) \longleftrightarrow S(f) = P(f)\cdot Z(f)$$

$$p(t) = 4\,\mathrm{sinc}^2(2t)\cos(6\pi t)$$

$$4\,\mathrm{sinc}^2(2t) \longleftrightarrow \frac{4}{2}\,\mathrm{tri}\left(\frac{f}{2}\right) = 2\,\mathrm{tri}\left(\frac{f}{2}\right)$$

$$P(f) = \frac{1}{2}\cdot\left[2\,\mathrm{tri}\left(\frac{f-3}{2}\right) + 2\,\mathrm{tri}\left(\frac{f+3}{2}\right)\right] = \mathrm{tri}\left(\frac{f-3}{2}\right) + \mathrm{tri}\left(\frac{f+3}{2}\right)$$

$$z(t) = \frac{d}{dt}\left[6\,\mathrm{sinc}(4t)\,\mathrm{sen}(6\pi t)\right]$$

$$6\,\mathrm{sinc}(4t) \longleftrightarrow \frac{6}{4}\,\mathrm{rect}\left(\frac{f}{4}\right) = \frac{3}{2}\,\mathrm{rect}\left(\frac{f}{4}\right)$$

$$6\,\mathrm{sinc}(4t)\cdot\mathrm{sen}(6\pi t) \longleftrightarrow \frac{1}{2j}\left[\frac{3}{2}\,\mathrm{rect}\left(\frac{f-3}{4}\right) - \frac{3}{2}\,\mathrm{rect}\left(\frac{f+3}{4}\right)\right]$$

$$= \frac{3}{4j}\left[\mathrm{rect}\left(\frac{f-3}{4}\right) - \mathrm{rect}\left(\frac{f+3}{4}\right)\right]$$

$$Z(f) = \cancel{j}\cancel{2}\pi f\cdot\frac{3}{{}_2\cancel{4}\cancel{j}}\left[\mathrm{rect}\left(\frac{f-3}{4}\right) - \mathrm{rect}\left(\frac{f+3}{4}\right)\right] = \frac{3}{2}\pi f\cdot\left[\mathrm{rect}\left(\frac{f-3}{4}\right) - \mathrm{rect}\left(\frac{f+3}{4}\right)\right]$$

$$S(f) = \left[\mathrm{tri}\left(\frac{f-3}{2}\right) + \mathrm{tri}\left(\frac{f+3}{2}\right)\right]\cdot\frac{3}{2}\pi f\cdot\left[\mathrm{rect}\left(\frac{f-3}{4}\right) - \mathrm{rect}\left(\frac{f+3}{4}\right)\right]$$

$$= \frac{3}{2}\pi f\left[\mathrm{tri}\left(\frac{f-3}{2}\right)\cdot\mathrm{rect}\left(\frac{f-3}{4}\right) - \cancel{\mathrm{tri}\left(\frac{f-3}{2}\right)\cdot\mathrm{rect}}\left(\frac{f+3}{4}\right)\right.$$

$$\left. + \,\mathrm{tri}\left(\frac{f+3}{2}\right)\cancel{\cdot\,\mathrm{rect}\left(\frac{f-3}{4}\right)} - \mathrm{tri}\left(\frac{f+3}{2}\right)\cdot\mathrm{rect}\left(\frac{f+3}{4}\right)\right]$$

$$= \frac{3}{2}\pi f\left[\mathrm{tri}\left(\frac{f-3}{2}\right) - \mathrm{tri}\left(\frac{f+3}{2}\right)\right]\cdot\cancel{\frac{j}{j}} \cancel{= 3j\pi f\;\mathrm{tri}\left(\frac{f}{2}\right)\cdot\mathrm{sen}(6\pi f)}$$

15/05/2013

ES. 1

$s(t) = \left[\mathrm{sinc}^2\left(2\left(t+\frac{3}{2}\right)\right) e^{j4\pi t}\right] * \left[\mathrm{sinc}\left(2\left(t-\frac{1}{2}\right)\right) e^{j10\pi t}\right]$

$s(t) = p(t) * z(t) \;<\!-\!> \; S(f) = P(f)\cdot Z(f)$

$p(t) = \mathrm{sinc}^2(2t+3)\, e^{j4\pi t}$

$\mathrm{sinc}^2(2t+3) \;<\!-\!>\; \frac{1}{2}\,\mathrm{tri}\left(\frac{f}{2}\right)\cdot e^{j\cancel{2}\pi \frac{3}{\cancel{2}} f} = \frac{1}{2}\,\mathrm{tri}\left(\frac{f}{2}\right) e^{j3\cdot\pi f}$

$P(f) = \frac{1}{2}\,\mathrm{tri}\left(\frac{f-2}{2}\right) e^{j3\pi(f-2)}$

$z(t) = \mathrm{sinc}(2t-1)\, e^{j10\pi t}$

$\mathrm{sinc}(2t-1) \;<\!-\!>\; \frac{1}{2}\,\mathrm{rect}\left(\frac{f}{2}\right) e^{-j\cdot\pi f}$

$Z(f) = \frac{1}{2}\,\mathrm{rect}\left(\frac{f-5}{2}\right) e^{-j\cdot\pi(f+5)}$

$S(f) = \frac{1}{2}\,\mathrm{tri}\left(\frac{f-2}{2}\right) e^{j3\pi(f-2)} \cdot \frac{1}{2}\,\mathrm{rect}\left(\frac{f-5}{2}\right) e^{-j\cdot\pi(f+5)}$

$= \frac{1}{4}\,\mathrm{tri}\left(\frac{f-2}{2}\right)\mathrm{rect}\left(\frac{f-5}{2}\right)\cdot e^{j3\pi(f-2) - j\cdot\pi(f+5)}$

$= -\frac{1}{4}\,\mathrm{tri}\left(\frac{f-2}{2}\right)\mathrm{rect}\left(\frac{f-5}{2}\right)\cdot e^{j\cdot 2\pi f} \quad = \emptyset$

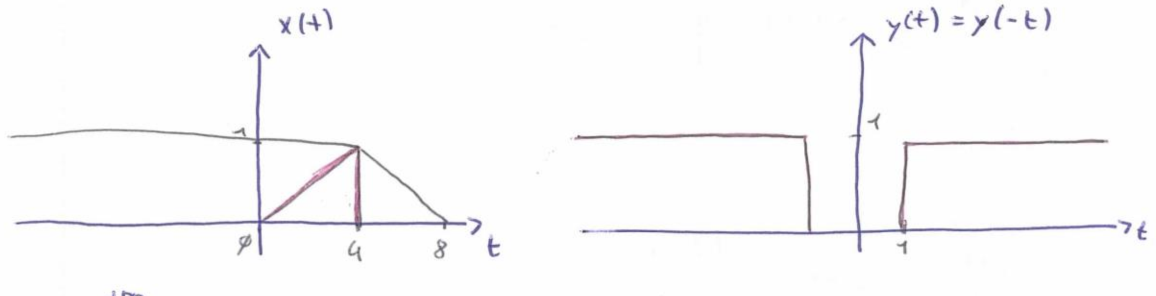
6/05/14 PROVA A

ES. 2

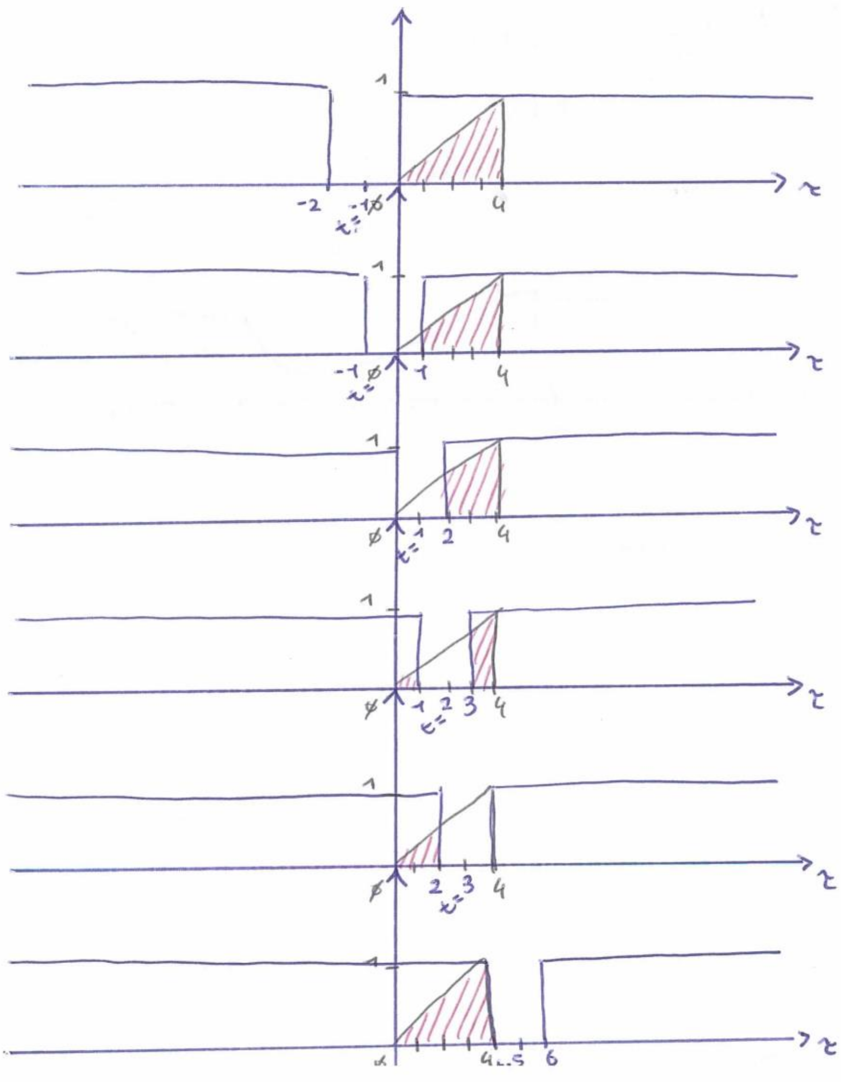
$$x(t) = \text{tri}\left(\frac{t-4}{4}\right) u(4-t)$$

$$y(t) = u(-t-1) + u(t-1)$$

$$z(t) = x(t) * y(t)$$



$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$



$t \leq -1 \rightarrow z(t) = 2$

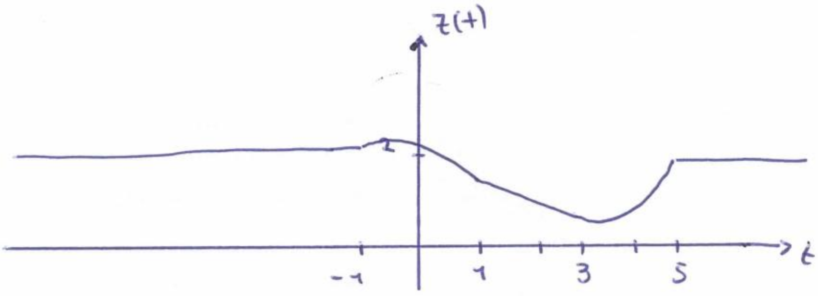
$-1 < t \leq 1 \rightarrow z(t) = \int_{t+1}^{4} \frac{1}{4} \tau \, d\tau = -\frac{t^2}{8} - \frac{1}{4} t + \frac{15}{8}$

$1 < t \leq 3 \rightarrow z(t) = \int_{0}^{t-1} \frac{1}{4} \tau \, d\tau + \int_{t+1}^{4} \frac{1}{4} \tau \, d\tau = -\frac{1}{2} t + 2$

$3 < t \leq 5 \rightarrow z(t) = \int_{0}^{t-1} \frac{1}{4} \tau \, d\tau = \frac{t^2}{8} - \frac{1}{4} t + \frac{1}{8}$

$t > 5 \rightarrow z(t) = 2$.

$$
z(t) = \begin{cases} 2 & t \leq -1 \\ -\frac{t^2}{8} - \frac{1}{4} t + \frac{15}{8} & -1 < t \leq 1 \\ -\frac{1}{2} t + 2 & 1 < t \leq 3 \\ \frac{t^2}{8} - \frac{1}{4} t + \frac{1}{8} & 3 < t \leq 5 \\ 2 & t > 5 \end{cases}
$$



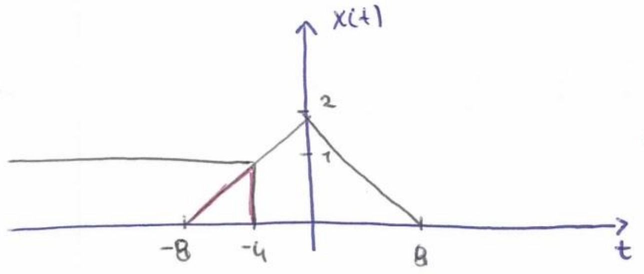
6/05/14 PROVA B

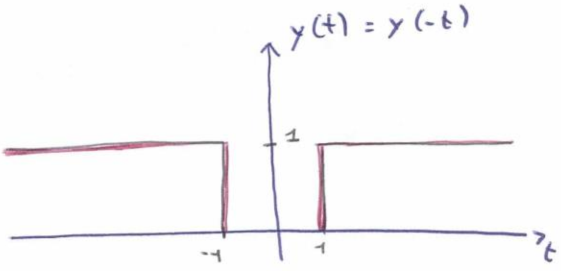
ES. 2

$x(t) = 2 \cdot \mathrm{tri}\left(\frac{t}{8}\right) \cdot u(-t-4)$

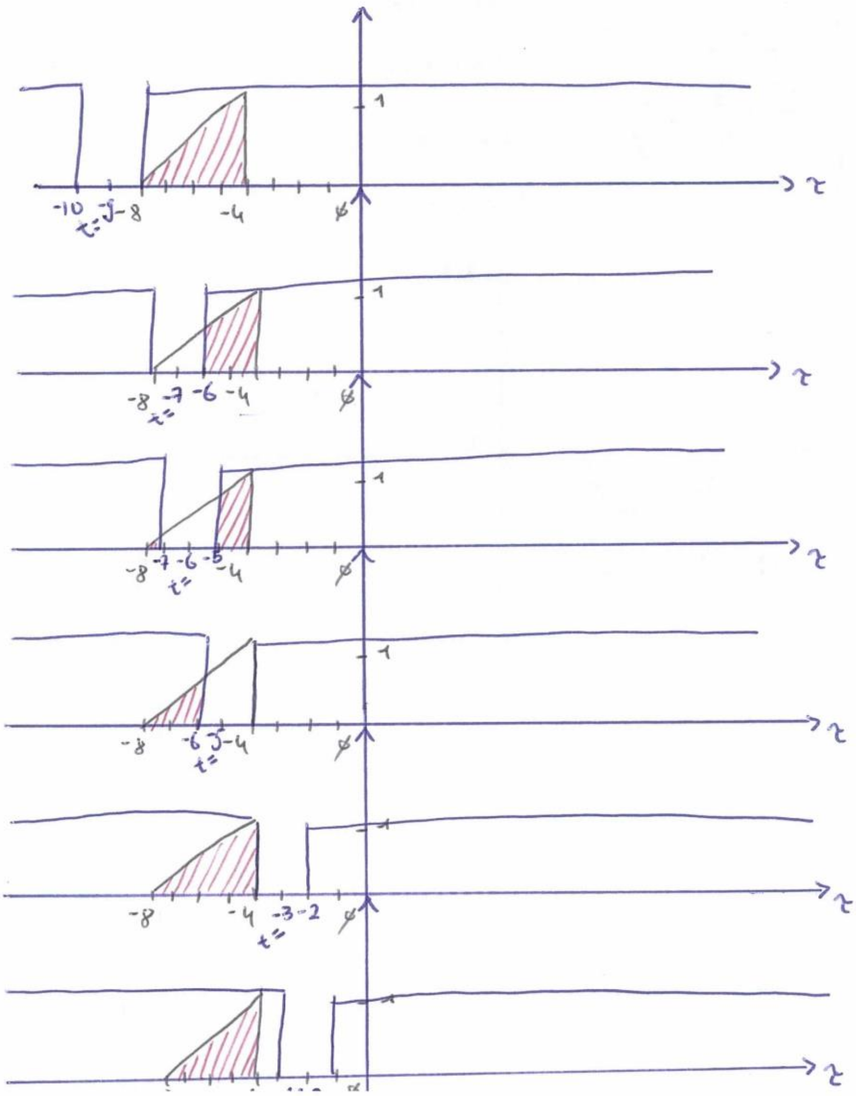
$y(t) = u(-t-1) + u(t-1)$

$z(t) = x(t) * y(t)$.





$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$



$t \le -9 \to z(t) = 2$

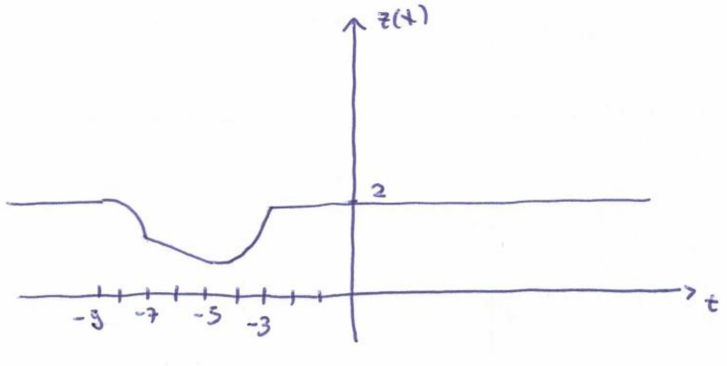
$-9 < t \le -7 \to z(t) = \int_{t+1}^{-4} \frac{1}{4}\tau + 2 \, d\tau = -\frac{t^2}{8} - \frac{9}{4}t - \frac{65}{8}$

$-7 < t \le -5 \to z(t) = \int_{-8}^{t-1} \frac{1}{4}\tau + 2 \, d\tau + \int_{t+1}^{-4} \frac{1}{4}\tau + 2 \, d\tau = -\frac{1}{2}t - 2$

$-5 < t \le -3 \to z(t) = \int_{-8}^{t-1} \frac{1}{4}\tau + 2 \, d\tau = \frac{t^2}{8} + \frac{7}{4}t + \frac{49}{8}$

$t > -3 \to z(t) = 2$

$$z(t) = \begin{cases} 2 & t \le -9 \\ -\frac{t^2}{8} - \frac{9}{4}t - \frac{65}{8} & -9 < t \le -7 \\ -\frac{1}{2}t - 2 & -7 < t \le -5 \\ \frac{t^2}{8} + \frac{7}{4}t + \frac{49}{8} & -5 < t \le -3 \\ 2 & t > -3 \end{cases}$$

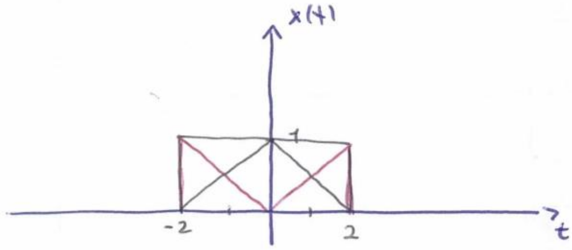


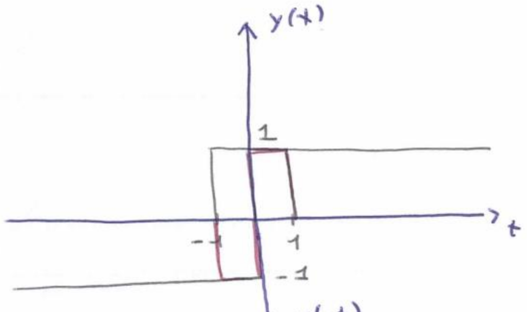
25/06/14

ES. 2

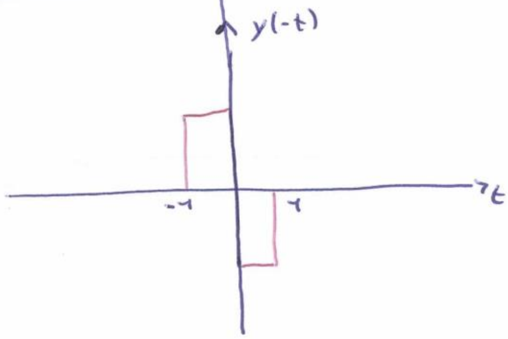
$x(t) = \mathrm{rect}\left(\frac{t}{4}\right) - \mathrm{tri}\left(\frac{t}{2}\right)$

