

FISICA 2

[Appunti Di Esercizi]

A CURA DI ALESSANDRO PAGHI

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LINK AL CORSO:

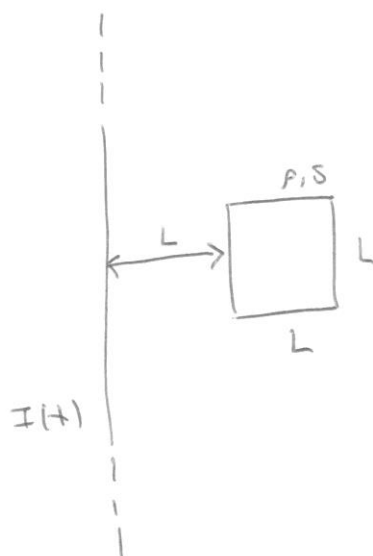
<http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=55107&aa=2015>

FREQUENTAZIONE: Consigliata.

10/12/2014

ES. 1

$$I(t) = I_1 \sin(\omega t) + I_2 \sin(2\omega t)$$



? $P(t)$, P dissipata dalla spira

$$\vec{B} = \frac{\mu_0 I(t)}{2\pi r}$$

$$\Phi(\vec{B}) = \int_L^{2L} \vec{B} \cdot d\vec{s} = \int_L^{2L} \frac{\mu_0 I(t)}{2\pi r} L \cdot dr = \frac{\mu_0 I(t)}{2\pi} \ln(2) \cdot L$$

$$\mathcal{E}(t) = - \frac{d}{dt} \Phi(\vec{B}) = \frac{\mu_0 L}{2\pi} \cdot \dot{I}(t) \ln(2)$$

$$P(t) = \frac{\mathcal{E}^2(t)}{R \frac{4L}{S}}$$

$$\langle P_m \rangle = \frac{1}{T} \int_0^T P(t) dt$$

$\frac{2\pi}{\omega}$ (period)
 $\frac{1}{T}$ (average factor)

ES. 2

$$r = 1,25 \text{ cm}$$

$$\eta = 100 \text{ pice/m}$$

$$I = 2,5 \text{ A}$$

$$? \mathcal{E}(t)$$

$$\mathcal{E}(t) = -\frac{d}{dt} \Phi(\vec{B})$$

$$\Phi(\vec{B}) = \int \vec{B} \cdot d\vec{S}$$

$$\vec{B} = \mu_0 \eta I$$

Area settore

$$A = \frac{\theta}{2\pi} \pi R^2 = \frac{\theta}{2} R^2$$

$$e = \sqrt{r^2 - (r-x)^2} = \sqrt{r^2 - r^2 + x^2 + 2rx} = \sqrt{x^2 + 2rx} = r \cdot \sin\left(\frac{\theta}{2}\right)$$

$$\text{Area} = A - \cancel{\frac{\theta}{2}} r \sin\left(\frac{\theta}{2}\right) \cdot \frac{(r-x)}{\cancel{2}} = \frac{\theta}{2} r^2 - (r-x) r \sin\left(\frac{\theta}{2}\right)$$

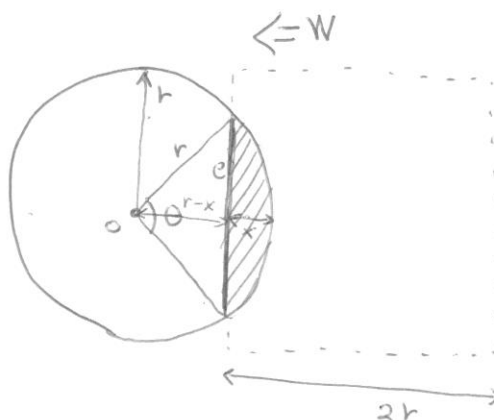
$$= \frac{\theta}{2} r^2 - r^2 \cos\left(\frac{\theta}{2}\right) \sin\left(\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2} r^2 - r^2 \frac{1}{2} \sin(2 \cdot \frac{\theta}{2}) = r^2 \left(\frac{\theta}{2} - \sin \theta \right)$$