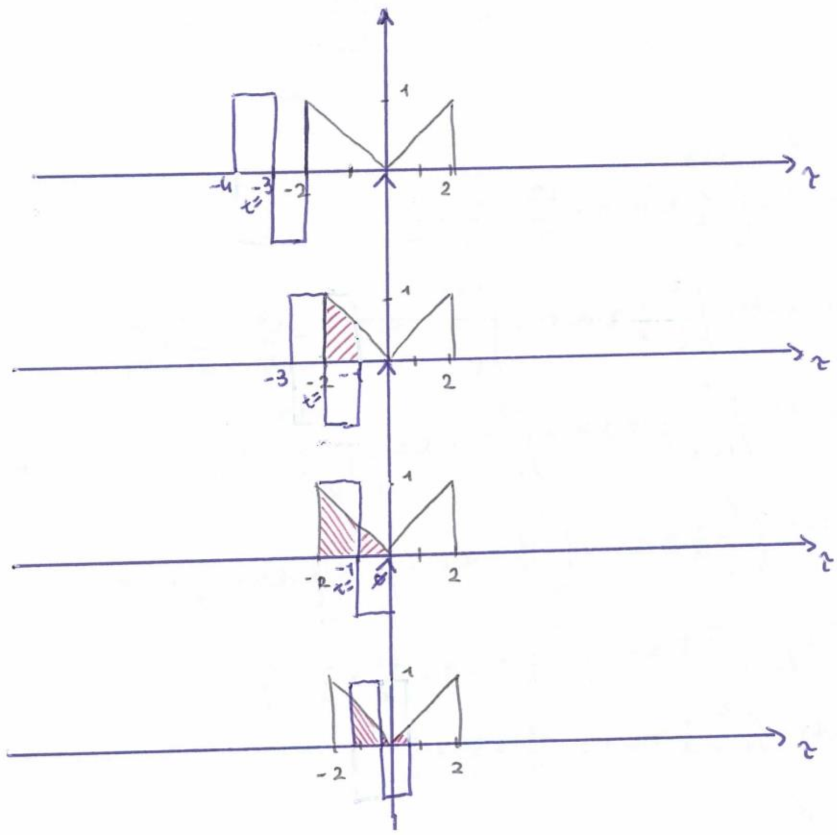
$y(t) = \mathrm{rect}\left(\frac{t}{2}\right)\,\mathrm{sign}(t)$ $\qquad z(t) = x(t) * y(t)$

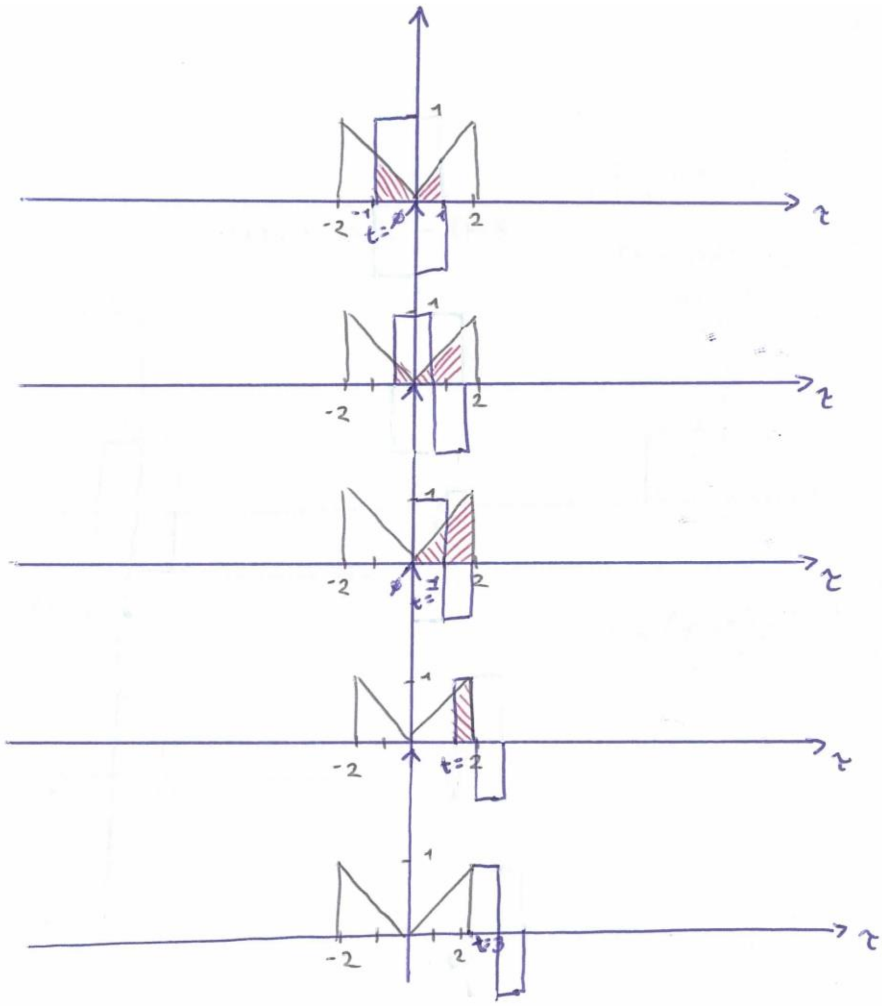




$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$







$t \leq -3 \rightarrow z(t) = \emptyset$

$-3 < t \leq -2 \rightarrow z(t) = \int_{-2}^{t+1} \frac{1}{2} \tau \, d\tau = \frac{t^2}{4} + \frac{t}{2} - \frac{3}{4}$

$-2 < t \leq -1 \rightarrow z(t) = \int_{-2}^{t} -\frac{1}{2} \tau \, d\tau + \int_{t}^{t+1} \frac{1}{2} \tau \, d\tau = -\frac{t^2}{4} + \frac{t}{2} + \frac{5}{4}$

$-1 < t \leq \emptyset \rightarrow z(t) = \int_{t-1}^{t} -\frac{1}{2} \tau \, d\tau + \int_{t}^{\emptyset} \frac{1}{2} \tau \, d\tau - \int_{\emptyset}^{t+1} \frac{1}{2} \tau \, d\tau = -\frac{1}{2} t^2 - t$

$\emptyset < t \leq 1 \rightarrow z(t) = \int_{t-1}^{\emptyset} -\frac{1}{2} \tau \, d\tau + \int_{\emptyset}^{t} \frac{1}{2} \tau \, d\tau - \int_{t}^{t+1} \frac{1}{2} \tau \, d\tau = \frac{1}{2} t^2 - t$

$1 < t \leq 2 \rightarrow z(t) = \int_{t-1}^{t} \frac{1}{2} \tau \, d\tau - \int_{t}^{2} \frac{1}{2} \tau \, d\tau = \frac{t^2}{4} + \frac{t}{2} - \frac{5}{4}$

$2 < t \leq 3 \rightarrow z(t) = \int_{t-1}^{2} \frac{1}{2} \tau \, d\tau = -\frac{t^2}{4} + \frac{t}{2} + \frac{3}{4}$

$t > 3 \rightarrow z(t) = \emptyset$.

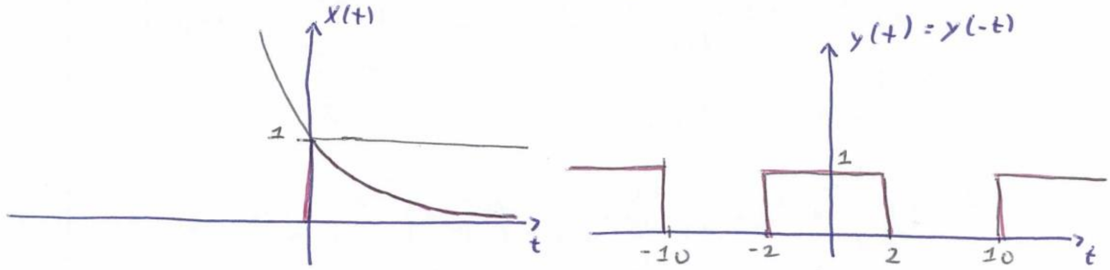
15/05/13

ES. 2

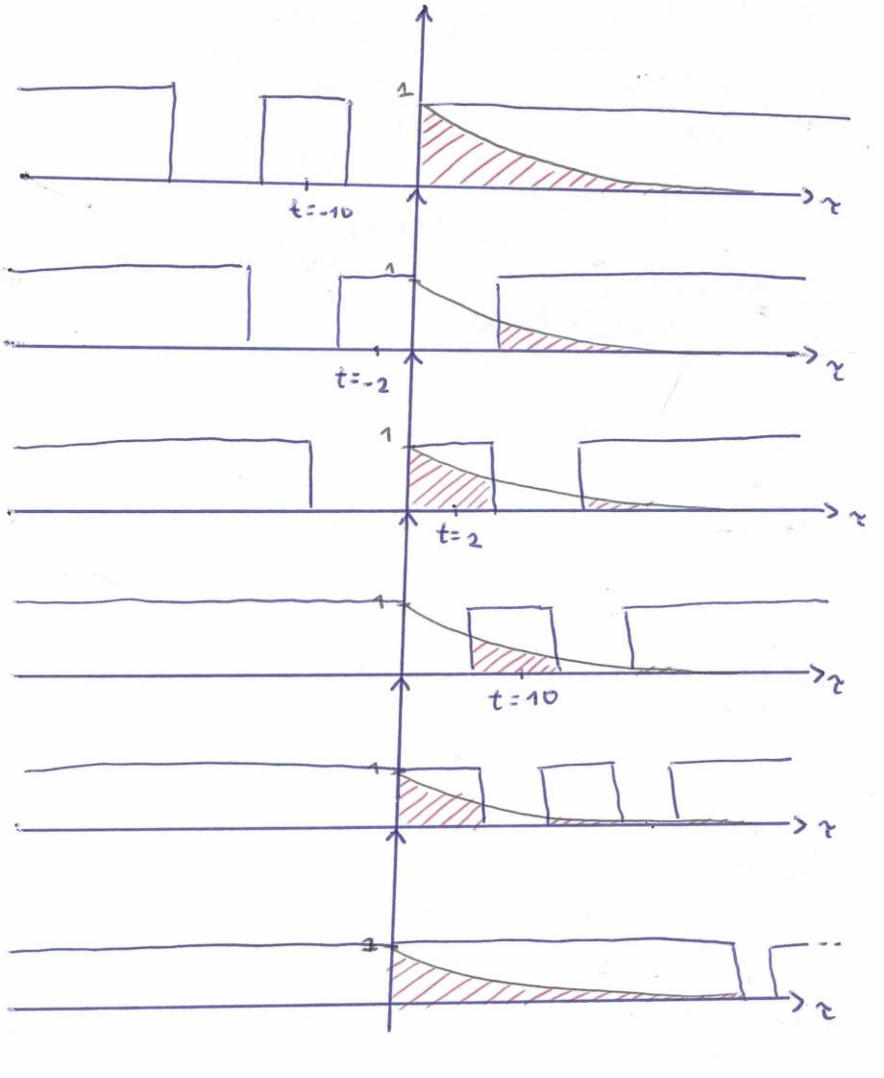
$$x(t) = e^{-t} u(t)$$

$$y(t) = u(-t-10) + rect\left(\frac{t}{4}\right) + u(t-10)$$

$$z(t) = x(t) * y(t)$$



$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$



$t \le -10 \rightarrow z(t) = \int_{\emptyset}^{+\infty} e^{-\tau} d\tau = \left[-e^{-\tau} \right]_{\emptyset}^{+\infty} = 1$

$-10 < t \le -2 \rightarrow z(t) = \int_{t+10}^{+\infty} e^{-\tau} d\tau = \left[-e^{-\tau} \right]_{t+10}^{+\infty} = e^{-(t+10)}$

$-2 < t \le 2 \rightarrow z(t) = \int_{\emptyset}^{t+2} e^{-\tau} d\tau + \int_{t+10}^{+\infty} e^{-\tau} d\tau = \left[-e^{-\tau} \right]_{\emptyset}^{t+2} + e^{-(t+10)}$

$= -e^{-(t+2)} + e^{-(t+10)} + 1$

$2 < t \le 10 \rightarrow z(t) = \int_{t-2}^{t+2} e^{-\tau} d\tau + \int_{t+10}^{+\infty} e^{-\tau} d\tau = \left[-e^{-\tau} \right]_{t-2}^{t+2} + e^{-(t+10)}$

$= -e^{-(t+2)} + e^{-(t-2)} + e^{-(t+10)} \quad \cancel{= -e^{-(t+2)} + e^{-(2t+8)}}$

$t > 10 \rightarrow z(t) = \int_{\emptyset}^{t-10} e^{-\tau} d\tau + \int_{t-2}^{t+2} e^{-\tau} d\tau + \int_{t+10}^{+\infty} e^{-\tau} d\tau$

$= \left[-e^{-\tau} \right]_{\emptyset}^{t-10} - e^{-(t+2)} + e^{-(t-2)} + e^{-(t+10)}$

$= -e^{-(t-10)} - e^{-(t+2)} + e^{-(t-2)} + e^{-(t+10)} + 1$

$\cancel{= -e^{-(2t-8)} + e^{-(2t+8)}}$

6/05/14 PROVA A

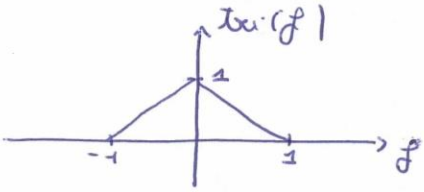
ES. 3

$s(t) = \mathrm{sinc}^2(t-1)$

$E_s = \int_{-\infty}^{+\infty} |s(t)|^2 \, dt = \int_{-\infty}^{+\infty} |S(f)|^2 df$

$S(f) = \mathrm{tri}(f) \cdot e^{-j2\pi f}$

$|S(f)| = \mathrm{tri}(f)$.



$E_s = \int_{-\infty}^{+\infty} \mathrm{tri}^2(f) \, df = \int_{-1}^{\emptyset} (1+f)^2 df + \int_{\emptyset}^{1} (1-f)^2 df$

$= \int_{-1}^{\emptyset} 1 + f^2 + 2f \, df + \int_{\emptyset}^{1} 1 + f^2 - 2f \, df = \left[\frac{f^3}{3} + f^2 + f\right]_{-1}^{\emptyset} + \left[\frac{f^3}{3} - f^2 + f\right]_{\emptyset}^{1}$

$= +\frac{1}{3} + 1 - 1 + \frac{1}{3} - 1 + 1 = \frac{2}{3}$

6/05/14 PROVA B

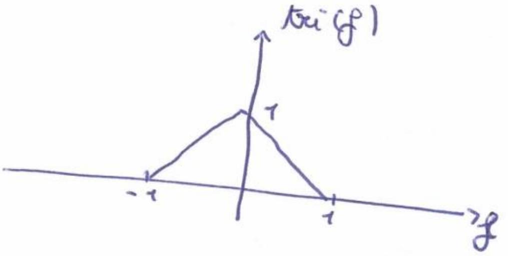
Es. 3

$s(t) = \text{sinc}^2(t+2)$

$$E_s = \int_{-\infty}^{+\infty} |s(t)|^2 \, dt = \int_{-\infty}^{+\infty} |S(f)|^2 \, df$$

$S(f) = \text{tri}(f)\, e^{+j4\pi f}$

$|S(f)| = \text{tri}(f)$



$$E_s = \int_{-\infty}^{+\infty} \text{tri}^2(f) = \int_{-1}^{0} (1+f)^2 \, df + \int_{0}^{1} (1-f)^2 \, df$$

$$= \int_{-1}^{0} \left[1 + f^2 + 2f \right] df + \int_{0}^{1} \left[1 + f^2 - 2f \right] df$$

$$= \left[\frac{f^3}{3} + f^2 + f \right]_{-1}^{0} + \left[\frac{f^3}{3} - f^2 + f \right]_{0}^{1}$$

$$= -\left(-\frac{1}{3} + 1 - 1 + \frac{1}{3} \right) - 1 + 1 = \frac{2}{3}$$

25/06/14

ES. 3

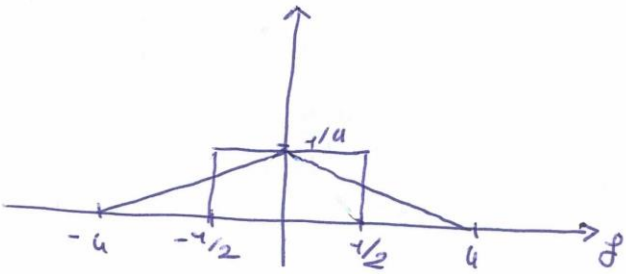
$$\int_{-\infty}^{+\infty} \operatorname{sinc}^2(4t-8)\,\operatorname{sinc}(t-2)\,dt$$

PARSEVAL GENERALIZZATO:

$$\int_{-\infty}^{+\infty} x(t)\,y^*(t)\,dt = \int_{-\infty}^{+\infty} X(f)\,Y^*(f)\,df$$

$$x(t) = \operatorname{sinc}^2(4(t-2))$$

$$X(f) = \frac{1}{4}\operatorname{tri}\left(\frac{f}{4}\right)\cdot e^{-j4\pi f}$$



$$y(t) = \operatorname{sinc}(t-2)$$

$$Y(f) = \operatorname{rect}(f)\cdot e^{-j4\pi f}$$

$$Y^*(f) = \operatorname{rect}(f)\cdot e^{j4\pi f}$$

$$\rightarrow \int_{-\infty}^{+\infty} \frac{1}{4}\operatorname{tri}\left(\frac{f}{4}\right)\cdot \cancel{e^{-j4\pi f}}\cdot \operatorname{rect}(f)\cdot \cancel{e^{j4\pi f}}\,df$$

$$= \int_{-\infty}^{+\infty} \frac{1}{4}\operatorname{tri}\left(\frac{f}{4}\right)\cdot \operatorname{rect}(f)\,df = \int_{-\frac{1}{2}}^{\frac{1}{2}} \frac{1}{4}\operatorname{tri}\left(\frac{f}{4}\right)df = \frac{15}{64}.$$

$$= \frac{1}{4}\left\{\int_{-1}^{0}\left(1+\frac{1}{4}f\right)(1+f)\,df + \int_{0}^{1}\left(1-\frac{1}{4}f\right)(1-f)\,df\right\}$$

$$= \frac{1}{4}\left\{\int_{-1}^{0} 1 + f + \frac{1}{4}f + \frac{1}{4}f^2\,df + \int_{0}^{1} 1 - f - \frac{1}{4}f + \frac{1}{4}f^2\,df\right\}$$

$$= \frac{1}{4}\left\{\left[f + \frac{5}{4}\cdot\frac{f^2}{2} + \frac{1}{4}\cdot\frac{f^3}{3}\right]_{-1}^{0} + \left[f - \frac{5}{4}\cdot\frac{f^2}{2} + \frac{1}{4}\cdot\frac{f^3}{3}\right]_{0}^{1}\right\}$$

$$= \frac{1}{4}\left\{-1 + \frac{5}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{-1}{3} + 1 - \frac{5}{4}\cdot\frac{1}{2} + \frac{1}{4}\cdot\frac{1}{3}\right\} = 0$$

15/05/13

ES. 3

$$\int_{-\infty}^{+\infty} \mathrm{sinc}^4(4t)\, e^{j8\pi t}\, dt = \int_{-\infty}^{+\infty} \mathrm{sinc}^2(4t)\, \mathrm{sinc}^2(4t)\, e^{j8\pi t}$$

$$\int_{-\infty}^{+\infty} x(t)\, y^*(t) = \int_{-\infty}^{+\infty} X(f)\, Y^*(f)$$

$$x(t) = \mathrm{sinc}^2(4t)\, e^{j8\pi t}$$

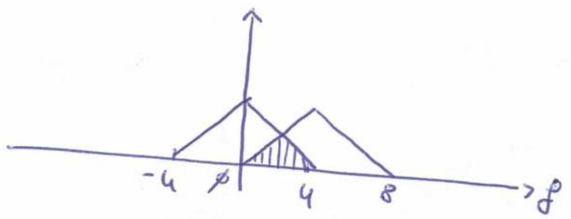
$$\mathrm{sinc}^2(4t) \longleftrightarrow \frac{1}{4}\, \mathrm{tri}\left(\frac{f}{4}\right)$$

$$X(f) = \frac{1}{4}\, \mathrm{tri}\left(\frac{f-4}{4}\right)$$

$$y(t) = \mathrm{sinc}^2(4t)$$

$$Y(f) = \frac{1}{4}\, \mathrm{tri}\left(\frac{f}{4}\right) = Y^*(f)$$

$$\int_{-\infty}^{+\infty} \frac{1}{4}\, \mathrm{tri}\left(\frac{f-4}{4}\right) \cdot \frac{1}{4}\, \mathrm{tri}\left(\frac{f}{4}\right) df = \frac{1}{16} \int_{-\infty}^{+\infty} \mathrm{tri}\left(\frac{f-4}{4}\right) \cdot \mathrm{tri}\left(\frac{f}{4}\right) df$$



$$= \int_{0}^{4} \frac{1}{4} f \cdot \left(-\frac{1}{4} f + 1\right) df = \int_{0}^{4} -\frac{1}{16} f^2 + \frac{1}{4} f\, df = \left[-\frac{1}{16} \cdot \frac{f^3}{3} + \frac{1}{4} \cdot \frac{f^2}{2}\right]_{0}^{4}$$

$$= \frac{2}{3} \cdot \frac{1}{16} = \frac{1}{24}$$

6/05/14 PROVA A

ES. 4

$s(t) = 3\,\mathrm{sinc}^2(3t)\cos(6\pi t)$

$h(t) = 6\,\mathrm{sinc}(3t)$

$y(t) = \int_{-\infty}^{+\infty} h(t-\tau)\,s(\tau)\,d\tau \quad , \quad H(f)\cdot S(f) = Y(f)$

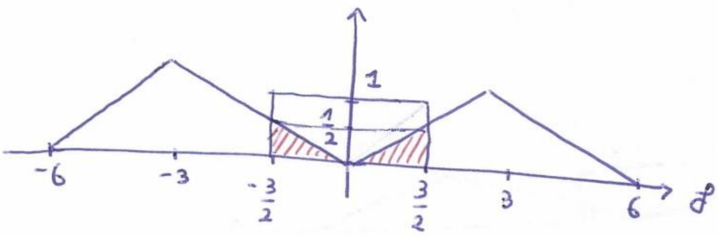
$H(f) = \frac{6}{3}\,\mathrm{rect}\left(\frac{f}{3}\right) = 2\,\mathrm{rect}\left(\frac{f}{3}\right)$

$3\,\mathrm{sinc}^2(3t) \longleftrightarrow \frac{3}{3}\,\mathrm{tri}\left(\frac{f}{3}\right) = \mathrm{tri}\left(\frac{f}{3}\right)$

$S(f) = \frac{1}{2}\left[\mathrm{tri}\left(\frac{f-3}{3}\right) + \mathrm{tri}\left(\frac{f+3}{3}\right)\right]$

$Y(f) = \cancel{2}\,\mathrm{rect}\left(\frac{f}{3}\right)\cdot\cancel{\frac{1}{2}}\left[\mathrm{tri}\left(\frac{f-3}{3}\right) + \mathrm{tri}\left(\frac{f+3}{3}\right)\right]$

$= \mathrm{rect}\left(\frac{f}{3}\right)\cdot\left[\mathrm{tri}\left(\frac{f-3}{3}\right) + \mathrm{tri}\left(\frac{f+3}{3}\right)\right]$



$= \frac{1}{2}\left\{\mathrm{rect}\left(\frac{f}{3}\right) - \mathrm{tri}\left(\frac{f}{\frac{3}{2}}\right)\right\} = \frac{1}{2}\left\{\mathrm{rect}\left(\frac{f}{3}\right) - \mathrm{tri}\left(\frac{2}{3}f\right)\right\}$

$y(t) = \frac{1}{2}\cdot 3\,\mathrm{sinc}(3t) - \frac{1}{2}\cdot\frac{3}{2}\cdot\mathrm{sinc}^2\left(\frac{3}{2}f\right)$

$= \frac{3}{2}\,\mathrm{sinc}(3t) - \frac{3}{4}\,\mathrm{sinc}^2\left(\frac{3}{2}t\right).$

6/05/14 PROVA B

ES. 4

$s(t) = 4\,\mathrm{sinc}^2(4t)\cdot\cos(8\pi t)$

$h(t) = 8\,\mathrm{sinc}(2t)$

$y(t) = \int_{-\infty}^{+\infty} h(t-\tau)\,s(\tau)\,d\tau \quad , \quad Y(f) = H(f)\cdot S(f)$

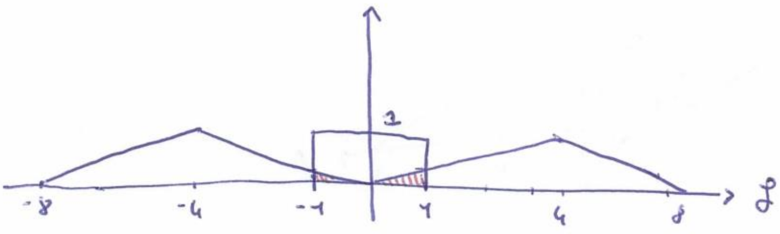
$H(f) = \frac{8}{2}\,\mathrm{rect}\left(\frac{f}{2}\right) = 4\,\mathrm{rect}\left(\frac{f}{2}\right)$

$4\,\mathrm{sinc}^2(4t) \longleftrightarrow \frac{\cancel{4}}{\cancel{4}}\,\mathrm{tri}\left(\frac{f}{4}\right) = \mathrm{tri}\left(\frac{f}{4}\right)$

$S(f) = \frac{1}{2}\cdot\left[\mathrm{tri}\left(\frac{f-4}{4}\right) + \mathrm{tri}\left(\frac{f+4}{4}\right)\right]$

$Y(f) = 2\cancel{4}\,\mathrm{rect}\left(\frac{f}{2}\right)\cdot\frac{1}{\cancel{2}}\left[\mathrm{tri}\left(\frac{f-4}{4}\right) + \mathrm{tri}\left(\frac{f+4}{4}\right)\right]$

$= 2\,\mathrm{rect}\left(\frac{f}{2}\right)\cdot\left[\mathrm{tri}\left(\frac{f-4}{4}\right) + \mathrm{tri}\left(\frac{f+4}{4}\right)\right]$



$= 2\cdot\left\{\frac{1}{4}\,\mathrm{rect}\left(\frac{f}{2}\right) - \frac{1}{4}\,\mathrm{tri}(f)\right\} = \frac{1}{2}\left\{\mathrm{rect}\left(\frac{f}{2}\right) - \mathrm{tri}(f)\right\}$

$y(t) = \frac{1}{2}\cdot 2\cdot\mathrm{sinc}(2t) - \frac{1}{2}\cdot\mathrm{sinc}^2(t) = \mathrm{sinc}(2t) - \frac{1}{2}\mathrm{sinc}^2(t).$

25/06/14

ES. 4

$s(t) = \mathrm{rect}\left(\frac{t-2}{4}\right)$

$h(t) = 2 \cdot \mathrm{rect}\left(\frac{t}{4}\right)$

$y(t) = \int_{-\infty}^{+\infty} h(t-\tau)\cdot s(\tau)\, d\tau \,, \quad Y(f) = H(f)\cdot S(f)$

$H(f) = 2 \cdot 4\,\mathrm{sinc}(4f) = 8\,\mathrm{sinc}(4f)$

$S(f) = 4\,\mathrm{sinc}(4f) \cdot e^{-j4\pi f}$

$Y(f) = 32\,\mathrm{sinc}(4f)\cdot \mathrm{sinc}(4f) \cdot e^{-j4\pi f}$

$\quad = 32 \cdot \mathrm{sinc}^2(4f) \cdot e^{-j4\pi f}$

$y(t) = 32 \cdot \frac{1}{4}\,\mathrm{tri}\left(\frac{t-2}{4}\right) = 8\,\mathrm{tri}\left(\frac{t-2}{4}\right)$

$E_s = \int_{-\infty}^{+\infty} |s(t)|^2 dt = \int_{-\infty}^{+\infty} 64\,\mathrm{tri}^2\left(\frac{t-2}{4}\right) dt$

$= 64 \left\{ \int_{-2}^{2} \left(\frac{1}{2}t\right)^2 dt + \int_{2}^{6} \left(-\frac{1}{2}t + 2\right)^2 dt \right\} = 64 \left\{ \int_{-2}^{2} \frac{1}{4}t^2 dt + \int_{2}^{6} \frac{1}{4}t^2 + 4 - 2t\, dt \right\} = \frac{512}{3}$

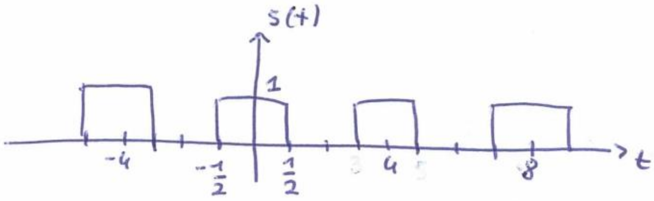
$\cancel{= 64 \left\{ \left[\frac{1}{4}\cdot\frac{t^3}{3}\right]_0^2 + \left[\frac{1}{4}\,\frac{t^3}{3} - 2\cdot\frac{t^2}{2} + 4t\right]_2^4 \right\}}$

$\cancel{= 64 \left\{ \frac{1}{4}\cdot\frac{8}{3} + \frac{1}{4}\cdot\frac{64}{3} - 2\cdot\frac{16}{2} + 4\cdot 4 - \frac{1}{4}\cdot\frac{8}{3} + 2\cdot\frac{4}{2} - 8 \right\} = \frac{256}{3}}$

15/05/13

ES. 4

$$s(t) = \sum_{m=-\infty}^{+\infty} \mathrm{rect}(t - 4m)$$



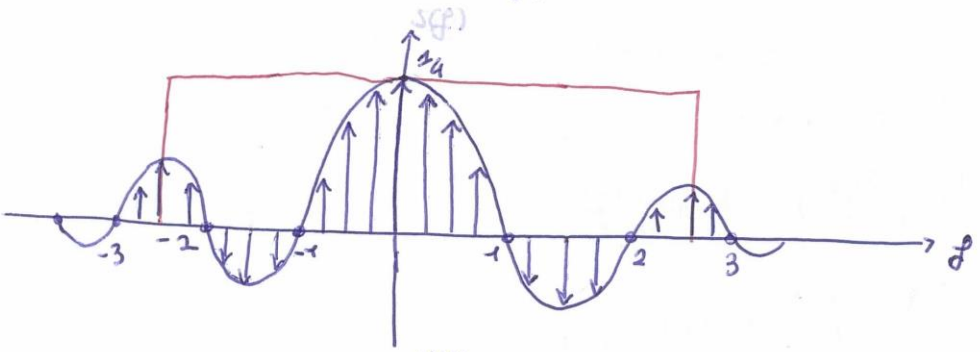
$S_T = \mathrm{rect}(t)$

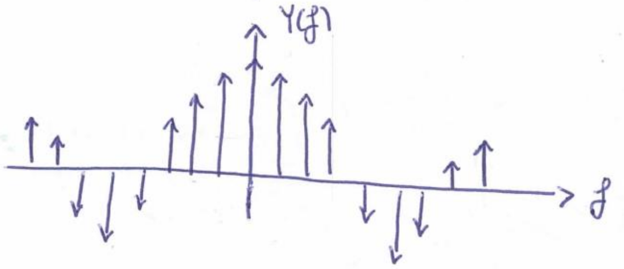
$$h(t) = \mathrm{sinc}(5t)$$

$$y(t) = \int_{-\infty}^{+\infty} h(t-\tau) \cdot s(\tau)\, d\tau \quad , \quad Y(f) = H(f) \cdot S(f)$$

$$H(f) = \frac{1}{5} \mathrm{rect}\left(\frac{f}{5}\right)$$

$$S(f) = \frac{1}{4} \cdot \sum_{m=-\infty}^{+\infty} \mathrm{sinc}\left(\frac{m}{4}\right) \cdot \delta\left(f - \frac{m}{4}\right)$$





$$y(t) = \frac{1}{20} + \frac{1}{10} \cdot \mathrm{sinc}\left(\frac{1}{4}\right) \cdot \cos\left(2\pi \frac{t}{4}\right) + \frac{1}{10} \cdot \mathrm{sinc}\left(\frac{2}{4}\right) \cdot \cos\left(2\pi \frac{2}{4} t\right)$$

$$+ \frac{1}{10} \cdot \mathrm{sinc}\left(\frac{3}{4}\right) \cdot \cos\left(2\pi \frac{3}{4} t\right) + \frac{1}{10} \cdot \mathrm{sinc}\left(\frac{5}{4}\right) \cdot \cos\left(2\pi \frac{5}{4} t\right)$$

$$+ \frac{1}{10} \cdot \mathrm{sinc}\left(\frac{6}{4}\right) \cdot \cos\left(2\pi \frac{6}{4} t\right) + \frac{1}{10} \cdot \mathrm{sinc}\left(\frac{7}{4}\right) \cdot \cos\left(2\pi \frac{7}{4} t\right)$$

$$+ \frac{1}{10} \cdot \mathrm{sinc}\left(\frac{9}{4}\right) \cdot \cos\left(2\pi \frac{9}{4} t\right) + \frac{1}{10} \cdot \mathrm{sinc}\left(\frac{10}{4}\right) \cdot \cos\left(2\pi \frac{10}{4} t\right).$$

6/05/14 PROVA A

ES. 5

$s(t) = \mathrm{sinc}(8Bt)$

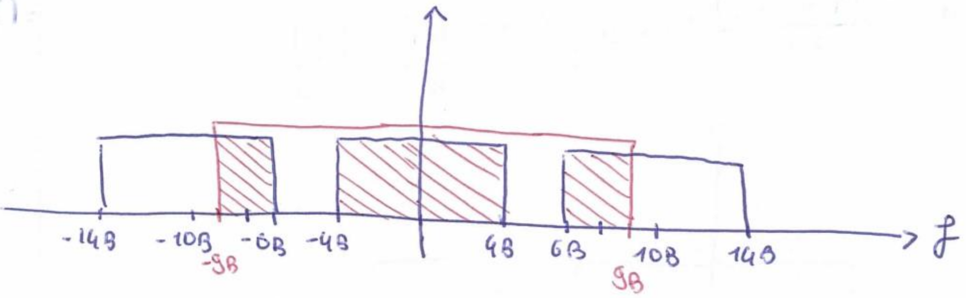
$f_c = 10B$

$F(f) = \mathrm{rect}\left(\frac{f}{18B}\right)$

$$S(f) = \frac{1}{8B} \cdot \mathrm{rect}\left(\frac{f}{8B}\right)$$

$$S_p(f) = 10B \cdot \sum_{n=-\infty}^{+\infty} \frac{1}{8B} \cdot \mathrm{rect}\left(\frac{f - n10B}{8B}\right)$$

$$= \frac{5}{4} \cdot \sum_{n=-\infty}^{+\infty} \mathrm{rect}\left(\frac{f - n10B}{8B}\right)$$



$$Y(f) = \frac{5}{4} \left\{ \mathrm{rect}\left(\frac{f}{8B}\right) + \mathrm{rect}\left(\frac{f + \frac{15}{2}B}{3B}\right) + \mathrm{rect}\left(\frac{f - \frac{15}{2}B}{3B}\right) \right\}$$

$$y(t) = \frac{5}{4} \cdot \left\{ 8B\,\mathrm{sinc}(8Bt) + \cos\left(2\pi \frac{15}{2} tB\right) \cdot 3B\,\mathrm{sinc}(3Bt) \right\}$$

6/05/14 PROVA B

ES. 5

$s(t) = \mathrm{sinc}(2Bt) \cdot \cos(2\pi Bt)$

$f_c = 6B$

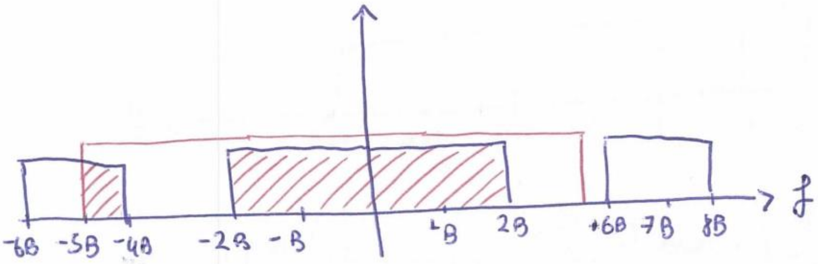
$F(f) = \mathrm{rect}\left(\frac{f}{10B}\right)$

$$S(f) = \frac{1}{2} \cdot \left[\frac{1}{2B} \cdot \mathrm{rect}\left(\frac{f-B}{2B}\right) + \frac{1}{2B} \cdot \mathrm{rect}\left(\frac{f+B}{2B}\right)\right]$$

$$= \frac{1}{4B} \cdot \left[\mathrm{rect}\left(\frac{f-B}{2B}\right) + \mathrm{rect}\left(\frac{f+B}{2B}\right)\right].$$

$$S_p(f) = 6B \cdot \sum_{m=-\infty}^{+\infty} \frac{1}{4B} \cdot \left[\mathrm{rect}\left(\frac{f - m6B - B}{2B}\right) + \mathrm{rect}\left(\frac{f - m6B + B}{2B}\right)\right].$$

$$= \frac{3}{2} \cdot \sum_{m=-\infty}^{+\infty} \left[\mathrm{rect}\left(\frac{f - B(m \cdot 6 + 1)}{2B}\right) + \mathrm{rect}\left(\frac{f + B(-m \cdot 6 + 1)}{2B}\right)\right].$$



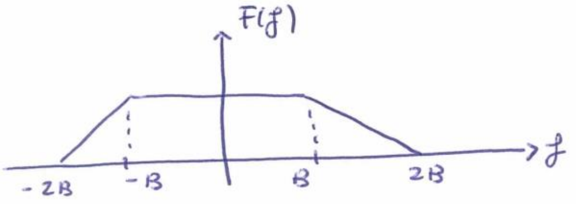
$$Y(f) = \frac{3}{2} \cdot \left\{\mathrm{rect}\left(\frac{f + \frac{9}{2}}{B}\right) + \mathrm{rect}\left(\frac{f}{4B}\right)\right\}$$

$$y(t) = \frac{3}{2} \cdot \left\{B\,\mathrm{sinc}(Bt) \cdot e^{-j2\pi \frac{9}{2} t} + 4B\,\mathrm{sinc}(4Bt)\right\}$$

25/06/14

ES. 5

$s(t) = \text{sinc}^2(100t) \cdot \cos(140\pi t)$



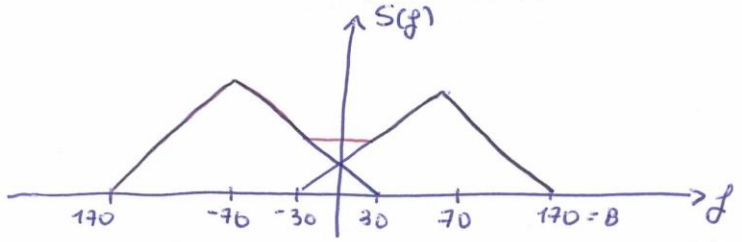
$n_{bit/c} = 10$ bit

$B = ?$

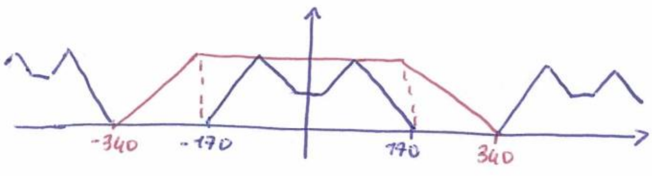
$f_c = ?$

$n_{bit/2R} = ?$

$$S(f) = \frac{1}{200} \cdot \left[tri\left(\frac{f-70}{100}\right) + tri\left(\frac{f+70}{100}\right) \right].$$



$B = 170$ Hz



$f_c = 510$ Hz

$n_{c/1s} = 510$

$n_{bit/2R} = 510 \cdot 2 \cdot 3600 \cdot 10 = 36.720.000$ bit

15/05/13

ES. 5

$S(t) = \text{sinc}(500t) \cdot \cos(500\pi t)$

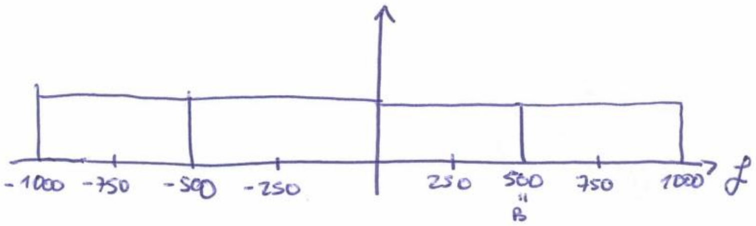
$f_c = 2B$

$n_{bit/c} = 15$ bit

In 2 secondi vengono trasmessi sulla linea 20 min di segnale

$V_T = ?$

$$S(f) = \frac{1}{1000} \cdot \left[\text{rect}\left(\frac{f-250}{500}\right) + \text{rect}\left(\frac{f+250}{500}\right) \right]$$



$B = 500$ Hz $\rightarrow f_c = 1000$ Hz.

$n_{c/1s} = 1000$

$n_{c/20min} = 1000 \cdot 20 \cdot 60 = 1.200.000$

$n_{bit/20\,min} = n_{c/20min} \cdot n_{bit/c} = 18.000.000$ bit

$V_T = \frac{n_{bit/20min}}{2s} = 9.000.000 = 9$ Mbit/s

6/05/16 PROVA A

ES. 6

$f_A(a) = \frac{1}{8}\,\mathrm{rect}\left(\frac{a-2}{4}\right) + \frac{1}{4}\,\mathrm{tri}\left(\frac{a-2}{2}\right)$

$E = \{A \geq 3\}$

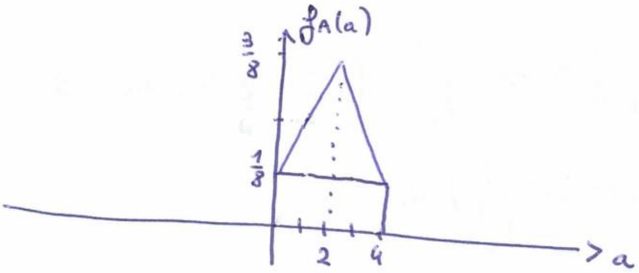
$E(a)$?

$\sigma^2(a)$?

$P(E)$?

$P(E) = \int_3^4 f_A(a)\,da = \int_3^4 -\frac{1}{8}a + \frac{5}{8}\,da = \frac{3}{16}$.