$$\Phi(\vec{B}) = \mu_0 \eta I \cdot \text{Area}$$

$$\mathcal{E}(t) = -\frac{d}{dt} \Phi(\vec{B})$$

$$P(t) = \frac{\mathcal{E}^2(t)}{R} \quad ; \quad F = \frac{P(t)}{W}$$



$$0 < \theta < 180^\circ$$

$$\theta = 0^\circ \Rightarrow A = 0$$

$$\theta = 180^\circ \Rightarrow A = \frac{\pi r^2}{2}$$

$$x = w \cdot t$$

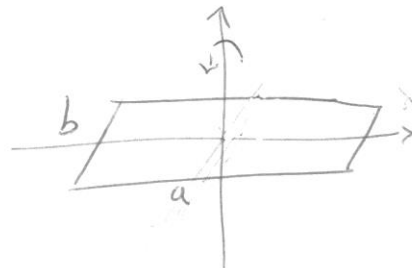
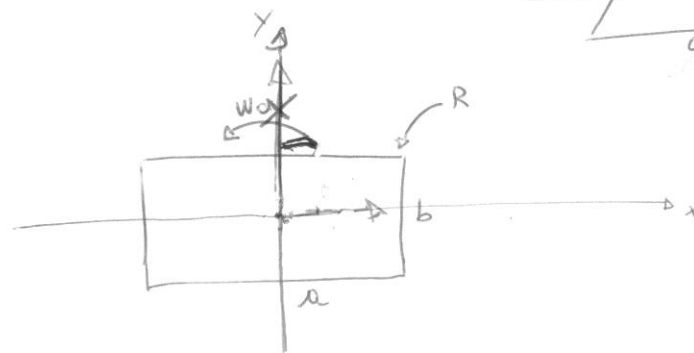
$$r \cdot \cos\left(\frac{\theta}{2}\right) = r-x$$

$$\cos\left(\frac{\theta}{2}\right) = \frac{r-x}{r}$$

$$\theta = 2 \arccos\left(\frac{r-x}{r}\right)$$

$$x = w \cdot t$$

ES. 3



? f.e.m., $P(t)$, τ

$$\phi(\vec{B}) = \int \vec{B} \cdot d\vec{S} =$$

$$= B_0 \vec{\mu}_0 \cdot \vec{\mu}_m \cdot a \cdot b \cos(\omega_0 t) = B_0 \cos(30^\circ) \cdot a b \cdot \cos(\omega_0 t)$$

$$\vec{\mu}_0 \cdot \vec{\mu}_m = \cos(30^\circ) = \sin(60^\circ)$$

$$= B_0 \frac{\sqrt{3}}{2} \cdot a b \cdot \cos(\omega_0 t)$$

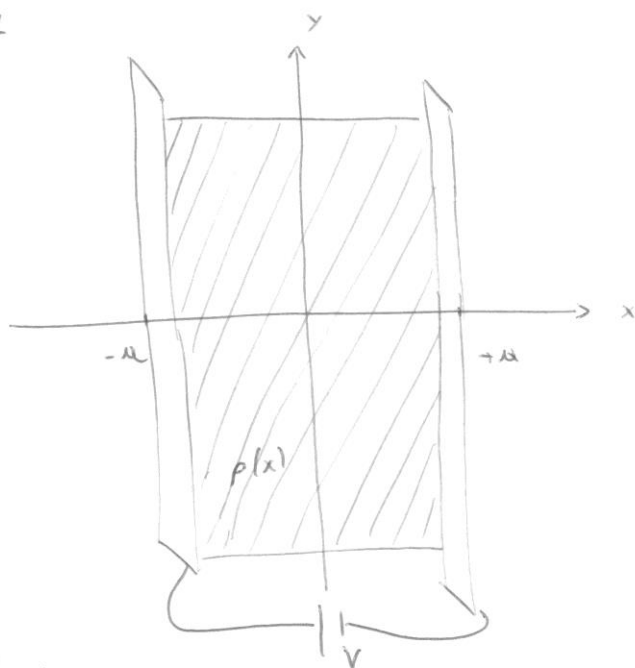
$$\mathcal{E}(t) = -\frac{d}{dt} \phi(\vec{B})$$