$E(a) = \int_{-\infty}^{+\infty} a\, f_A(a)\, da$



$= \int_{\emptyset}^{4} a\, f_A(a)\,da = \int_{\emptyset}^{2} a\cdot\left(\frac{1}{8}a + \frac{1}{8}\right)da + \int_2^4 a\cdot\left(-\frac{1}{8}a + \frac{5}{8}\right)da$

$= \left[\frac{1}{8}\cdot\frac{a^3}{3} + \frac{1}{8}\cdot\frac{a^2}{2}\right]_{\emptyset}^{2} + \left[-\frac{1}{8}\cdot\frac{a^3}{3} + \frac{5}{8}\cdot\frac{a^2}{2}\right]_2^4$

$= \frac{1}{\cancel{8}}\cdot\frac{\cancel{8}}{3} + \frac{1}{2\cancel{8}}\cdot\frac{\cancel{4}}{2} + \left[-\frac{1}{\cancel{8}}\cdot\frac{\cancel{64}^{8}}{3} + \frac{5}{\cancel{8}_1}\cdot\frac{\cancel{16}^{2}}{\cancel{2}} - \left(-\frac{1}{\cancel{8}}\cdot\frac{8}{3} + \frac{5}{\cancel{8}_2}\cdot\frac{\cancel{4}^{1}}{\cancel{2}}\right)\right] = \frac{1}{3} + \frac{1}{4} - \frac{8}{3} + 5 + \frac{1}{3} - \frac{5}{4} = 2$

$E(a^2) = \int_{-\infty}^{+\infty} a^2 f_A(a)\,da = \int_{\emptyset}^{2} a^2\cdot\left(\frac{1}{8}a + \frac{1}{8}\right)da + \int_2^4 a^2\cdot\left(-\frac{1}{8}a + \frac{5}{8}\right)da$

$= \left[\frac{1}{8}\cdot\frac{a^4}{4} + \frac{1}{8}\cdot\frac{a^3}{3}\right]_{\emptyset}^{2} + \left[-\frac{1}{8}\cdot\frac{a^4}{4} + \frac{5}{8}\cdot\frac{a^3}{3}\right]_2^4$

$= \frac{1}{\cancel{8}}\cdot\frac{\cancel{16}^{2^{1}}}{\cancel{4}_2} + \frac{1}{\cancel{8}}\cdot\frac{\cancel{8}}{3} + -\frac{1}{\cancel{8}}\cdot\frac{\cancel{256}^{\cancel{32}^{8}}}{\cancel{4}_1} + \frac{5}{\cancel{8}_1}\cdot\frac{\cancel{64}^{8}}{3} - \left(-\frac{1}{\cancel{8}_1}\cdot\frac{\cancel{16}^{2^{1}}}{\cancel{4}_2} + \frac{5}{\cancel{8}}\cdot\frac{\cancel{8}}{3}\right)$

$= \frac{1}{2} + \frac{1}{3} - 8 + \frac{40}{3} + \frac{1}{2} - \frac{5}{3} = 5$

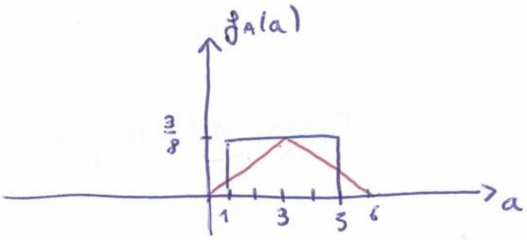
$\sigma^2(a) = E(a^2) - E(a)^2 = 1$.

6/05/14 PROVA B

ES. 6

$f_A(a) = \frac{3}{8}\,\text{rect}\left(\frac{a-3}{4}\right)\text{tri}\left(\frac{a-3}{3}\right)$ $E(a)?$ $P(E)?$ $\sigma^2(a)?$

$E = \{A \geq 3\}$



$$E(a) = \int_{-\infty}^{+\infty} a\, f_A(a)\, da = \int_{1}^{3} a \cdot \frac{1}{8} \cdot a\, da + \int_{3}^{5} a \cdot \left(-\frac{1}{8} \cdot a + \frac{6}{8}\right) da = 3$$

$$E(a^2) = \int_1^3 a^2 \cdot \frac{1}{8} a\, da + \int_3^5 a^2 \cdot \left(-\frac{1}{8} a + \frac{6}{8}\right) da$$

$$= \left[\frac{1}{8} \cdot \frac{a^4}{4}\right]_1^3 + \left[-\frac{1}{8} \cdot \frac{a^4}{4} + \frac{6}{8} \cdot \frac{a^3}{3}\right]_3^5$$

$$= \frac{1}{8} \cdot \frac{81}{4} - \frac{1}{8} \cdot \frac{1}{4} + -\frac{1}{8} \cdot \frac{625}{5} + \frac{6}{8} \cdot \frac{125}{3} - \left(-\frac{1}{8} \cdot \frac{81}{4} + \frac{6}{8} \cdot \frac{27}{3}\right) = 10$$

$$\sigma^2(a) = E(a^2) - E(a)^2 = 10 - 9 = 1$$

$$P(E) = \int_3^5 -\frac{1}{8} a + \frac{6}{8}\, da = \frac{1}{2}$$

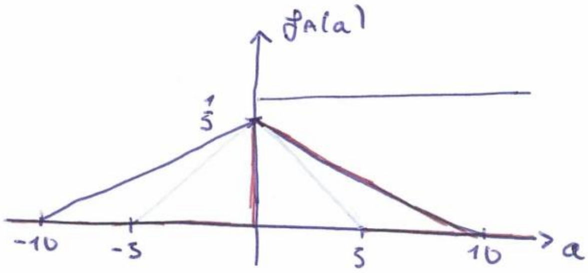
15/05/13

ES. 6

$f_A(a) = \frac{1}{5}\, tri\left(\frac{a}{10}\right) u(a)$ 	$E(a)$? 	$P(E)$?

$\sigma^2(a)$?

$E = \{A \geq 5\}$



$$E(a) = \int_{-\infty}^{+\infty} a \cdot f_A(a)\, da = \int_{\emptyset}^{10} a \cdot \left(-\frac{1}{50} a + \frac{1}{5}\right) da = \left[-\frac{1}{50} \cdot \frac{a^3}{3} + \frac{1}{5} \cdot \frac{a^2}{2}\right]_{\emptyset}^{5} = \frac{10}{3}$$

$$E(a^2) = \int_{\emptyset}^{10} a^2 \left(-\frac{1}{50} a + \frac{1}{5}\right) da = \left[-\frac{1}{50} \cdot \frac{a^4}{4} + \frac{1}{5} \cdot \frac{a^3}{3}\right]_{\emptyset}^{10} = \frac{50}{3}$$

$$\sigma^2(a) = E(a^2) - E(a)^2 = \frac{50}{9}$$

$$P(E) = \int_{5}^{10} -\frac{1}{50} \cdot a + \frac{1}{5}\, da = \frac{1}{4}.$$

25/06/14

ES. 6

A, B v.a. indip.

$f_A(a) = \frac{1}{8} \cdot \text{rect}\left(\frac{a-3}{8}\right)$, $f_B(b) = \frac{1}{8} \text{tri}\left(\frac{b-3}{8}\right)$

$\Psi_{NN}(f) = \frac{N_0}{2}$, $\mu_N = \emptyset$

$y(k,t) = (A-B) n(k,t)$, $\Psi_{YY}(f)$.

$R_{YY}(t_1,t_2) = E[y(k,t_1) \cdot y(k,t_2)] = E[(A-B) n(k,t_1) \cdot (A-B) n(k,t_2)]$

$= \left\{E[A] \cdot E[n(k,t_1)] - E[B] \cdot E[n(k,t_1)]\right\} \cdot \left\{E[A] \cdot E[n(k,t_2)] - E[B] \cdot E[n(k,t_2)]\right\}$

$= E[(A-B)^2] \cdot E[n(k,t_1) n(k,t_2)]$

$= \left\{E[A^2] + E[B^2] - 2E[A]E[B]\right\} E[n(k,t_1) n(k,t_2)]$

$E[A] = 3$

$E[B] = 3$

$\rightarrow \left\{A_1 + B_1 - 18\right\} \frac{N_0}{2} \cdot \delta(\tau)$

$\Psi_{YY}(f) = \left\{A_1 \delta(f) + B_1 \delta(f) - 18 \delta(f)\right\} \frac{N_0}{2}$

6/05/14 PROVA A

ES. 7

$X(k,t) = A\cos(2\pi f_1 t - 2\theta) + B\cos(2\pi f_2 t - 2\theta)$

A, B v.a. indip.

$f_A(a) = \frac{1}{4}\,\mathrm{rect}\left(\frac{a}{4}\right)$, $f_B(b) = \frac{1}{8}\,\mathrm{rect}\left(\frac{b}{8}\right)$

θ unif. distr. fra $\emptyset \div 4\pi$

$E(A) = \emptyset$

$E(B) = \emptyset$

Studiare SSL e $\Psi_{xx}(f)$.

SSL :

$$\begin{cases} \mu_x = \text{costante} \\ R_{xx}(t_1,t_2) = R_{xx}(t_1 - t_2) = R_{xx}(\tau). \end{cases}$$

$\mu_x = E[x(k,t)] = E[A\cdot\cos(2\pi f_1 t - 2\theta) + B\cos(2\pi f_2 t - 2\theta)]$

$= E[A]\cdot E[\cos(2\pi f_1 t - 2\theta)] + E[B]\cdot E[\cos(2\pi f_2 t - 2\theta)] = \emptyset$

$R_{xx}(t_1,t_2) = E[x(k,t_1)\cdot x(k,t_2)]$

$= E[(A\cdot\cos(2\pi f_1 t_1 - 2\theta) + B\cos(2\pi f_2 t_1 - 2\theta))\cdot$

$(A\cdot\cos(2\pi f_1 t_2 - 2\theta) + B\cos(2\pi f_2 t_2 - 2\theta))]$

$= E[A^2\cdot\cos(2\pi f_1 t_1 - 2\theta)\cdot\cos(2\pi f_1 t_2 - 2\theta) + \cancel{AB\cos(2\pi f_1 t_1 - 2\theta)\cos(2\pi f_2 t_2 - 2\theta)} \to \emptyset$

$+ \cancel{AB\cos(2\pi f_2 t_1 - 2\theta)\cdot\cos(2\pi f_1 t_2 - 2\theta)} \to \emptyset + B^2\cos(2\pi f_2 t_1 - 2\theta)\cos(2\pi f_2 t_2 - 2\theta)]$

$= E[A^2]\cdot E[\cancel{\frac{1}{2}\cos(2\pi f_1(t_1+t_2) - 4\theta)} \to \emptyset + \frac{1}{2}\cos(2\pi f_1(t_1 - t_2))] +$

$$+E[B^2]\cdot E\left[\frac{1}{2}\cos(2\pi f_2(\cancel{t_1+t_2})-4\theta)+\frac{1}{2}\cos(2\pi f_2(t_1-t_2))\right]$$

$$=E[A^2]\cdot E\left[\frac{1}{2}\cos(2\pi f_1(t_1-t_2))\right]+E[B^2]\cdot E\left[\frac{1}{2}\cos(2\pi f_2(t_1-t_2))\right].$$

$$=\frac{4}{3}\cdot\frac{1}{2}\cos(2\pi f_1\tau)+\frac{16}{3}\cdot\frac{1}{2}\cos(2\pi f_2\tau)=\frac{2}{3}\cos(2\pi f_1\tau)+\frac{8}{3}\cos(2\pi f_2\tau).$$

$x(k,t)$ è SSL!

$$\varphi_{xx}(f)=\frac{2}{3}\cdot\frac{1}{2}\left\{\delta(f-f_1)+\delta(f+f_1)\right\}+\frac{8}{3}\cdot\frac{1}{2}\left\{\delta(f+f_2)+\delta(f-f_2)\right\}$$

$$=\frac{1}{3}\left\{\delta(f-f_1)+\delta(f+f_1)\right\}+\frac{4}{3}\left\{\delta(f-f_2)+\delta(f+f_2)\right\}.$$

6/05/14 PROVA B

ES. 7

$$x(k,t) = 3A\cdot\cos(2\pi f_1 t - 2\theta) + 2B\cdot\cos(2\pi f_2 t - 2\theta)$$

A, B v.a. indip.

$$f_A(a) = \frac{1}{2}\,\mathrm{rect}\left(\frac{a}{2}\right), \quad f_B(b) = \frac{1}{3}\,\mathrm{rect}\left(\frac{b}{3}\right)$$

$E[A] = \emptyset$

$E[B] = \emptyset$

$E[A^2] = 1/3$

$E[B^2] = 3/4$

θ unif. distr. in $\emptyset \div 2\pi$.

SSL? $\varphi_{xx}(f)$?

μ_x = costante

$$\mu_x = E[3A\cdot\cos(2\pi f_1 t - 2\theta) + 2B\cdot\cos(2\pi f_2 t - 2\theta)]$$

$$= 3\cdot E[A]\cdot E[\cancel{\cos(2\pi f_1 t - 2\theta)}] + 2\cdot E[B]\cdot E[\cancel{\cos(2\pi f_2 t - 2\theta)}] = \emptyset$$

$$R_{xx}(\tau) = R_{xx}(t_1 - t_2) = E[x(k,t_1)\cdot x(k,t_2)]$$

$$= E[(3A\cdot\cos(2\pi f_1 t_1 - 2\theta) + 2B\cdot\cos(2\pi f_2 t_1 - 2\theta))\cdot(3A\cdot\cos(2\pi f_1 t_2 - 2\theta) + 2B\cdot\cos(2\pi f_2 t_2 - 2\theta))]$$

$$= E[9A^2\cdot\cos(2\pi f_1 t_1 - 2\theta)\cdot\cos(2\pi f_1 t_2 - 2\theta) + \cancel{6AB\cdot\cos(2\pi f_1 t_1 - 2\theta)\cdot\cos(2\pi f_2 t_2 - 2\theta)} + \cancel{6AB\cdot\cos(2\pi f_2 t_1 - 2\theta)\cdot\cos(2\pi f_1 t_2 - 2\theta)} + 4B^2\cdot\cos(2\pi f_2 t_1 - 2\theta)\cdot\cos(2\pi f_2 t_2 - 2\theta)]$$

$$= 9\cdot E[A^2]\cdot E\left[\frac{1}{2}\left(\cancel{\cos(2\pi f_1(t_1+t_2) - 4\theta)} + \cos(2\pi f_1(t_1 - t_2))\right)\right] + 4\cdot E[B^2]\cdot E\left[\frac{1}{2}\left(\cancel{\cos(2\pi f_2(t_1+t_2) - 4\theta)} + \cos(2\pi f_2(t_1 - t_2))\right)\right]$$

$$= \frac{3}{2}\cdot\cos(2\pi f_1 \overbrace{(t_1 - t_2)}^{\tau}) + \frac{3}{2}\cdot\cos(2\pi f_2 \overbrace{(t_1 - t_2)}^{\tau}).$$

$x(k,t)$ è SSL.

$$\varphi_{xx}(f) = \frac{3}{4}\cdot\left\{\delta(f - f_1) + \delta(f + f_1)\right\} + \frac{3}{4}\cdot\left\{\delta(f - f_2) + \delta(f + f_2)\right\}$$

25/6/14

ES. 7

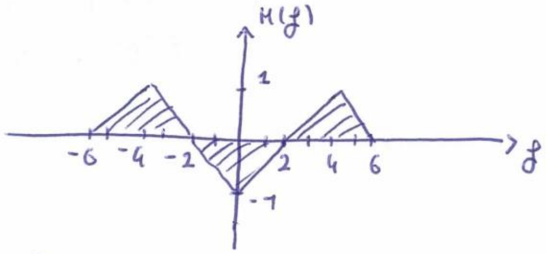
$\varphi_{NN}(f) = \frac{N_0}{2}$.

$h(t) = 4\,\mathrm{sinc}^2(2t)\cos(8\pi t) - 2\,\mathrm{sinc}^2(2t)$

$\varphi_{yy}(f) = ?$

$P_y = ?$

$H(f) = \mathrm{tri}\left(\frac{f-4}{2}\right) + \mathrm{tri}\left(\frac{f+4}{2}\right) - \mathrm{tri}\left(\frac{f}{2}\right)$



$$\varphi_{yy}(f) = \frac{N_0}{2}\cdot\left[\mathrm{tri}\left(\frac{f-4}{2}\right) + \mathrm{tri}\left(\frac{f+4}{2}\right) - \mathrm{tri}\left(\frac{f}{2}\right)\right]^2$$

$$P_y = \int_{-\infty}^{+\infty}\varphi_{yy}(f)\,df = \frac{N_0}{2}\int_2^6 \mathrm{tri}\left(\frac{f+4}{2}\right)df = \int_2^4 \frac{1}{2}f - 1\,df + \int_4^6 -\frac{1}{2}f + 3\,df$$

$$= \left[\frac{1}{2}\cdot\frac{f^2}{2} - f\right]_2^4 + \left[-\frac{1}{2}\cdot\frac{f^2}{2} + 3f\right]_4^6 = \frac{1}{2}\cdot\frac{16}{2} - 4 - \left(\frac{1}{2}\cdot\frac{4}{2} - 2\right)$$

$$+\left(-\frac{1}{2}\cdot\frac{36}{2} + 18\right) - \left(-\frac{1}{2}\cdot\frac{16}{2} + 12\right) = 2\cdot\frac{N_0}{2} = N_0$$

$$P_y = \int_{-\infty}^{+\infty}\varphi_{yy}(f)\,df = 3\left\{\int_2^4\left(\frac{1}{2}f - 1\right)^2 df + \int_4^6\left(-\frac{1}{2}f + 3\right)df\right\}\cdot\frac{N_0}{2} = 2N_0$$

www.alessandropaghi.it

15/05/13

ES. 7

$x(k,t) = AB \cdot \cos(2\pi f_0 t - 2\theta)$

$E[A] = \emptyset$

$E[B] = \emptyset$

A, B v.a. indip.

$f_A(a) = \frac{1}{8} \mathrm{rect}\left(\frac{a}{8}\right)$, $f_B(b) = \frac{1}{2} \mathrm{rect}\left(\frac{b}{2}\right)$

θ unif. distr. in $\emptyset \div 8\pi$

$\varphi_{NN}(f) = \frac{N_0}{2} \rightarrow R_{NN}(\tau) = \frac{N_0}{2} \cdot \delta(\tau)$

$y(k,t) = n(k,t) + x(k,t)$ è SSL? $\varphi_{yy}(f) = ?$

$\mu_y = E[n(k,t) + x(k,t)] = E[n(k,t)] + E[A] \cdot E[B] \cdot E[\cos(2\pi f_0 t - 2\theta)] = \emptyset$

$R_{yy}(\tau) = R_{xx}(t_1,t_2) = E[(n(k,t_1) + AB\cos(2\pi f_0 t_1 - 2\theta)) \cdot (n(k,t_2) + AB\cos(2\pi f_0 t_2 - 2\theta))]$

$= E[n(k,t_1) \cdot n(k,t_2)] + E[n(k,t_1) \cdot AB \cancel{\cos(2\pi f_0 t_2 - 2\theta)}^{\to\emptyset}]$ $+ E[AB\cancel{\cos(2\pi f_0 t_1 - 2\theta)}^{\to\emptyset} \cdot n(k,t_2)] + E[AB\cos(2\pi f_0 t_1 - 2\theta) \cdot AB\cos(2\pi f_0 t_2 - 2\theta)]$

$= \frac{N_0}{2} \cdot \delta(\tau) + E[A^2] \cdot E[B^2] \cdot E[\frac{1}{2}(\cancel{\cos(2\pi f_0 (t_1 + t_2) - 4\theta)}^{\to\emptyset} + \cos(2\pi f_0 (t_1 - t_2))]$

$= \frac{N_0}{2} \cdot \delta(\tau) + \frac{16}{18} \cos(2\pi f_0 \tau)$

$y(k,t)$ è SSL!

$\varphi_{yy}(f) = \frac{N_0}{2} + \frac{16}{36} \{ \delta(f - f_0) + \delta(f + f_0) \}$

6/05/14 PROVA A

ES. 8

$x(k,t)$ SSL.

$P_x = ?$

$R_{xx}(\tau) = 4\,\mathrm{sinc}(4\tau)$

$h(t) = 8\,\mathrm{sinc}(4t)\cdot\cos(2\pi t)$

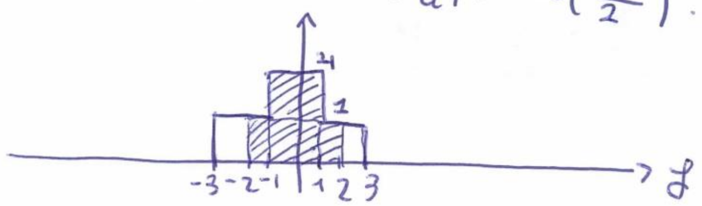
$\varphi_{xx}(f) = \frac{4}{4}\cdot \mathrm{rect}\left(\frac{f}{4}\right) = \mathrm{rect}\left(\frac{f}{4}\right)$

$H(f) = \frac{1}{2}\cdot\left[\frac{8}{4}\,\mathrm{rect}\left(\frac{f-1}{4}\right) + \frac{8}{4}\,\mathrm{rect}\left(\frac{f+1}{4}\right)\right]$

$= \mathrm{rect}\left(\frac{f-1}{4}\right) + \mathrm{rect}\left(\frac{f+1}{4}\right).$

$\varphi_{yx}(f) = \varphi_{xx}(f)\cdot|H(f)|^2 = \mathrm{rect}\left(\frac{f}{4}\right)\cdot\left[\mathrm{rect}\left(\frac{f}{6}\right) + 3\,\mathrm{rect}\left(\frac{f}{2}\right)\right]$

$= \mathrm{rect}\left(\frac{f}{4}\right)\cdot\mathrm{rect}\left(\frac{f}{6}\right) + \mathrm{rect}\left(\frac{f}{4}\right) 3\,\mathrm{rect}\left(\frac{f}{2}\right).$



$= \mathrm{rect}\left(\frac{f}{4}\right) + 3\,\mathrm{rect}\left(\frac{f}{2}\right)$

$P_y = \int_{-\infty}^{+\infty} \varphi_{xy}(f) = 10.$

6/05/14 PROVA B

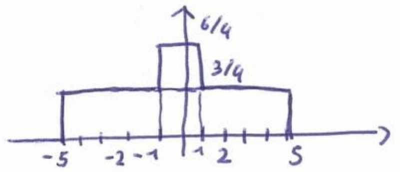
ES. 8

$\varphi_{NN}(f) = \frac{N_0}{2}$

$P_y = ?$

$h(t) = 8\,\mathrm{sinc}(6t)\cdot\cos(4\pi t)$

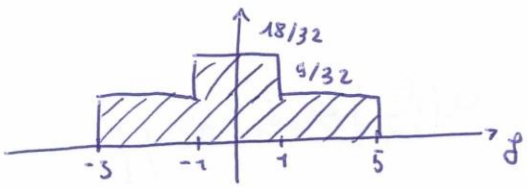
$H(f) = \frac{3}{4}\left\{\mathrm{rect}\left(\frac{f-2}{6}\right) + \mathrm{rect}\left(\frac{f+2}{6}\right)\right\}$



$\frac{3}{4}\,\mathrm{rect}\left(\frac{f}{10}\right) + \frac{3}{4}\,\mathrm{rect}\left(\frac{f}{2}\right)$

$\varphi_{YY}(f) = \varphi_{NN}(f)\cdot|H(f)|^2 = \frac{N_0}{2}\cdot\left\{\frac{9}{16}\,\mathrm{rect}\left(\frac{f}{10}\right) + \frac{9}{16}\,\mathrm{rect}\left(\frac{f}{2}\right)\right\}$

$= \frac{9}{32}N_0\cdot\mathrm{rect}\left(\frac{f}{10}\right) + \frac{9}{32}N_0\cdot\mathrm{rect}\left(\frac{f}{2}\right)$



$P_y = \int_{-\infty}^{+\infty}\varphi_{XX}(f)\,df = 10\cdot\frac{9}{32} + 2\cdot\frac{9}{32} = \frac{27}{8}.$

25/06/14

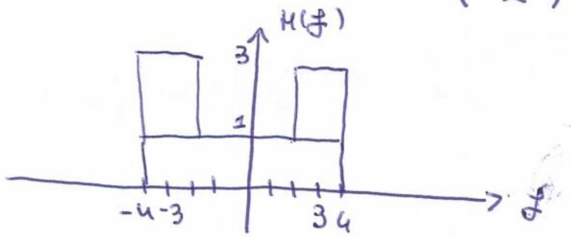
ES. 8

$R_{xx}(\tau) = 8\,\mathrm{sinc}^2(4\tau)$ $\qquad P_y = ?$

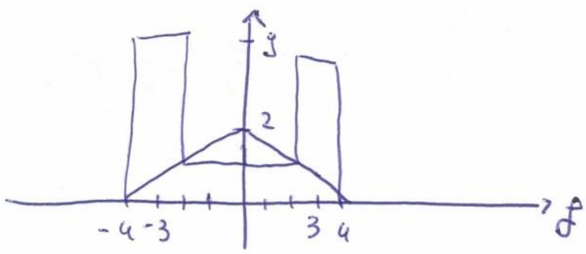
$h(t) = 8\,\mathrm{sinc}(8t) + 8\,\mathrm{sinc}(2t)\cos(6\pi t)$

$\Phi_{xx}(f) = \frac{8}{4}\cdot \mathrm{tri}\left(\frac{f}{4}\right) = 2\,\mathrm{tri}\left(\frac{f}{4}\right)$

$H(f) = \mathrm{rect}\left(\frac{f}{8}\right) + 2\,\mathrm{rect}\left(\frac{f-3}{2}\right) + 2\,\mathrm{rect}\left(\frac{f+3}{2}\right)$



$\Phi_{yx}(f) = 2\,\mathrm{tri}\left(\frac{f}{4}\right)\cdot\left\{\mathrm{rect}\left(\frac{f}{8}\right) + 8\,\mathrm{rect}\left(\frac{f+3}{2}\right) + 8\,\mathrm{rect}\left(\frac{f-3}{2}\right)\right\}$



$P_y = 9\int_{-4}^{-2} \quad df + \int_{-3}^{0} \quad df + \int_{0}^{2} \quad df + 9\int_{2}^{4} \quad df$

$= 2\left[\int_{0}^{2} 2-\frac{1}{2}f\,df + 9\int_{2}^{4} 2-\frac{1}{2}f\,df\right] =$

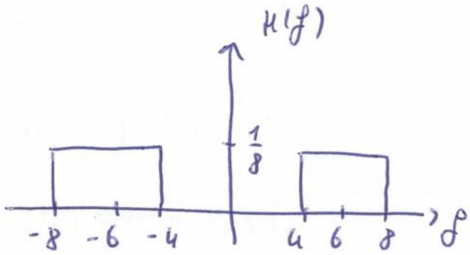
$2\cdot\left[2f+\frac{1}{2}\cdot\frac{f^2}{2}\right]_0^2 + 18\cdot\left[2f-\frac{1}{2}\cdot\frac{f^2}{2}\right]_2^4 = 9$

15/05/13

ES. 8

$R_{xx}(\tau) = 8\,\mathrm{sinc}^2(4\tau)\cos(8\pi\tau)$ $\qquad P_y = ?$

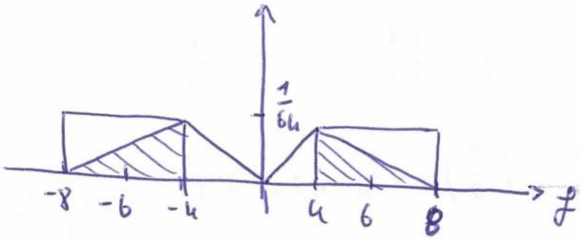
$h(t) = \mathrm{sinc}(4t)\cos(12\pi t)$



$\Psi_{xx}(f) = \mathrm{tri}\left(\frac{f-4}{4}\right) + \mathrm{tri}\left(\frac{f+4}{4}\right)$

$H(f) = \frac{1}{8}\cdot\left\{\mathrm{rect}\left(\frac{f-6}{4}\right) + \mathrm{rect}\left(\frac{f+6}{4}\right)\right\}$

$\Psi_{yy}(f) = \left\{\mathrm{tri}\left(\frac{f-4}{4}\right) + \mathrm{tri}\left(\frac{f+4}{4}\right)\right\}\frac{1}{64}\left\{\mathrm{rect}\left(\frac{f-6}{4}\right) + \mathrm{rect}\left(\frac{f+6}{4}\right)\right\}$



$P_y = \frac{2}{64}\int_4^8 1 - \frac{f-4}{4}\;df = \frac{1}{32}\cdot\int_4^8 1 - \frac{f}{4} + 1\;df = \frac{1}{32}\left[-\frac{1}{4}\cdot\frac{f^2}{2} + 2\cdot f\right]_4^8 = \frac{1}{16}.$

20/06/12

ES. 1

$$s(t) = \text{tri}\left(\frac{t+2}{3}\right) \cdot \cos(20\pi t) + \frac{d}{dt} \text{sinc}(4t)\, e^{j8\pi t}$$

$$s(t) = k(t) + p(t)$$

$$S(f) = K(f) + P(f)$$

$$\text{tri}\left(\frac{t+2}{3}\right) \longleftrightarrow 3\,\text{sinc}^2(3f) \cdot e^{j4\pi f}$$

$$K(f) = \frac{1}{2}\left\{3\,\text{sinc}^2[3(f-10)] \cdot e^{j4\pi(f-10)} + 3\,\text{sinc}^2[3(f+10)]e^{j4\pi(f+10)}\right\}$$

$$= \frac{3}{2}\left\{\text{sinc}^2[3(f-10)] \cdot e^{j4\pi(f-10)} + \text{sinc}^2[3(f+10)]\, e^{j4\pi(f+10)}\right\}$$

$$P(f) = j2\pi f \cdot \frac{1}{4} \text{rect}\left(\frac{f-4}{4}\right) = \frac{1}{2} j\pi f\, \text{rect}\left(\frac{f-4}{4}\right)$$

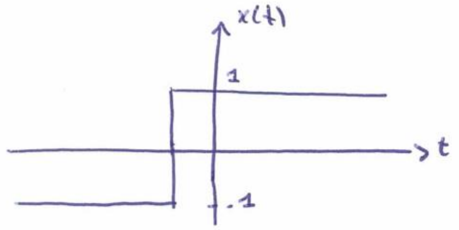
$$S(f) = \frac{3}{2}\left\{\text{sinc}^2[3(f-10)]\, e^{j4\pi(f-10)} + \text{sinc}^2[3(f+10)]\, e^{j4\pi(f+10)}\right\} + \frac{1}{2} j\pi f\, \text{rect}\left(\frac{f-4}{4}\right).$$

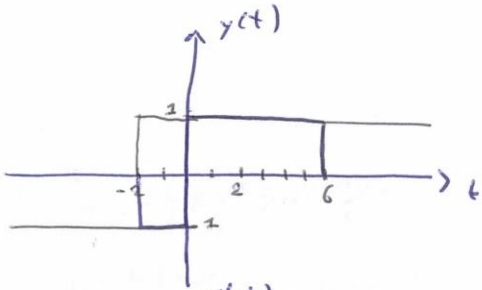
Es. 2

$x(t) = \text{sign}(t+1)$

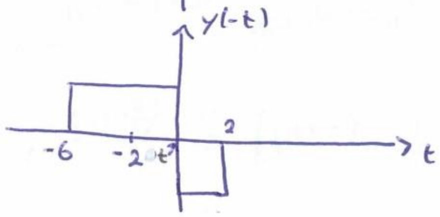
$y(t) = \text{rect}\left(\frac{t-2}{8}\right)\text{sign}(t)$

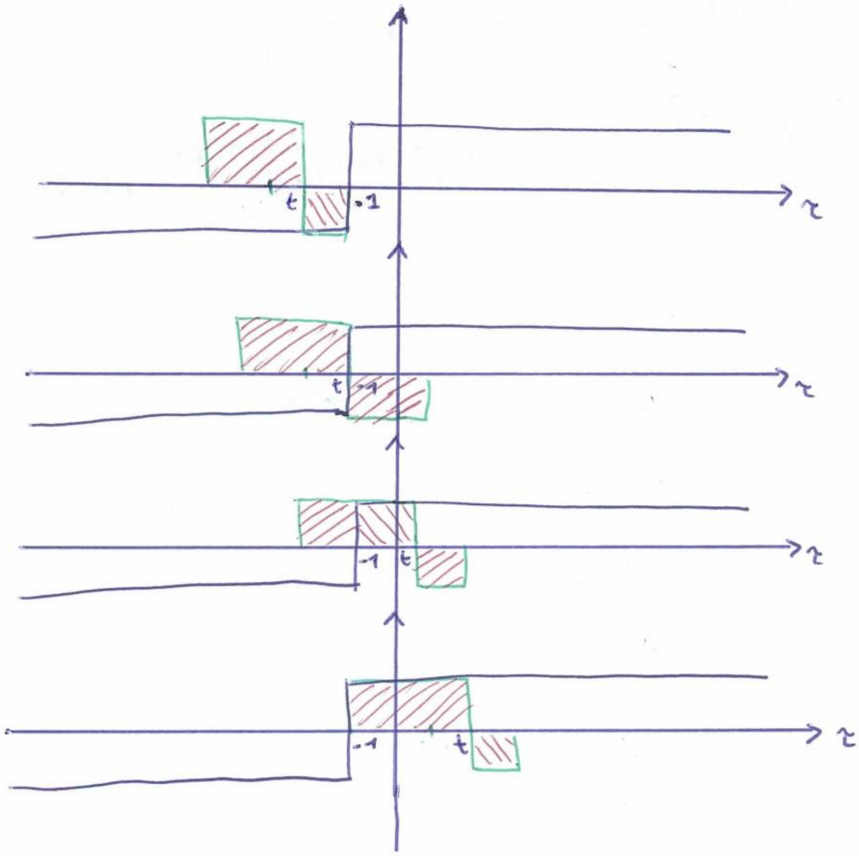
$z(t) = x(t) * y(t)$



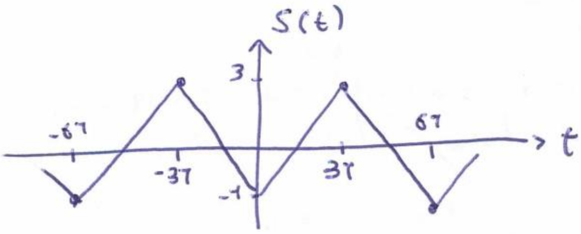


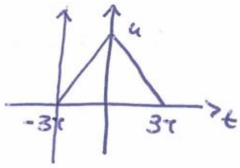
$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$





ES. 3





$$S_p(t) = \sum_{m=-\infty}^{+\infty} S_T(t - mT)$$

$$S_T = 4\,\mathrm{tri}\left(\frac{t-3\tau}{3\tau}\right)$$

$$\updownarrow$$

$$4 \cdot 3\tau \cdot \mathrm{sinc}^2(3\tau f) \cdot e^{-j2\pi 3\tau f} = 12\tau\,\mathrm{sinc}^2(3\tau f)\,e^{-j6\tau\pi f}$$

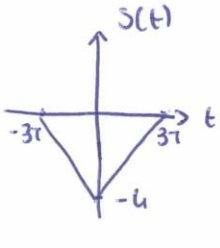
$$S_p(f) = \frac{1}{T} \cdot \sum_{m=-\infty}^{+\infty} S_T\left(\frac{m}{T}\right) \cdot \delta\left(f - \frac{m}{T}\right)$$

$$S_p(t) = \left[4 \cdot \sum_{m=-\infty}^{+\infty} \mathrm{tri}\left(\frac{t - 3\tau - m6\tau}{3\tau}\right)\right] - 1$$

$$= \left[4 \cdot \sum_{m=-\infty}^{+\infty} \mathrm{tri}\left(\frac{t - 3\tau(1+2m)}{3\tau}\right)\right] - 1$$

$$S_p(f) = \left[\frac{1}{6\tau} \cdot \sum_{m=-\infty}^{+\infty} 12\tau\,\mathrm{sinc}^2\left(3\tau \cdot \frac{m}{6\tau}\right) \cdot e^{-j6\tau\pi\left(\frac{m}{6\tau}\right)} \cdot \delta\left(f - \frac{m}{6\tau}\right)\right] - \delta(f)$$

$$= \left[\frac{1}{6\tau} \cdot \sum_{m=-\infty}^{+\infty} 12\tau \cdot \mathrm{sinc}^2\left(\frac{m}{2}\right) \cdot e^{-jm\pi} \cdot \delta\left(f - \frac{m}{6\tau}\right)\right] - \delta(f).$$

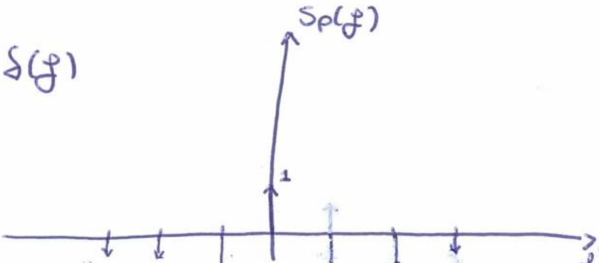


$$S_T = -4\,\mathrm{tri}\left(\frac{t}{3\tau}\right) \longleftrightarrow -4 \cdot 3\tau \cdot \mathrm{sinc}^2(3\tau \cdot f).$$

$$S_p(t) = \left[\sum_{m=-\infty}^{+\infty} -4 \cdot \mathrm{tri}\left(\frac{t - m6\tau}{3\tau}\right)\right] + 3$$

$$S_p(f) = \left[\frac{1}{6\tau} \cdot \sum_{m=-\infty}^{+\infty} -12\tau \cdot \mathrm{sinc}^2\left(3\tau \cdot \frac{m}{6\tau}\right) \cdot \delta\left(f - \frac{m}{6\tau}\right)\right] + 3\delta(f)$$

$$= \left[-2 \cdot \sum_{m=-\infty}^{+\infty} \mathrm{sinc}^2\left(\frac{m}{2}\right) \cdot \delta\left(f - \frac{m}{6\tau}\right)\right] + 3\delta(f)$$

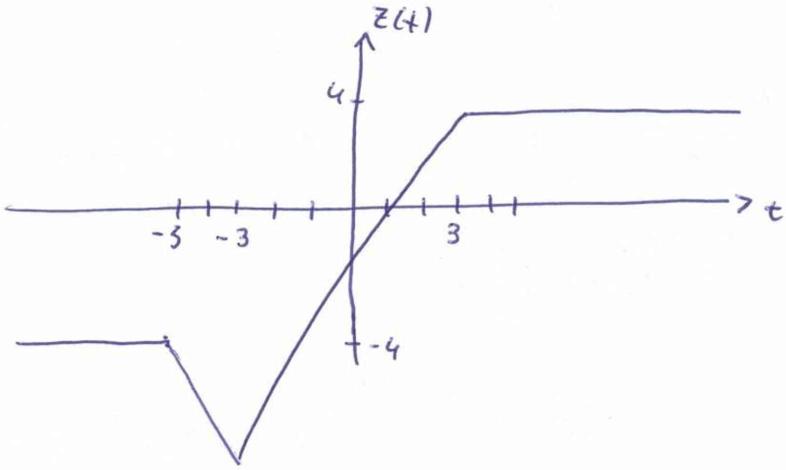


$t \le -3 \to z(t) = -4$

$-3 < t \le -1 \to z(t) = \int_{t-4}^{t+2} -1 \, d\tau + \int_{t+2}^{-1} 1 \, d\tau + \int_{-1}^{t+4} -1 \, d\tau = -2t - 14$

$-3 < t \le 3 \to z(t) = \int_{t-4}^{-1} -1 \, d\tau + \int_{-1}^{t+2} 1 \, d\tau + \int_{t+2}^{t+4} -1 \, d\tau = 2t - 2$

$t > 3 \to z(t) = 4$



Es. 4

$s(t) = 100 \operatorname{sinc}^2(10t)$

$h(t) = \operatorname{sinc}(2t)(1 + \cos(18\pi t))$ $\qquad P_y = ?$

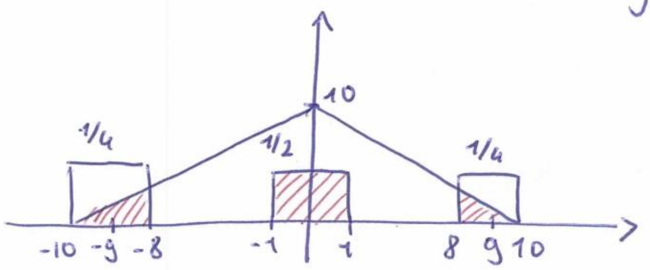
$= \operatorname{sinc}(2t) + \operatorname{sinc}(2t) \cdot \cos(18\pi t)$

$Y(f) = H(f) \cdot S(f)$

$$H(f) = \frac{1}{2} \operatorname{rect}\left(\frac{f}{2}\right) + \frac{1}{2}\left[\frac{1}{2} \operatorname{rect}\left(\frac{f-9}{2}\right) + \frac{1}{2} \operatorname{rect}\left(\frac{f+9}{2}\right)\right]$$

$$S(f) = \frac{100}{10} \operatorname{tri}\left(\frac{f}{10}\right)$$

$$Y(f) = \left\{\frac{1}{2} \operatorname{rect}\left(\frac{f}{2}\right) + \frac{1}{4} \operatorname{rect}\left(\frac{f-9}{2}\right) + \frac{1}{4} \operatorname{rect}\left(\frac{f+9}{2}\right)\right\} \cdot 10 \operatorname{tri}\left(\frac{f}{10}\right)$$



Es finita $\rightarrow P_y = \emptyset$

$$E_s = \left\{\int_{10}^{-8} \frac{1}{16}(f+10)^2 \, df + \int_{-1}^{\emptyset} \frac{1}{4}(f+10)^2 \, df\right\} \cdot 2 = \frac{91}{2}.$$

ES. 5

$S(t) = 200 \cdot \mathrm{sinc}(200t)$

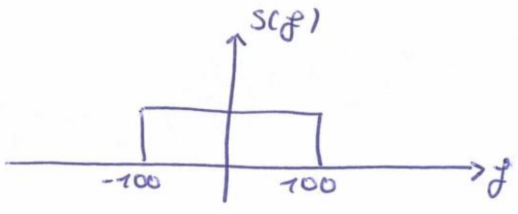
$f_c = 2B$

$n_{bit/c} = 10\ bit$

24 h di segnale trasmesso in 43,2 secondi

$V_T = ?$

$S(f) = \mathrm{rect}\left(\frac{f}{200}\right)$



$B = 100\ Hz$

$f_c = 200\ Hz \rightarrow n_{c/s} = 200$

$n_{c/24h} = n_{c/s} \cdot 24 \cdot 3600 = 17280000$

$n_{bit/24h} = n_{c/24h} \cdot n_{bit/c} = 172.800.000$

$V_T = \frac{n_{bit/24h}}{43,2\ s} = 4.000.000 = 4\ Mbit/s$.