$$P(t) = \frac{\mathcal{E}^2(t)}{R}, \quad \tau = \frac{P(t)}{\omega_0}$$

$$\Phi(\vec{v}) = \int \vec{v} \cdot \vec{\mu}_m \cdot d\vec{r}$$

24/11/2014

ES. 1



$$\rho(x) = \rho_0 \left[1 - \left(\frac{x}{2a} \right)^2 \right]$$

$E(x)$ $t = \phi$, $t = \infty$

$t = \phi$

E unif, D non unif.

$$V_0 = \int E \cdot dS = E \cdot 2a \Rightarrow E = \frac{V_0}{2a}, t = \phi$$

$t = \infty$

E non unif, D unif

$$E(x) = \rho(x) \cdot D$$

$$V_0 = \int_{-a}^{+a} E(x) \cdot dx = \int_{-a}^{+a} \rho_0 \left[1 - \left(\frac{x}{2a} \right)^2 \right] \cdot D \cdot dx$$

$$= \rho_0 D \int_{-a}^{+a} 1 - \frac{x^2}{4a^2} dx = \rho_0 D \left[x - \frac{x^3}{12a^2} \right]_{-a}^{+a}$$

$$= \rho_0 D \left[a - \frac{a^3}{12a^2} + a - \frac{a^3}{12a^2} \right] = \rho_0 D \cdot \frac{11}{6} a$$

$$D = \frac{V_0 \cdot 6}{11 \cdot \rho_0 \cdot a}$$

$$E(x) = \rho(x) \cdot D, t = \infty$$

? Equivalenza $\vec{E}$ presente elettrico.

$$E(x) = \rho(x) \cdot \gamma$$

$$\frac{d}{dx} E(x) = \frac{d}{dx} \rho(x) \cdot \gamma \rightarrow \rho(x) \cdot \frac{d}{dx} \gamma = \frac{\lambda}{\epsilon_0}$$

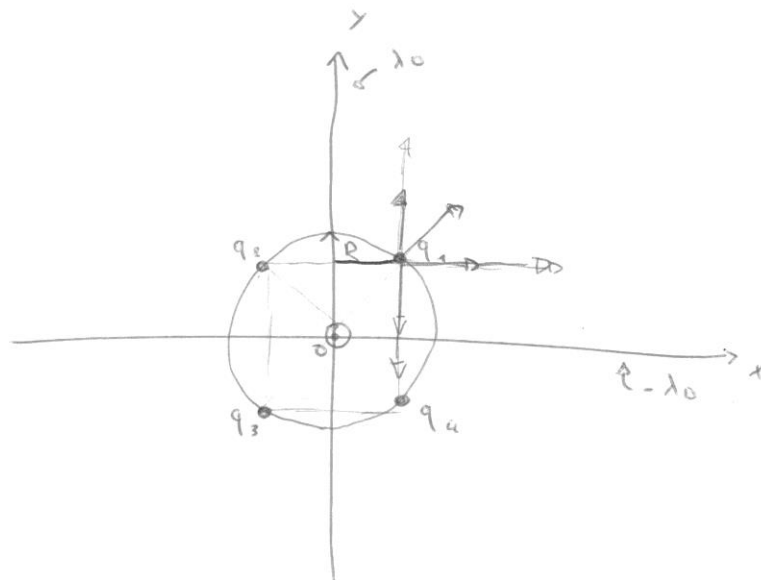
$$\lambda = \frac{d}{dx} \rho(x) \cdot \gamma \cdot \epsilon_0$$