Es. 7

$C_{xx}(\tau) = 8\,\text{sinc}^2(4\tau)$ $\qquad$ $\varphi_{xx}(f) = ?$

$\mu_x = 4$ $\qquad$ $P_x = ?$

$R_{xx}(\tau) = C_{xx}(\tau) + \mu_x^2 = 8\,\text{sinc}^2(4\tau) + 16$

$\varphi_{xx}(f) = 2\,\text{tri}\left(\frac{f}{4}\right) + 16\,\delta(f).$

$P_x = R_{xx}(\emptyset) = 8 + 16 = 24$

Es. 8

$R_{xx}(\tau) = 4\,\text{sinc}^2(4\tau)$

$\mu_x = \emptyset$ $\qquad$ $h(t) = ?$

$R_{yy}(\tau) = 8\,\text{sinc}(4\tau) + 4\,\text{sinc}^2(2\tau)$

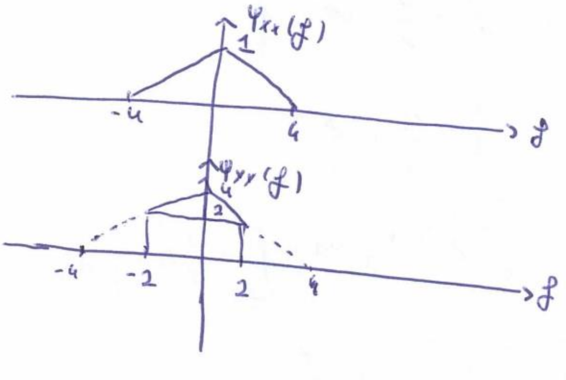
$\varphi_{xx}(f) = \text{tri}\left(\frac{f}{4}\right)$; $\varphi_{yy}(f) = 2\,\text{rect}\left(\frac{f}{4}\right) + 2\,\text{tri}\left(\frac{f}{2}\right).$

$\varphi_{yy}(f) = \varphi_{xx}(f) \cdot |H(f)|^2$

$\rightarrow |H(f)|^2 = \dfrac{\varphi_{yy}(f)}{\varphi_{xx}(f)} = 4\,\text{rect}\left(\frac{f}{4}\right)$

$\rightarrow H(f) = 2\,\text{rect}\left(\frac{f}{4}\right).$

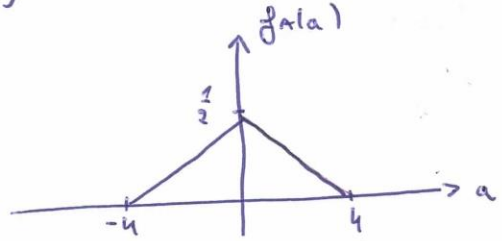
$h(t) = 8\,\text{sinc}(4t)$



ES. 6

$f_A(a) = \frac{1}{2} \operatorname{tri}\left(\frac{a}{4}\right) \cdot u(a)$ $\qquad$ $E(a)$? $\quad P(E)$? $\quad \sigma^2(a)$?

$E = \{A < 1,\ A > 3\}$



$$E(a) = \int_{\emptyset}^{4} a \cdot \left(-\frac{1}{8}a + \frac{1}{2}\right) da = \left[-\frac{1}{8}\cdot\frac{a^3}{3} + \frac{1}{2}\cdot\frac{a^2}{2}\right]_{\emptyset}^{4}$$

$$= -\frac{1}{\cancel{8}}\cdot\frac{\cancel{64}^{8}}{3} + \frac{1}{\cancel{2}_1}\cdot\frac{\cancel{16}^{\cancel{8}\,4}}{\cancel{2}_1} = -\frac{8}{3} + 4 = \frac{4}{3}$$

$$E(a^2) = \int_{\emptyset}^{4} a^2\left(-\frac{1}{8}\cdot a + \frac{1}{2}\right) da = \left[-\frac{1}{8}\cdot\frac{a^4}{4} + \frac{1}{2}\cdot\frac{a^3}{3}\right]_{\emptyset}^{4}$$

$$= -\frac{1}{{}_1\cancel{8}}\cdot\frac{\cancel{256}^{\cancel{32}\,8}}{\cancel{4}_1} + \frac{1}{\cancel{2}_1}\cdot\frac{\cancel{64}^{32}}{3} = \frac{8}{3}$$

$$\sigma^2(a) = \frac{8}{3} - \frac{16}{9} = \frac{8}{9}$$

$$P(E) = \int_{\emptyset}^{1} -\frac{1}{8}a + \frac{1}{2}\, da + \int_{3}^{4} -\frac{1}{8}a + \frac{1}{2}\, da = \left[-\frac{1}{8}\cdot\frac{a^2}{2} + \frac{1}{2}\cdot a\right]_{\emptyset}^{1} + \left[-\frac{1}{8}\cdot\frac{a^2}{2} + \frac{1}{2}\cdot a\right]_{3}^{4}$$

$$= -\frac{1}{8}\cdot\frac{1}{2} + \frac{1}{2} - \frac{1}{\cancel{8}}\cdot\frac{\cancel{16}^{2}}{2} + \frac{1}{\cancel{2}}\cdot\cancel{4}^{2} - \left(-\frac{1}{8}\cdot\frac{9}{2} + \frac{1}{2}\cdot 3\right) = \frac{1}{2}$$

16/05/12

ES. 1

$s(t) = [8\,\mathrm{sinc}(4t)\cos(8\pi t)] * [4\,\mathrm{sinc}(2t)\cos(10\pi t)]$

$s(t) = k(t) * p(t) \quad \leftrightarrow \quad S(f) = K(f) \cdot P(f)$

$k(t) = 8\,\mathrm{sinc}(4t)\cos(8\pi t)$

$8\,\mathrm{sinc}(4t) \leftrightarrow \frac{8}{4}\,\mathrm{rect}\left(\frac{f}{4}\right) = 2\,\mathrm{rect}\left(\frac{f}{4}\right)$

$K(f) = \frac{1}{2}\left[2\,\mathrm{rect}\left(\frac{f-4}{4}\right) + 2\,\mathrm{rect}\left(\frac{f+4}{4}\right)\right] = \mathrm{rect}\left(\frac{f-4}{4}\right) + \mathrm{rect}\left(\frac{f+4}{4}\right)$

$p(t) = 4\,\mathrm{sinc}(2t)\cos(10\pi t)$

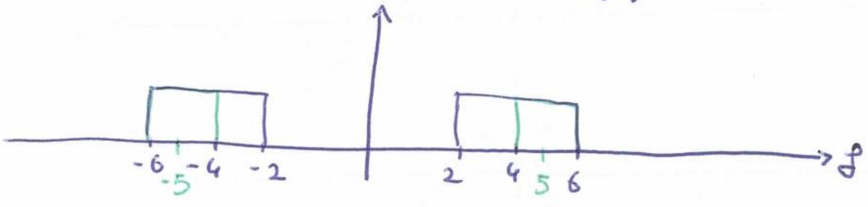
$4\,\mathrm{sinc}(2t) \leftrightarrow \frac{4}{2}\,\mathrm{rect}\left(\frac{f}{2}\right) = 2\,\mathrm{rect}\left(\frac{f}{2}\right)$

$P(f) = \frac{1}{2}\left[2\,\mathrm{rect}\left(\frac{f-5}{2}\right) + 2\,\mathrm{rect}\left(\frac{f+5}{2}\right)\right] = \mathrm{rect}\left(\frac{f-5}{2}\right) + \mathrm{rect}\left(\frac{f+5}{2}\right).$

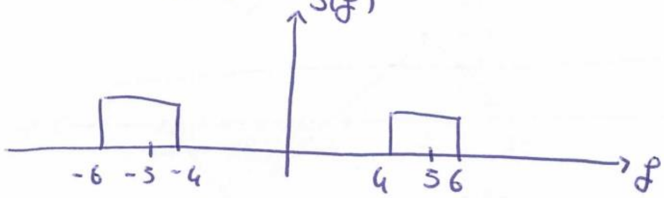
$S(f) = \left[\mathrm{rect}\left(\frac{f-4}{4}\right) + \mathrm{rect}\left(\frac{f+4}{4}\right)\right] \cdot \left[\mathrm{rect}\left(\frac{f-5}{2}\right) + \mathrm{rect}\left(\frac{f+5}{2}\right)\right]$

$= \mathrm{rect}\left(\frac{f-4}{4}\right)\mathrm{rect}\left(\frac{f-5}{2}\right) + \mathrm{rect}\left(\frac{f-4}{4}\right)\mathrm{rect}\left(\frac{f+5}{2}\right) +$

$+ \mathrm{rect}\left(\frac{f+4}{4}\right)\mathrm{rect}\left(\frac{f-5}{2}\right) + \mathrm{rect}\left(\frac{f+4}{4}\right)\mathrm{rect}\left(\frac{f+5}{2}\right).$



$= \mathrm{rect}\left(\frac{f-5}{2}\right) + \mathrm{rect}\left(\frac{f+5}{2}\right)$

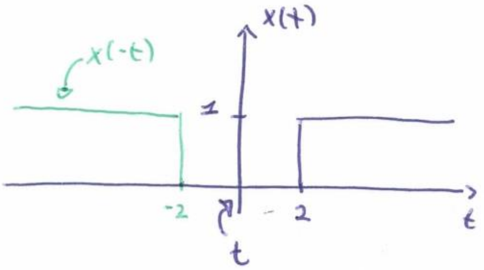


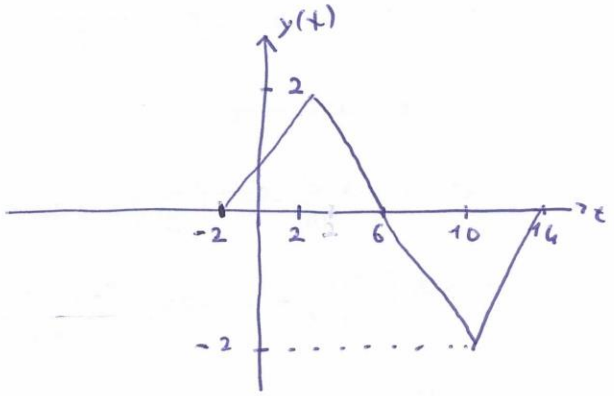
Es. 2

$$x(t) = u(t-2)$$

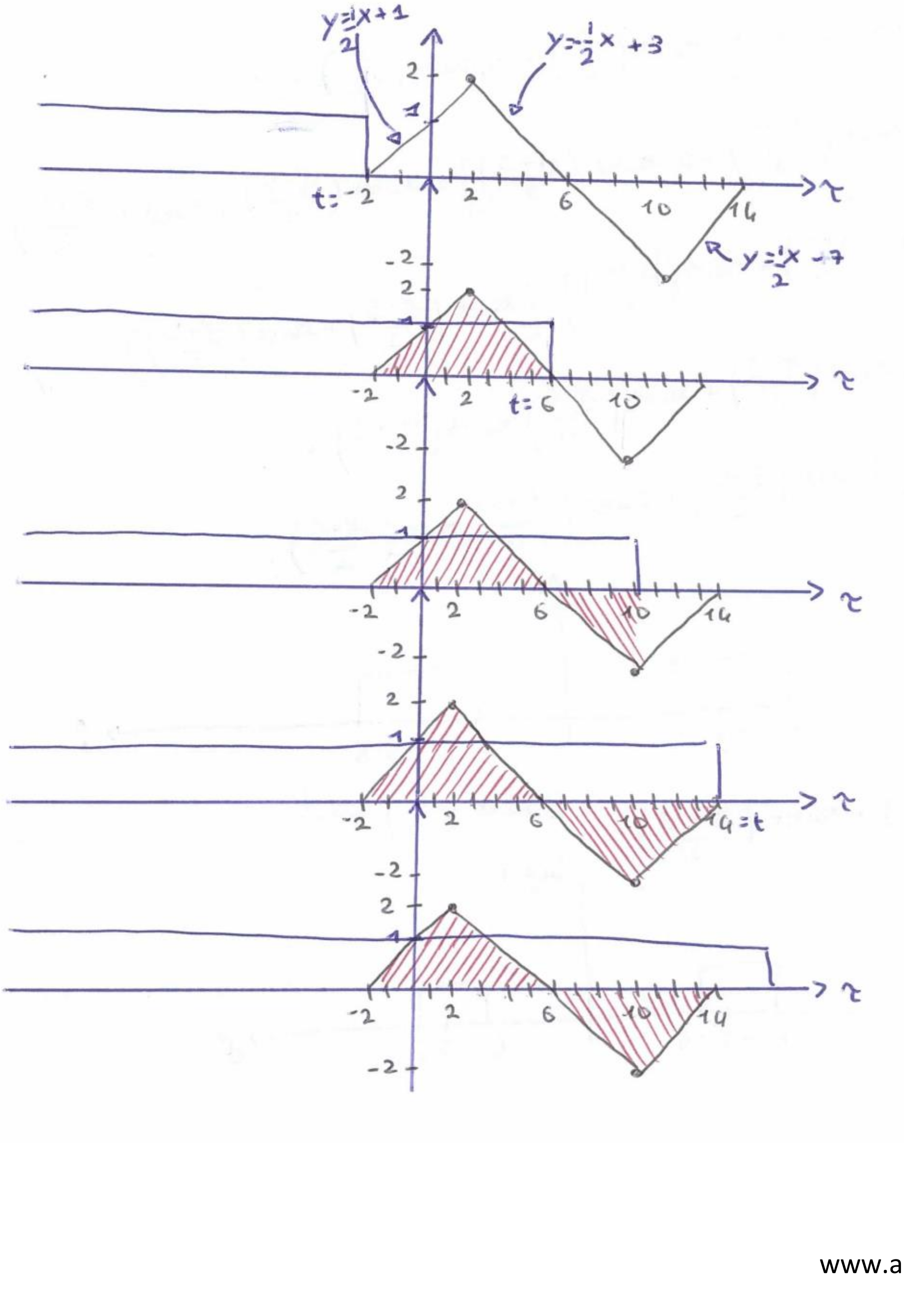
$$y(t) = 2\,\mathrm{tri}\left(\frac{t-2}{4}\right) - 2\,\mathrm{tri}\left(\frac{t-10}{4}\right)$$

$$z(t) = x(t) * y(t)$$





$$z(t) = \int_{-\infty}^{+\infty} y(\tau)\, x(t-\tau)\, d\tau$$



$t \leq -2 \rightarrow z(t) = \emptyset$

$-2 < t \leq 2 \rightarrow z(t) = \int_{-2}^{t} \frac{\tau}{2} + 1 \, d\tau = \frac{t^2}{4} + t + 1$

$2 < t \leq 6 \rightarrow z(t) = \int_{-2}^{2} \frac{\tau}{2} + 1 \, d\tau + \int_{2}^{t} -\frac{\tau}{2} + 3 \, d\tau = -\frac{t^2}{4} + 3t - 1$

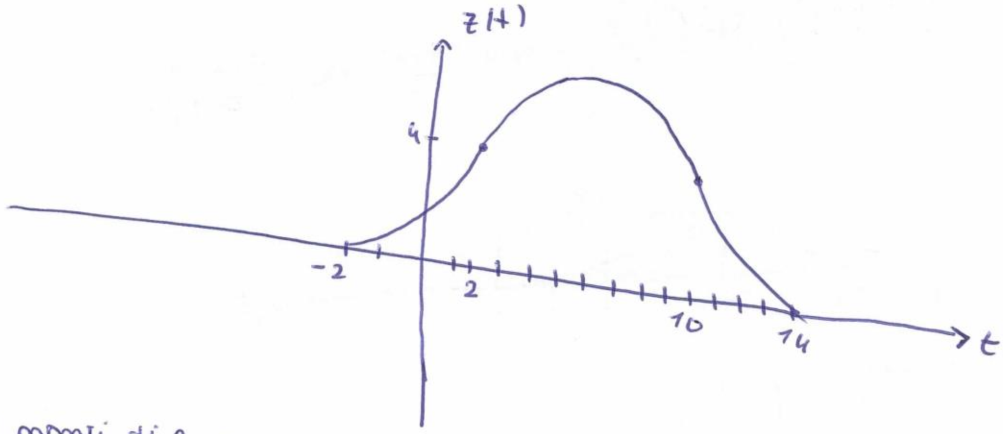
$6 < t \leq 10 \rightarrow z(t) = \int_{-2}^{2} \frac{\tau}{2} + 1 \, d\tau + \int_{2}^{6} -\frac{\tau}{2} + 3 \, d\tau = \int_{6}^{\tau} -\frac{\tau}{2} + 3 \, d\tau = -\frac{t^2}{4} + 3t - 1$

$10 < t \leq 14 \rightarrow z(t) = \int_{-2}^{2} \frac{\tau}{2} + 1 \, d\tau + \int_{2}^{10} -\frac{\tau}{2} + 3 \, d\tau + \int_{10}^{t} \frac{\tau}{2} - 7 \, d\tau = \frac{t^2}{4} - 7t + 49$

$t > 14 \rightarrow z(t) = \emptyset$

$$z(t) = \begin{cases} \emptyset & t \leq -2 \\ -\frac{t^2}{4} + 3t - 1 & 2 < t \leq 10 \\ \frac{t^2}{4} - 7t + 49 & 10 < t \leq 14 \\ \emptyset & t > 14 \end{cases}$$

$\longrightarrow \frac{t^2}{4} + t + 1 \quad -2 < t \leq 2$



tutto avanti di 2 $\rightarrow$

ES. 3

$s(t) = 8\,\mathrm{sinc}^2(4t)\, e^{-j4\pi t}$

$h(t) = \mathrm{sinc}(2t)\cos(2\pi t)$

$y(t) = \int_{-\infty}^{+\infty} h(t-\tau)\cdot s(\tau)\, d\tau \quad , \; Y(f) = H(f)\cdot S(f)$

$\mathrm{sinc}(2t) \longleftrightarrow \frac{1}{2}\mathrm{rect}\left(\frac{f}{2}\right)$

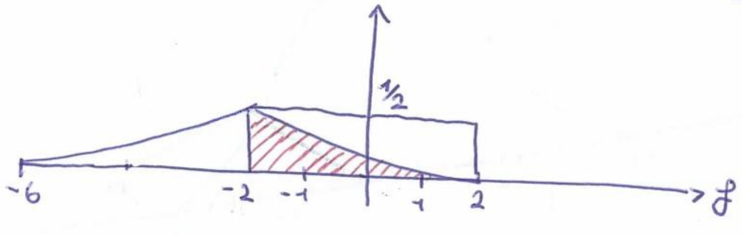
$H(f) = \frac{1}{2}\cdot\left[\frac{1}{2}\mathrm{rect}\left(\frac{f-1}{2}\right) + \frac{1}{2}\mathrm{rect}\left(\frac{f+1}{2}\right)\right] = \frac{1}{4}\left[\mathrm{rect}\left(\frac{f-1}{2}\right) + \mathrm{rect}\left(\frac{f+1}{2}\right)\right]$

$8\,\mathrm{sinc}^2(4t) \longleftrightarrow \frac{8}{4}\mathrm{tri}\left(\frac{f}{4}\right) = 2\,\mathrm{tri}\left(\frac{f}{4}\right)$

$S(f) = 2\,\mathrm{tri}\left(\frac{f+2}{4}\right).$

$Y(f) = \frac{1}{4}\left[\mathrm{rect}\left(\frac{f-1}{2}\right) + \mathrm{rect}\left(\frac{f+1}{2}\right)\right]\cdot 2\,\mathrm{tri}\left(\frac{f+2}{4}\right)$

$= \frac{1}{2}\cdot\left[\mathrm{rect}\left(\frac{f-1}{2}\right)\cdot\mathrm{tri}\left(\frac{f+2}{4}\right) + \mathrm{rect}\left(\frac{f+1}{2}\right)\cdot\mathrm{tri}\left(\frac{f+2}{4}\right)\right].$



$\int_{-\infty}^{+\infty} |Y(f)|^2 df =$

$= \int_{-2}^{2}\left(-\frac{1}{8}f + \frac{1}{4}\right)^2 df = \frac{1}{16}\int_{-2}^{2} \frac{f^2}{4} + 1 - f\, df = \frac{1}{16}\left[\frac{f^3}{12} + f - \frac{f^2}{2}\right]_{-2}^{2} = \frac{1}{3}.$

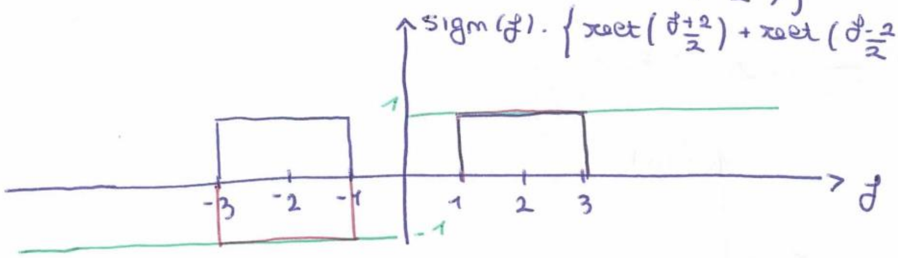
ES. 4

$$s(t) = \text{sinc}(2t)\, e^{-j4\pi t} + \text{sinc}(2t)\, e^{j4\pi t}$$

$$\hat{s}(t) = \left[\text{sinc}(2t)\, e^{-j4\pi t} + \text{sinc}(2t)\, e^{j4\pi t}\right] * \frac{1}{\pi t}$$

$$S(f) = \frac{1}{2}\,\text{rect}\left(\frac{f+2}{2}\right) + \frac{1}{2}\,\text{rect}\left(\frac{f-2}{2}\right) = \frac{1}{2}\left\{\text{rect}\left(\frac{f+2}{2}\right) + \text{rect}\left(\frac{f-2}{2}\right)\right\}.$$

$$\hat{S}(f) = \frac{1}{2}\left\{-j\,\text{sisgn}(f)\right\}\cdot\left\{\text{rect}\left(\frac{f+2}{2}\right) + \text{rect}\left(\frac{f-2}{2}\right)\right\}$$



$$\hat{S}(f) = \frac{1}{2j}\cdot\left[+\text{rect}\left(\frac{f-2}{2}\right) - \text{rect}\left(\frac{f+2}{2}\right)\right]$$

$$\hat{s}(t) = 2\,\text{sinc}(2t)\cdot\text{sen}(4\pi t)$$

ES. 5

$S(t) = \mathrm{sinc}^2(60t)\cos(100\pi t)$

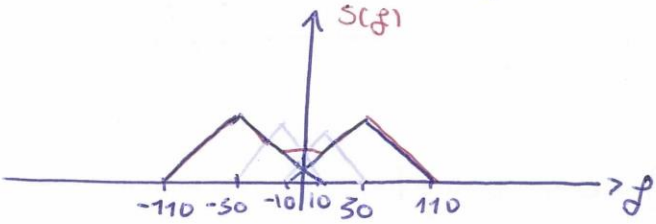
$f_c = 2B$

$n_{bit/c} = 12$ bit

$V_T = 2$ Mbit/s

$T_T = ?$ per trasmettere 45 min di segnale.

$$S(f) = \frac{1}{120} \cdot \left[\mathrm{tri}\left(\frac{f-50}{60}\right) + \mathrm{tri}\left(\frac{f+50}{60}\right) \right]$$



$B = 110$ Hz

$f_c = 220$ Hz , $n_{c/1s} = 220$.