? Densità di Potenza in funzione di x , $t=0$, $t=\infty$.

$$t=0 \Rightarrow P = \frac{E(x)^2}{\rho(x)}$$

$$t=\infty \Rightarrow P = \gamma^2 \cdot \rho(x)$$

ES. 2



? Te ou q_1

$$E = \frac{K \cdot q}{(\sqrt{2} \cdot R)^2} + \frac{K \cdot q}{(\sqrt{2} \cdot R)^2} + \frac{K \cdot q}{(2R)^2} + \frac{2K \cdot \lambda_0}{\frac{\sqrt{2}}{2} R} + \frac{2K \cdot (-\lambda_0)}{\frac{\sqrt{2}}{2} R}$$

q_2 :

$$E_2 = \frac{K \cdot q}{(\sqrt{2} \cdot R)^2} \cdot \mu_x$$

q_4 :

$$E_4 = \frac{K \cdot q}{(\sqrt{2} \cdot R)^2} \cdot \mu_y$$

q_3 :

$$E_3 = \frac{K \cdot q}{(2R)^2} \begin{pmatrix} \frac{\sqrt{2}}{2} \mu_x \\ \frac{\sqrt{2}}{2} \mu_y \end{pmatrix}$$

$$\lambda_0: \frac{2K \cdot \lambda_0}{\frac{\sqrt{2}}{2} R} \mu_x$$

$$-\lambda_0: E_{-\lambda_0} = -\frac{2K \lambda_0}{\frac{\sqrt{2}}{2} R} \cdot \mu_y$$

$$\begin{cases} E_{\text{tot } x} = E_2 + E_3(x) + E_{\lambda_0} \\ E_{\text{tot } y} = E_4 + E_3(y) + E_{-\lambda_0} \end{cases}$$

$$E_{\text{tot}} = \sqrt{E_{\text{tot } x}^2 + E_{\text{tot } y}^2}$$

com r_0 potencial
nulo

? V ou z.

$$V_{q_1}(z) = \frac{K \cdot q}{\sqrt{R^2 + z^2}}$$

filio r_0

$$\begin{aligned} V(r) &= \int_r^\infty E(r) dr, \quad V(\infty) = 0 \\ &= 2K\lambda \cdot \ln(r) \Big|_r^\infty = +2K\lambda \ln\left(\frac{r_0}{r}\right) \end{aligned}$$

3 due fili si elidono nei contributi

$$V(z) = u \cdot \frac{K \cdot q}{\sqrt{R^2 + z^2}}$$

$$z \rightarrow \infty$$

$$V(z) = \frac{u K q}{z} \quad \underline{\underline{Q = u q}}$$

ES.3

$$\vec{E}(x, y, z) = \vec{E}(z) = E_0 \left(e^{-\alpha|z|}, 0, 0 \right)$$

$$\vec{B}(x, y, z) = \vec{B}(z) = B_0 \left(0, 0, \beta z^2 \right)$$

? $\vec{E}$ possibile produrre in una regione di spazio

$$\text{div } \vec{E} = \frac{d}{dx} E_x + \frac{d}{dy} E_y + \frac{d}{dz} E_z = \frac{\rho}{\epsilon_0} = 0 \Rightarrow \rho = 0$$

affinché $\vec{E}$ possa esistere dobbiamo essere nel vuoto.

$$\text{rot } \vec{E} = 0$$

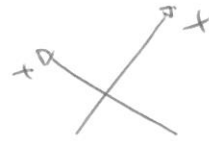
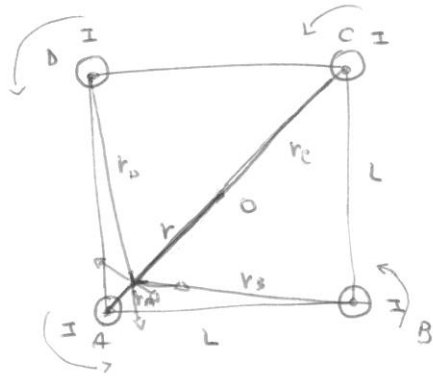
$$\frac{\partial}{\partial z} E_x = -\alpha E_0 e^{-\alpha|z|} \cdot \text{sgn}(z)$$

$\vec{E}$ non può esistere

$$\text{div } \vec{B} = \beta \cdot z^2 \neq 0, \quad \text{rot } \vec{B} = 0$$

$\vec{B}$ non può esistere

ES. 4



? Campo magnetico sulla diagonale AC

legge di Ampere

$$B(r) = \frac{\mu_0 I}{2\pi r}$$

$K(r)$ non noto da calcolare

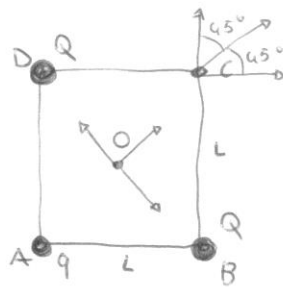
$$r_a = R - r$$

$$r_c = R + r$$

$$r_b = r_s = K(r)$$

22/01/2015

ES. 1



? q : $|E_o| = |E_e|$

E_o : contributo di B, D al elemento

$$E_o = k \cdot \frac{q}{\left(\frac{L\sqrt{2}}{2}\right)^2} = \frac{2k \cdot q}{L^2}$$

E_e : Si somma anche alla A e

$$E_e = k \cdot \frac{q}{(L\sqrt{2})^2} + \sqrt{\left(\frac{k \cdot Q}{L^2}\right)^2 + \left(\frac{k \cdot Q}{L^2}\right)^2} = \frac{k \cdot q}{2L^2} + \sqrt{\frac{k^2 Q^2}{L^4} \cdot 2}$$

$$= \frac{k \cdot q}{2L^2} + \frac{kQ}{L^2} \cdot \sqrt{2}$$

$|E_o| = |E_e|$

$E_o = +E_e$

$$\frac{2kq}{L^2} = \frac{kq}{2L^2} + \frac{kQ}{L^2} \cdot \sqrt{2}$$

$$2q = \frac{q}{2} + Q\sqrt{2}$$

$$q_1 = \frac{2}{3}\sqrt{2} \cdot Q$$

? ΔV_{oc}

$\Delta V_{oc} = V_o - V_e$

$$V_o = \frac{k \cdot q}{\frac{L\sqrt{2}}{2}} + \frac{k \cdot Q}{\frac{L\sqrt{2}}{2}} + \frac{k \cdot Q}{\frac{L\sqrt{2}}{2}}$$

$$V_e = \frac{k \cdot q}{L\sqrt{2}} + \frac{k \cdot Q}{L} + \frac{k \cdot Q}{L}$$