$n_{c/45\,min} = 220 \cdot 45 \cdot 60 = 594.000$

$n_{bit/45\,min} = 7.128.000$ bit

$$T_T = \frac{7.128.000}{2.000.000} = 3{,}564 \text{ s}.$$

ES. 6

$f_A(a) = 3e^{-3a} \cdot u(a)$

$\Psi_{NN}(f) = \frac{N_0}{2}$

$y(k,t) = A - 3n(k,t) \qquad \Psi_{YY}(f) = ?$

$R_{YY}(t_1,t_2) = E[(A - 3n(k,t_1)) \cdot (A - 3n(k,t_2))]$

$= E[A^2 - 3 \cdot A\, n(k,t_2) - 3A\, n(k,t_1) + 9\, n(k,t_1)\, n(k,t_2)]$

$= E[A^2] - 3 \cdot E[A] \cdot E[\cancel{n(k,t_2)}]^{\to \emptyset} - 3\, E[A] \cdot E[\cancel{n(k,t_1)}]^{\to \emptyset}$

$+ 9\, E[n(k,t_1)\, n(k,t_2)]$

$= 2/9 + \frac{9}{2} N_0$

$\Psi_{YY}(f) = 2/9\, \delta(f) + \frac{9}{2} N_0$

ES. 7

$x(k,t) = (2A+3B)\cos(2\pi f_0 t - 2\theta)$

A, B v.a. indip.

$E[A] = \emptyset$

$E[B] = 3$

$f_A(a) = \frac{1}{4}\,\mathrm{rect}\left(\frac{a}{4}\right)$, $f_B(b) = \frac{1}{8}\,\mathrm{rect}\left(\frac{b-3}{8}\right)$

$E[A^2] = \frac{4}{3}$

$\theta \sim$ unif. in $\emptyset \div 2\pi$

$E[B^2] = \frac{43}{3}$

SSL ? $\varphi_{YX}(f) = ?$

$\mu_x = E[2A\cos(2\pi f_0 t - 2\theta) + 3B\cos(2\pi f_0 t - 2\theta)]$

$= E[2A]\cdot E[\cos(2\pi f_0 t - 2\theta)] + E[3B]\cdot E[\cos(2\pi f_0 t - 2\theta)] = \emptyset.$

$R_{XX}(\tau) = E[(2A+3B)\cdot\cos(2\pi f_0 t_1 - 2\theta)\cdot(2A+3B)\cos(2\pi f_0 t_2 - 2\theta)]$

$= E[(2A+3B)^2\cdot\cos(2\pi f_0 t_1 - 2\theta)\cdot\cos(2\pi f_0 t_2 - 2\theta)]$

$= \{E[4A^2] + E[9B^2] + E[\cancel{12AB}]^{\to\emptyset}\}\,\frac{1}{2}E[\cos(2\pi f_0(\cancel{t_1+t_2})^{\to\emptyset} - 4\theta)\cos(2\pi f_0(t_1-t_2))]$

$= \frac{403}{3}\cdot\frac{1}{2}\cdot\cos(2\pi f_0(t_1-t_2)) = \frac{403}{6}\cdot\cos(2\pi f_0 \overbrace{(t_1-t_2)}^{\tau}).$

$x(k,t)$ è SSL

$\varphi_{xx}(f) = \frac{403}{12}\{\delta(f-f_0) + \delta(f+f_0)\}.$

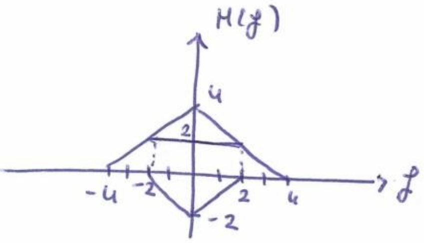
ES. 8

$\varphi_{NN}(f) = \frac{N_0}{2}$

$h(t) = 16\,\mathrm{sinc}^2(4t) - 4\,\mathrm{sinc}^2(2t)$

$H(f) = 4\,\mathrm{tri}\left(\frac{f}{4}\right) - 2\,\mathrm{tri}\left(\frac{f}{2}\right)$

$P_y = ?$



$\varphi_{yx}(f) = \frac{N_0}{2} \cdot \left\{ 4\,\mathrm{tri}\left(\frac{f}{4}\right) - 2\,\mathrm{tri}\left(\frac{f}{2}\right) \right\}^2$

$P_y = \int_{-\infty}^{+\infty} \varphi_{yx}(f)\, df = 2 \cdot \left\{ \int_0^4 (-f+4)^2 df + \int_0^2 (f-2)^2 df \right\}$

$= 2 \cdot \left\{ \left[\frac{f^3}{3} + 16f - 8\frac{f^2}{2} \right]_0^4 + \left[\frac{f^3}{3} + 4f - 4\frac{f^2}{2} \right]_0^2 \right\}$

$= 2 \cdot \left\{ \frac{64}{3} + \frac{20}{3} \right\} = 56.$

$= \frac{N_0}{2} 2 \cdot \left\{ \int_0^2 2^2\, df + \int_2^4 (-f+4)^2 df \right\} = N_0 \frac{32}{3}$

24/06/2011

ES. 1

$$s(t) = \frac{d}{dt}\left[\mathrm{rect}(2t+5)\cdot e^{j8\pi t} - \mathrm{sinc}(3t)\,\mathrm{sen}(8\pi t)\right]$$

$$S(f) = j2\pi f \cdot K(f)$$

$$K(f) \longleftrightarrow k(t) = z(t) + p(t)$$

$$z(t) = \mathrm{rect}\left(\overbrace{2t+5}^{2(t+\frac{5}{2})}\right)\cdot e^{j8\pi t}$$

$$Z(f) = \frac{1}{2}\cdot \mathrm{sinc}\left(\frac{f-4}{2}\right)\cdot e^{j2\pi\frac{5}{2}f}$$

$$P(f) = -\frac{1}{2j}\left[\frac{1}{3}\mathrm{rect}\left(\frac{f-4}{3}\right) - \frac{1}{3}\mathrm{rect}\left(\frac{f+4}{3}\right)\right]$$

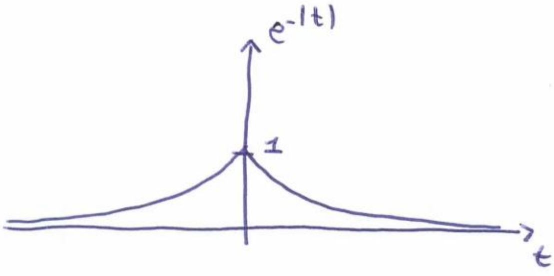
$$S(f) = j2\pi f\cdot\left\{-\frac{1}{2}\mathrm{sinc}\left(\frac{f-4}{2}\right)\cdot e^{j5\pi f} - \frac{1}{6j}\cdot\left[\mathrm{rect}\left(\frac{f-4}{3}\right) - \mathrm{rect}\left(\frac{f+4}{3}\right)\right]\right\}.$$

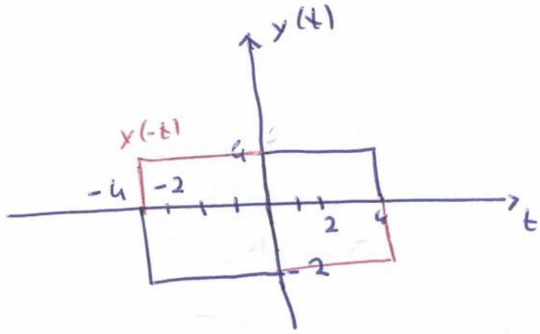
ES. 2

$$x(t) = e^{-|t|}$$

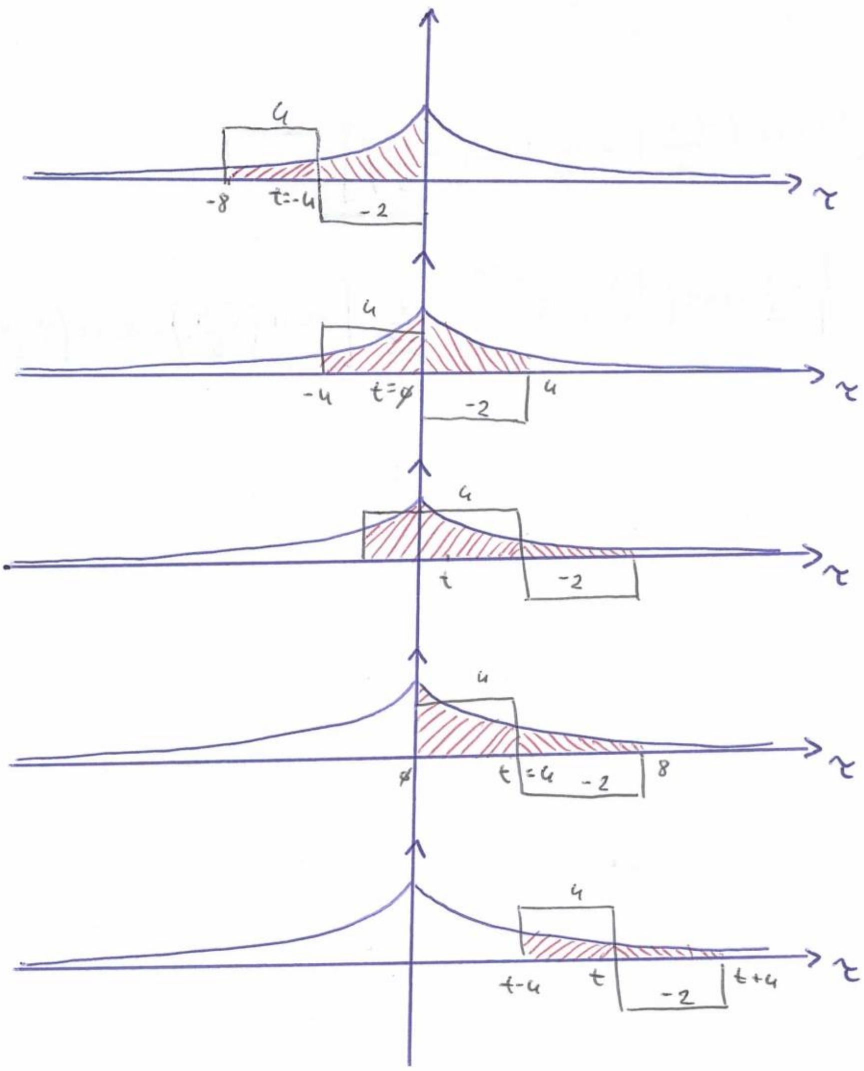
$$y(t) = 4 \cdot \mathrm{rect}\left(\frac{t-2}{4}\right) - 2\,\mathrm{rect}\left(\frac{t+2}{4}\right)$$

$$z(t) = x(t) * y(t).$$





$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau.$$



$$t \leq -4 \quad \rightarrow \quad z(t) = 4\int_{t-4}^{t} e^{t}\,dt - 2\int_{t}^{t+4} e^{t}\,dt = 4\left[e^{t}\right]_{t-4}^{t} - 2\left[e^{t}\right]_{t}^{t+4}$$

$$= 4e^{t} - 4e^{t-4} - 2e^{t+4} + 2e^{t} = 6e^{t} - 2e^{t+4} - 4e^{t-4}.$$

$$-4 < t \leq \emptyset \quad \rightarrow \quad z(t) = 4\int_{t-4}^{t} e^{t}\,dt - 2\int_{t}^{\emptyset} e^{t}\,dt - 2\int_{\emptyset}^{t+4} e^{-t}\,dt$$

$$= 4\left[e^{t}\right]_{t-4}^{t} - 2\left[e^{t}\right]_{t}^{\emptyset} - 2\left[-e^{-t}\right]_{\emptyset}^{t+4}$$

$$= 4e^{t} - 4e^{t-4} - 2 + 2e^{t} + 2e^{-t-4} - 2$$

$$= 6e^{t} + 2e^{-t-4} - 4e^{t-4} - 4.$$

$$\emptyset < t \leq 4 \quad \rightarrow \quad z(t) = 4\int_{t-4}^{\emptyset} e^{t}\,dt + 4\int_{\emptyset}^{t} e^{-t}\,dt - 2\int_{t}^{t+4} e^{-t}\,dt$$

$$= 4\left[e^{t}\right]_{t-4}^{\emptyset} + 4\left[-e^{-t}\right]_{\emptyset}^{t} - 2\left[-e^{-t}\right]_{t}^{t+4}$$

$$= 4 - 4e^{t-4} - 4e^{-t} + 4 + 2e^{-t-4} - 2e^{-t}$$

$$= -6e^{-t} + 2e^{-t-4} - 4e^{t-4} + 8.$$

$$t > 4 \quad \rightarrow \quad z(t) = 4\int_{t-4}^{t} e^{-t}\,dt - 2\int_{t}^{t+4} e^{-t}\,dt$$

$$= 4\left[-e^{-t}\right]_{t-4}^{t} - 2\left[-e^{-t}\right]_{t}^{t+4}$$

$$= -4e^{-t} + 4e^{-t+4} + 2e^{-t-4} - 2e^{-t}$$

$$= -6e^{-t} + 4e^{-t+4} + 2e^{-t-4}.$$

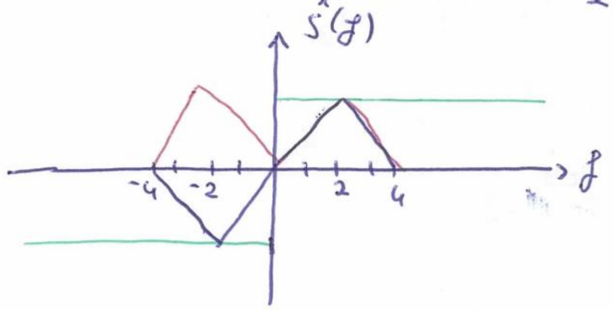
ES. 3

$s(t) = 2\,\mathrm{sinc}^2(2t)\,\mathrm{sen}(4\pi t)$

$\hat{s}(t) = 2\,\mathrm{sinc}^2(2t)\,\mathrm{sen}(4\pi t) * \frac{1}{\pi t}$

$S(f) = 2 \cdot \frac{1}{2} \cdot \frac{1}{2j} \cdot \left\{ \mathrm{tri}\left(\frac{f-2}{2}\right) - \mathrm{tri}\left(\frac{f+2}{2}\right) \right\}$

$\hat{S}(f) = -\frac{1}{2\cancel{j}} \cdot \cancel{j}\,\mathrm{segn}(f) \cdot \left\{ \mathrm{tri}\left(\frac{f-2}{2}\right) - \mathrm{tri}\left(\frac{f+2}{2}\right) \right\}$



$\hat{S}(f) = -\frac{1}{2} \left\{ \mathrm{tri}\left(\frac{f-2}{2}\right) + \mathrm{tri}\left(\frac{f+2}{2}\right) \right\}$.

ES. 5

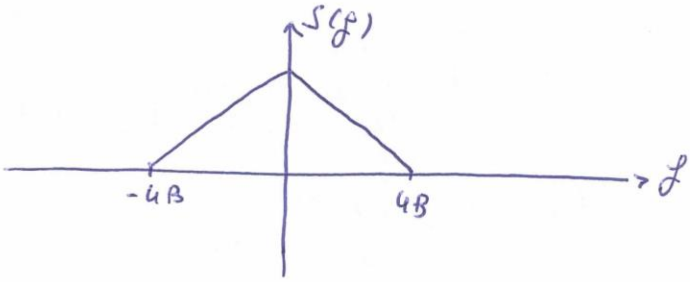
$s(t) = 4B \, \text{sinc}^2(4Bt)$

$f_c = 6B$

$F(f) = \text{rect}\left(\frac{f}{8B}\right)$

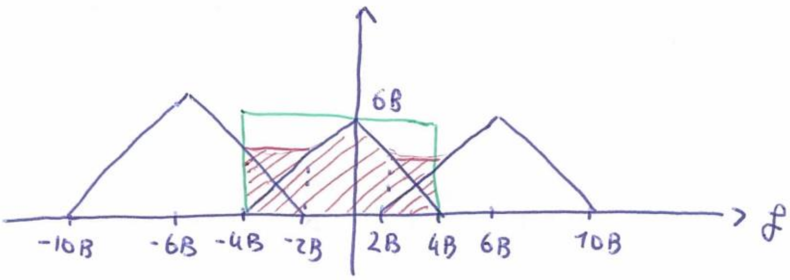
$S(f) = \text{tri}\left(\frac{f}{4B}\right)$

$Y(f) = ?$ $\quad y(t) = ?$



$B = 4B$

$f_c \geq 8B$ ma $f_c = 6B$



$Y(f) = 3B \, \text{rect}\left(\frac{f}{8B}\right) + 3B \, \text{tri}\left(\frac{f}{2B}\right)$

$y(t) = 24B^2 (\text{sinc}(8Bt) + 6B^2 \text{sinc}^2(2Bt)$

ES. 6

$f_A(a) = \frac{1}{2} \cdot e^{-2a} \cdot u(a)$

$E = \{ A \leq 2 \}$

$$E[a] = \int_{-\infty}^{+\infty} a \cdot f_A(a)\, da = \int_{\emptyset}^{+\infty} a \cdot \frac{1}{2} \cdot e^{-2a}\, da = \frac{1}{2} \int_{\emptyset}^{+\infty} a \cdot e^{-2a}\, da$$

$$= \left[-\frac{1}{4}(2a+1)\, e^{-2a} \right]_{\emptyset}^{+\infty} = \dots$$

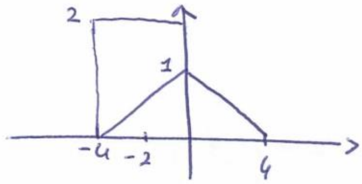
ES. 7

$\Psi_{NN}(f) = \frac{N_0}{2}$

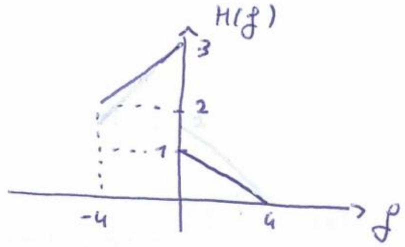
$R(t) = 4\,\text{sinc}^2(4t) + 8\,\text{sinc}(4t)\,e^{-j4\pi t}$

$P_y = ?$

$H(f) = \text{tri}\left(\frac{f}{4}\right) + 2\,\text{rect}\left(\frac{f+2}{4}\right)$



$\Psi_{yy}(f) = \frac{N_0}{2}\cdot\left|\text{tri}\left(\frac{f}{4}\right)+2\,\text{rect}\left(\frac{f+2}{4}\right)\right|^2$



$P_y = \int_{-\infty}^{+\infty}\Psi_{yy}(f)\,df = \left[\int_{-4}^{0}\left(\frac{1}{4} f+3\right)^2 df + \int_{0}^{4}\left(-\frac{1}{4} f+1\right)^2 df\right]\frac{N_0}{2} = N_0\cdot\frac{40}{3}$

$= \cancel{\frac{N_0}{2}\cdot\left[\frac{1}{4}\cdot\frac{f^2}{2}+3f\right]_{-4}^{0} + \frac{N_0}{2}\cdot\left[-\frac{1}{4}\frac{f^2}{2}+f\right]_{0}^{4}}$

$= \cancel{\frac{N_0}{2}\left(-\frac{1}{4}\cdot\frac{16}{2}+12\right)+\frac{N_0}{2}\left(-\frac{1}{4}\cdot\frac{16}{2}+4\right) = \frac{N_0}{2}\cdot 10 + \frac{N_0}{2}\cdot 2 = 6N_0}$