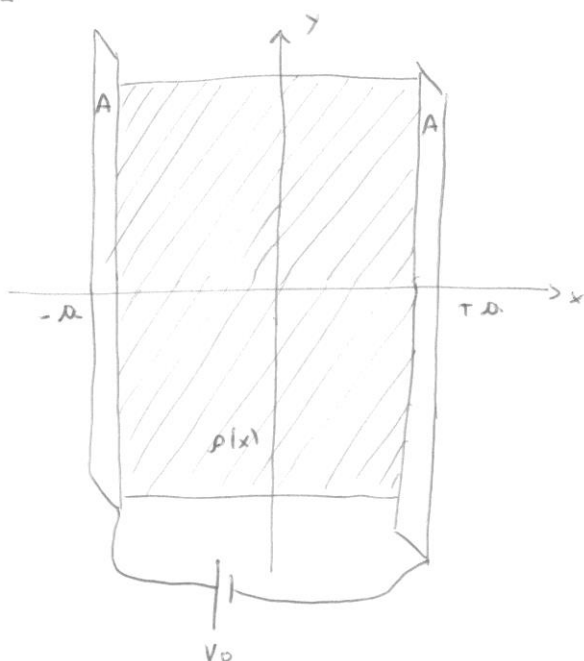
$E_o = -E_e$

$$\frac{2k \cdot q}{L^2} = -\frac{kq}{2L^2} - \frac{kQ}{L^2} \cdot \sqrt{2}$$

$$2q = -\frac{q}{2} - Q\sqrt{2}$$

$$q_2 = -\frac{2}{5}\sqrt{2} \cdot Q$$

Es. 2



$$\rho(x) = \rho_0 \cdot \cos(k \cdot x) \quad , \quad k = \frac{1}{a}$$

$$= \rho_0 \cdot \cos\left(\frac{1}{a} \cdot x\right)$$

? $E(x)$ a $t=0$; $t=+\infty$.

$t=0$: E uniforme, J non uniforme

$$V_0 = \int E \cdot d\epsilon = E \int d\epsilon = E \cdot 2a \Rightarrow E = \frac{V_0}{2a} \text{ a } t=0.$$

$t=\infty$: E non uniforme, J uniforme

$$E(x) = \rho(x) \cdot J = \rho_0 \cdot \cos\left(\frac{x}{a}\right) \cdot J$$

$$V_0 = \int_{-a}^{+a} E(x) dx = \rho_0 J \cdot \int_{-a}^{+a} \cos\left(\frac{x}{a}\right) dx = \rho_0 J \left[a \cdot \sin\left(\frac{x}{a}\right) \right]_{-a}^{+a}$$

$$= \rho_0 J \cdot 2a^2 \cdot \sin(1)$$

$$J = \frac{V_0}{2\rho_0 a^2 \sin(1)}$$

$$E(x) = \rho(x) \cdot J = \cos\left(\frac{x}{a}\right) \cdot \frac{1}{2a^2 \sin(1)}$$

? $t=\infty$, distribuita carica nel volume

$\text{div } E \neq 0$: E distribuita carica nel volume!

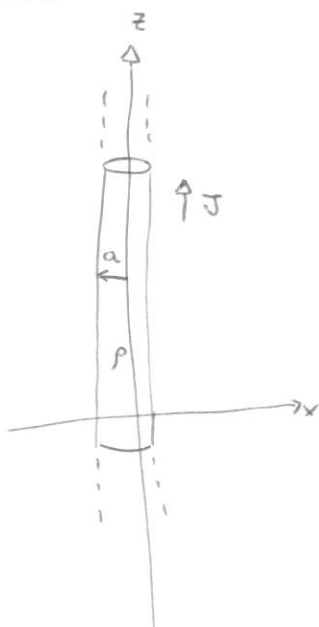
$$\frac{d}{dx} E(x) = \frac{d}{dx} \rho(x) \cdot J + \rho(x) \cdot \frac{d}{dx} J(x) = \frac{\lambda}{\epsilon_0}$$

$$\lambda = \frac{d}{dx} \rho(x) \cdot J \cdot \epsilon_0 \quad , \quad \text{carica distribuita come } \lambda.$$

? Corrente a I_∞

$$I_\infty = J_\infty \cdot A$$

ES. 3



$$? \frac{P}{\Delta y} \quad \frac{P}{\Delta V}$$

$$P = R \cdot J^2$$

$$R = \rho \cdot \frac{l}{S} = \rho \cdot \frac{\Delta y}{\pi a^2}$$

$$I = \iint J \cdot dS = J \pi a^2$$

$$P = \rho \cdot \frac{\Delta y}{\pi a^2} \cdot J^2 \cdot \pi a^2 = \rho \Delta y \cdot J^2 \cdot \pi \cdot a^2$$