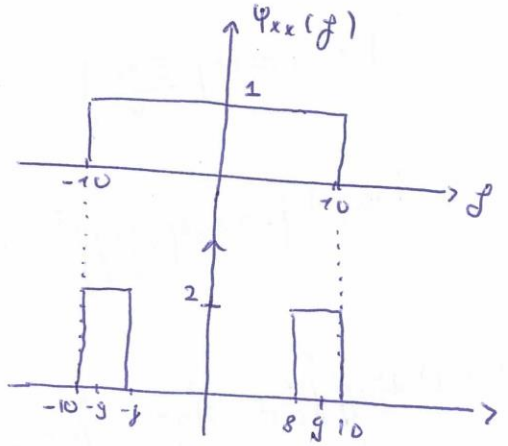
ES. 8

$R_{xx}(\tau) = 20 \, \text{sinc}(20\tau)$

$\varphi_{yy}(f) = 2\,\text{rect}\left(\frac{f-9}{2}\right) + 2\,\text{rect}\left(\frac{f+9}{2}\right)$

$h(t) = ?$

$\varphi_{xx}(f) = \text{rect}\left(\frac{f}{20}\right)$



$|H(f)|^2 = 2\,\text{rect}\left(\frac{f-9}{2}\right) + 2\,\text{rect}\left(\frac{f+9}{2}\right)$

$H(f) = \sqrt{2}\,\text{rect}\left(\frac{f-9}{2}\right) + \sqrt{2}\,\text{rect}\left(\frac{f+9}{2}\right)$

$h(t) = 2\sqrt{2}\,\text{sinc}(2t)\,e^{j18\pi t} + 2\sqrt{2}\,\text{sinc}(2t)\,e^{-j18\pi t}$

$= 4\sqrt{2}\,\text{sinc}(2t)\cos(18\pi t)$

www.alessandropaghi.it

10/05/2011

ES. 1

$$s(t) = \int_{-\infty}^{t} \text{tri}(2\tau+3) \cdot \cos(8\pi\tau + 4\pi)\, d\tau$$

$$S(f) = \frac{1}{j2\pi f} \cdot K(f)$$

$$K(f) \longleftrightarrow k(t) = \text{tri}(2t+3) \cdot \cos(8\pi t + 4\pi)$$
$$= \text{tri}(2t+3) \cdot \cos(8\pi t)$$

$$K(f) = \frac{1}{2} \cdot \left\{ \frac{1}{2} \text{sinc}^2\left(\frac{f-4}{2}\right) \cdot e^{j2\pi\frac{3}{2}(f-4)} + \frac{1}{2} \text{sinc}^2\left(\frac{f+4}{2}\right) e^{j2\pi\frac{3}{2}(f+4)} \right\}$$
$$= \frac{1}{4} \left\{ \text{sinc}^2\left(\frac{f-4}{2}\right) e^{j3\pi(f-4)} + \text{sinc}^2\left(\frac{f+4}{2}\right) e^{j3\pi(f+4)} \right\}$$

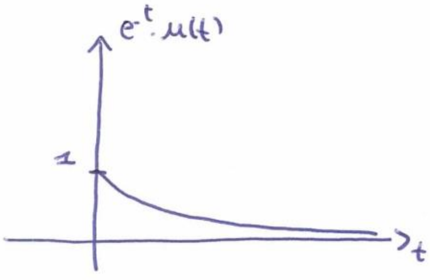
$$S(f) = \frac{1}{j8\pi f} \cdot \left\{ \text{sinc}^2\left(\frac{f-4}{2}\right) \cdot e^{j3\pi(f-4)} + \text{sinc}^2\left(\frac{f+4}{2}\right) \cdot e^{j3\pi(f+4)} \right\}$$

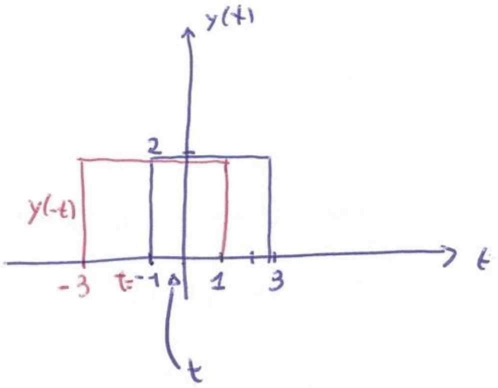
ES. 2

$$x(t) = e^{-t}u(t)$$

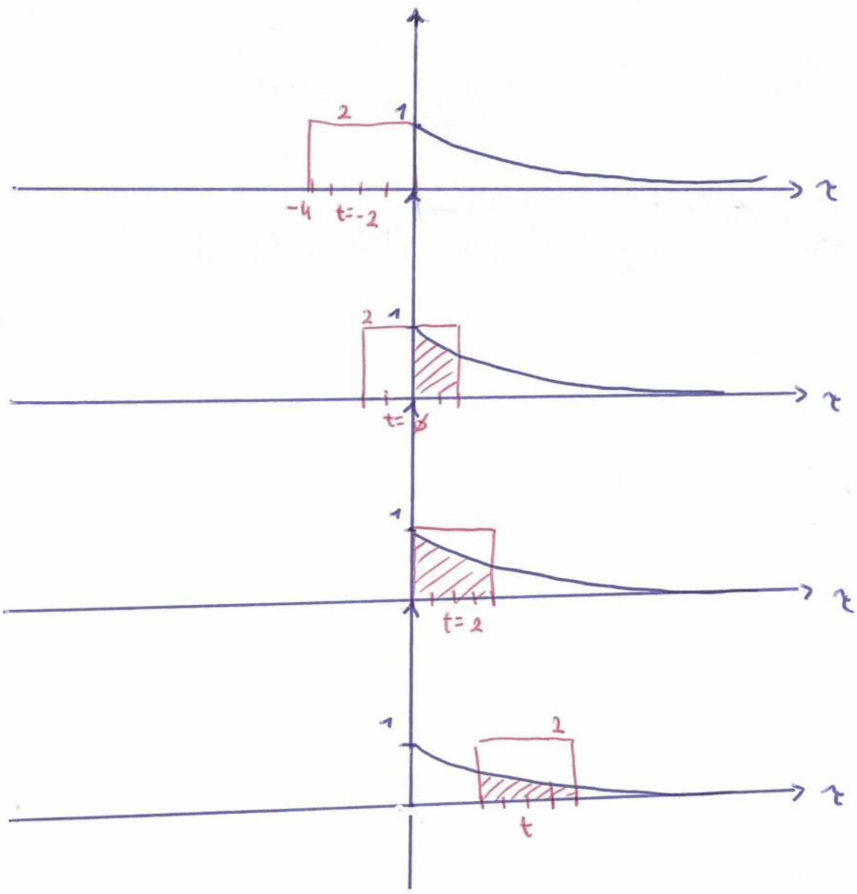
$$y(t) = 2\,\mathrm{rect}\left(\frac{t-1}{4}\right)$$

$$z(t) = x(t) * y(t)$$





$$z(t) = \int_{-\infty}^{+\infty} x(\tau)\, y(t-\tau)\, d\tau$$



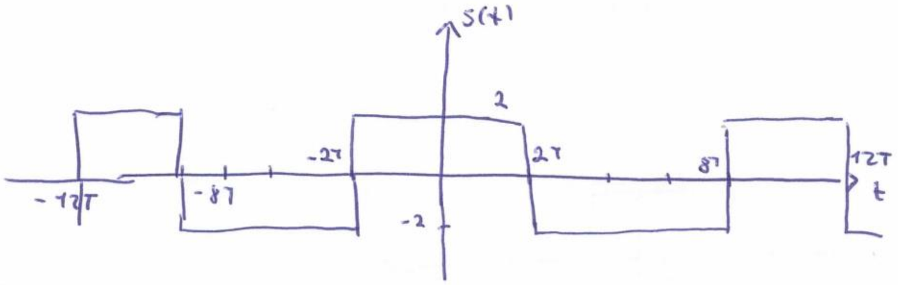
$t \leq -2 \;\rightarrow\; z(t) = \varnothing$

$-2 < t \leq 2 \;\rightarrow\; z(t) = 2\int_{\varnothing}^{t+2} e^{-t}\,dt = 2\left[-e^{-t}\right]_{\varnothing}^{t+2} = \left\{-e^{-(t+2)} + 1\right\}\cdot 2$
$= \left\{1 - e^{-t-2}\right\}\cdot 2$

$t > 2 \;\rightarrow\; z(t) = 2\int_{t-2}^{t+2} e^{-t}\,dt = 2\left[-e^{-t}\right]_{t-2}^{t+2} = \left\{-e^{-(t+2)} + e^{-(t-2)}\right\}\cdot 2$
$= \left\{e^{-t+2} - e^{-t-2}\right\}\cdot 2$

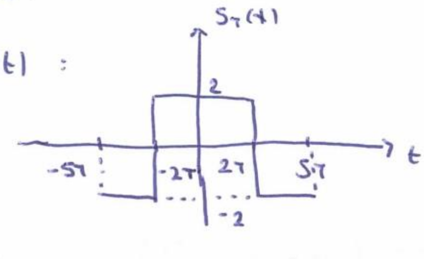
$$z(t) = \begin{cases} \varnothing & t \leq -2 \\ 2 - 2e^{-t-2} & -2 < t \leq 2 \\ 2e^{-t+2} - 2e^{-t-2} & t > 2 \end{cases}$$

ES. 3

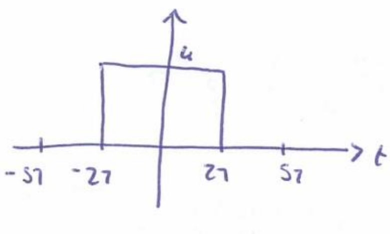


$T = 10\tau$

$S_T(t)$:



$\rightarrow$



$S_T(t) = 4\,\mathrm{rect}\left(\frac{t}{4\tau}\right) - 2 \;\longleftrightarrow\; 4\cdot 4\tau\,\mathrm{sinc}(4\tau\cdot f) - 2\delta(f)$

$$S_p(t) = \left[\sum_{m=-\infty}^{+\infty} 4\cdot \mathrm{rect}\left(\frac{t - m10\tau}{4\tau}\right)\right] - 2$$

$$S_p(f) = \left[\frac{1}{10\tau}\cdot\sum_{m=-\infty}^{+\infty} 16\tau\,\mathrm{sinc}\left(4\tau\cdot\frac{m}{10\tau}\right)\cdot\delta\left(f - \frac{m}{10\tau}\right)\right] - 2\delta(f)$$

$$= \left[\frac{8}{5}\cdot\sum_{m=-\infty}^{+\infty}\mathrm{sinc}\left(\frac{2}{5}\cdot m\right)\cdot\delta\left(f - \frac{m}{10\tau}\right)\right] - 2\delta(f)$$

ES. 4

$s(t) = \mathrm{sinc}(2t)\, e^{j2\pi t}$

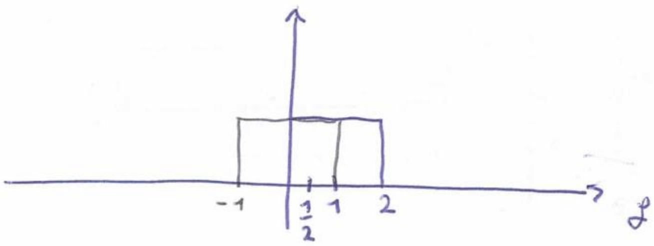
$h(t) = \mathrm{sinc}(2t+1) = \mathrm{sinc}\left(2\left(t+\frac{1}{2}\right)\right)$

$y(t) = \int_{-\infty}^{+\infty} s(\tau)\, h(t-\tau)\, d\tau \quad , \quad Y(f) = S(f) \cdot H(f)$

$S(f) = \frac{1}{2} \mathrm{rect}\left(\frac{f-1}{2}\right) \quad , \quad H(f) = \frac{1}{2} \mathrm{rect}\left(\frac{f}{2}\right) \cdot e^{+j2\pi \frac{1}{2} f}$

$Y(f) = \frac{1}{2} \mathrm{rect}\left(\frac{f-1}{2}\right) \cdot \frac{1}{2} \mathrm{rect}\left(\frac{f}{2}\right) \cdot e^{j2\pi \frac{1}{2} f}$

$= \frac{1}{4} \mathrm{rect}\left(\frac{f-1}{2}\right) \cdot \mathrm{rect}\left(\frac{f}{2}\right) \cdot e^{j\pi f}.$



$= \frac{1}{4} \mathrm{rect}\left(\frac{f-\frac{1}{2}}{1}\right) \cdot e^{j\pi f}$

$y(t) = \frac{1}{4} \cdot \mathrm{sinc}\left(\left(t+\frac{1}{2}\right)\right) \cdot e^{-j2\pi \frac{1}{2} t} = \frac{1}{4} \mathrm{sinc}\left[t+\frac{1}{2}\right] e^{-j\pi t}.$

Es. 5

$s(t) = \operatorname{sinc}^2(200t)\, e^{-j400\pi t} + \operatorname{sinc}^2(200t) \cdot e^{j400\pi t}$

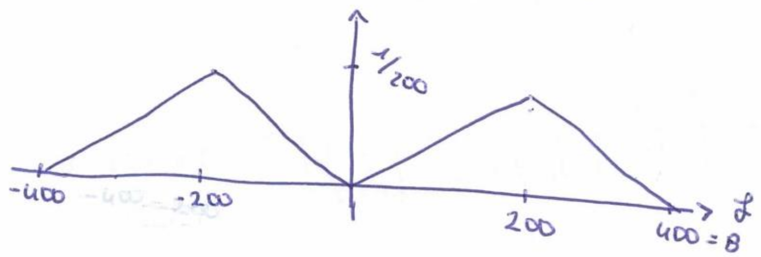
$f_c = 2B$

$n_{bit/c} = 16$ bit

$V_T = 1$ Mbit/s

secondi di segnale trasmessi in 1 secondo sulla linea.

$S(f) = \frac{1}{200} \operatorname{tri}\left(\frac{f+200}{200}\right) + \frac{1}{200} \operatorname{tri}\left(\frac{f-200}{200}\right)$



$B = 400$ Hz , $f_c = 800$ Hz , $n_{c/sec} = 800$.

$n_{bit/tempo} = V_T \cdot \text{Tempo Trasmissione} = 1\ \text{Mbit} = 1.00{:}000.0$ bit

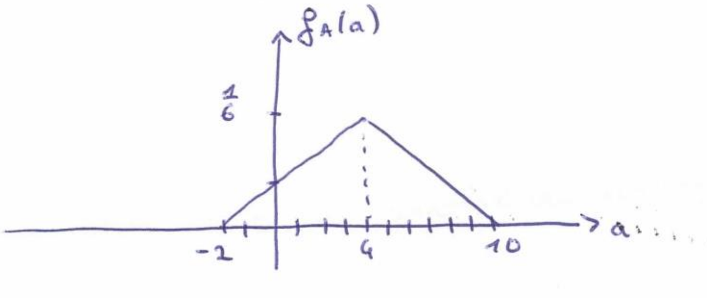
$n_{b/sec} = 800 \cdot 16 = 12800$ bit

$n_{bit/tempo} = n_{b/sec} \cdot \text{tempo} \rightarrow \text{tempo} = \frac{n_{bit/tempo}}{n_{bit/sec}} = 78{,}125$ sec .

ES. 6

$f_A(a) = \frac{1}{6} \, \mathrm{tri}\left(\frac{a-4}{6}\right)$ $\qquad E(a)$? $\qquad \sigma^2(a)$? $\qquad P(E)$?

$E = \{A \leq 5\}$



$E(a) = 4$

$$E(a^2) = \int_{-2}^{4} a^2 \cdot \left(\frac{1}{36}a + \frac{1}{18}\right) da + \int_{4}^{10} a^2 \cdot \left(-\frac{1}{36}a + \frac{5}{18}\right) da$$

$$= \left[\frac{a^4}{4} \cdot \frac{1}{36} + \frac{a^3}{3} \cdot \frac{1}{18}\right]_{-2}^{4} + \left[\frac{a^4}{4} \cdot \left(-\frac{1}{36}\right) + \frac{a^3}{3} \cdot \left(\frac{5}{18}\right)\right]_{4}^{10} = 3 + 19 = 22$$

$\sigma^2(a) = 22 - 16 = 6$.

$$P(E) = \int_{-2}^{4} \frac{1}{36}a + \frac{1}{18} \, da + \int_{4}^{5} -\frac{1}{36}a + \frac{5}{18} \, da$$

$$= \left[\frac{1}{36} \cdot \frac{a^2}{2} + \frac{1}{18} \cdot a\right]_{-2}^{4} + \left[-\frac{1}{36} \cdot \frac{a^2}{2} + \frac{5}{18} a\right]_{4}^{5} = \frac{47}{72} .$$

ES. 7

$x(k,t) = (A-B)\cdot\cos(2\pi f_0 t - 2\theta)$

$f_A(a) = \frac{1}{2}\,\mathrm{rect}\left(\frac{a}{2}\right)$, $f_B(b) = \frac{1}{6}\,\mathrm{rect}\left(\frac{b-2}{6}\right)$, $\Theta \sim 0 \div 2\pi$

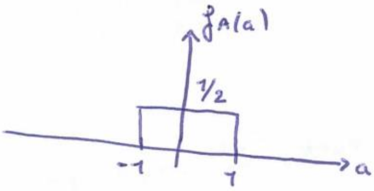
$x(k,t)$ SSL ? $\quad R_{xx}(\tau) = ?$

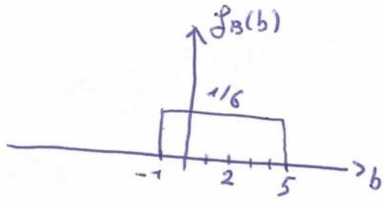
$\mu_x = E[x(k,t)] = E[(A-B)\cos(2\pi f_0 t - 2\theta)]$

$= \{E[A] - E[B]\}\cdot E[\cos(2\pi f_0 t - 2\theta)] = 0.$

$E[A] = 0$

$E[B] = 2$





$E[A^2] = \int_{-1}^{1} a^2\cdot\frac{1}{2}\,da = \left[\frac{a^3}{6}\right]_{-1}^{1} = \frac{1}{6}+\frac{1}{6} = \frac{1}{3}$

$E[B^2] = \int_{-1}^{5} b^2\cdot\frac{1}{6}\,db = \left[\frac{b^3}{18}\right]_{-1}^{5} = \frac{125}{18}+\frac{1}{18} = 7$

$R_{xx}(t_1,t_2) = E[x(k,t_1)\cdot x(k,t_2)] =$

$= E[(A-B)\cos(2\pi f_0 t_1 - 2\theta)\cdot(A-B)\cos(2\pi f_0 t_2 - 2\theta)]$

$= E[(A-B)^2\cdot\cos(2\pi f_0 t_1 - 2\theta)\cos(2\pi f_0 t_2 - 2\theta)]$

$= \{E[A^2] + E[B^2] - 2\cancel{E[A]E[B]}^{\,0}\}\cdot\frac{1}{2}\{E[\cancel{\cos(2\pi f_0(t_1+t_2)-4\theta)}^{\,0} + \cos(2\pi f_0(t_1-t_2))]$

$= \frac{7}{6}\cos(2\pi f_0 \tau).$

ES. 8

$C_{xx}(\tau) = \mathrm{sinc}^2(12\tau)$

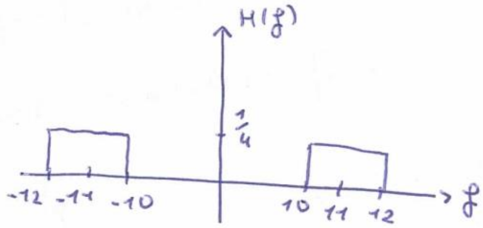
$\mu_x = \emptyset$ $\qquad\qquad\qquad\qquad$ $P_y = ?$

$h(t) = \mathrm{sinc}(2t)\cos(22\pi t)$

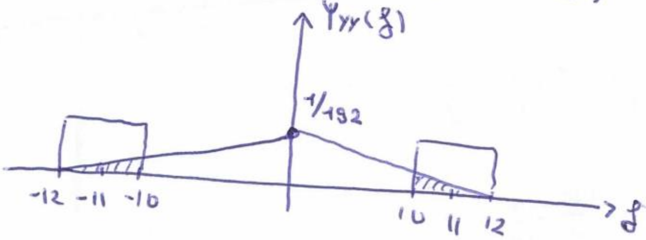
$R_{xx}(\tau) = C_{xx}(\tau) + \mu_x^2 = \mathrm{sinc}^2(12\tau)$

$\Psi_{xx}(f) = \frac{1}{12}\,\mathrm{tri}\left(\frac{f}{12}\right).$

$H(f) = \frac{1}{4}\left\{\mathrm{rect}\left(\frac{f-11}{2}\right) + \mathrm{rect}\left(\frac{f+11}{2}\right)\right\}$



$\Psi_{yy}(f) = \Psi_{xx}(f)\cdot|H(f)|^2 = \frac{1}{192}\,\mathrm{tri}\left(\frac{f}{12}\right)\cdot\left\{\mathrm{rect}\left(\frac{f-11}{2}\right) + \mathrm{rect}\left(\frac{f+11}{2}\right)\right\}$



$P_y = \int_{-\infty}^{+\infty}\Psi_{yy}(f)\,df = 2\cdot\int_{10}^{12} -\frac{1}{2304}\cdot f + \frac{1}{192}\,df = 2\cdot\left[-\frac{1}{2304}\cdot\frac{f^2}{2} + \frac{1}{192}\cdot f\right]_{10}^{12} = \frac{1}{576}.$