$$\frac{P}{\Delta y} = \rho J^2 \pi a^2$$

$$\frac{P}{\Delta V} = \rho J^2$$

? $\vec{B}$ a distanza x

Legge di Ampere

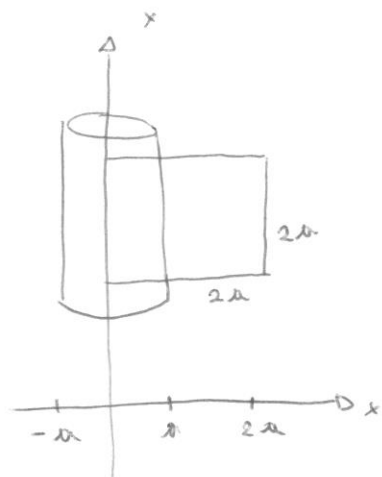
$$x < a$$

$$\mu_0 \cdot I = B_i \cdot 2\pi x \Rightarrow B_i = \frac{\mu_0 \cdot I}{2\pi x} = \frac{\mu_0 \cdot J \cdot \pi x^2}{2\pi x} = \frac{\mu_0 J x}{2}$$

$$x > a$$

$$\mu_0 \cdot I = B_e \cdot 2\pi x \Rightarrow B_e = \frac{\mu_0 I}{2\pi x} = \frac{\mu_0 \cdot J \cdot \pi a^2}{2\pi x} = \frac{\mu_0 J a^2}{2x}$$

? Flusso campo attraverso la spirale



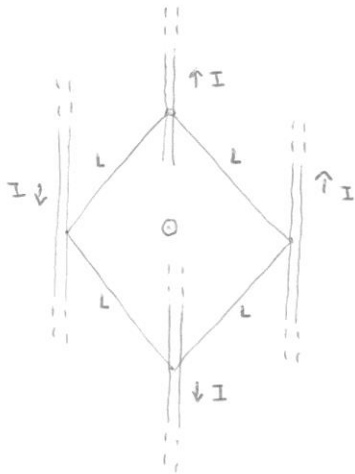
$$\phi(\vec{B}) = \phi(\vec{B}_i) + \phi(\vec{B}_e)$$

$$\phi(\vec{B}_i) = \int \vec{B}_i \cdot d\vec{S} = \int_0^a B_i \cdot 2a \cdot dx = \mu_0 J \cdot \frac{a^3}{2}$$

$$\phi(\vec{B}_e) = \int \vec{B}_e \cdot d\vec{S} = \int_a^{2a} B_e \cdot 2a \cdot dx = \mu_0 J a^3 \ln(2)$$

22/01/2015

ES.4



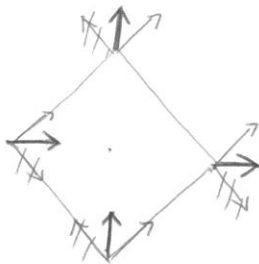
Una piccola spira circolare di raggio $r \ll L$ e resistenza R ruota con velocità angolare ω_0 costante intorno ad un suo diametro parallelo ai quattro fili e passa dal centro del quadrato.

? Trascurando gli effetti di autoinduzione, calcolare la potenza dissipata sulla spira, ed il momento necessario a mantenere costante la ω angolare.

Per la legge di Ampere, ogni filo genera in O un campo magnetico:

$$\mu_0 I = B_l \cdot \frac{\sqrt{2} L \pi}{2} \Rightarrow B_l = \frac{\mu_0 I}{\sqrt{2} L \pi}$$

$2\pi r$: circonferenza
 $r = \frac{\sqrt{2} L}{2}$



Si sommano solo le componenti parallele!

Componente iniziale: $\frac{L\sqrt{2}}{2}$
Proiezione: $\frac{L}{2}$

Moltiplico quindi la componente iniziale per $\frac{1}{\sqrt{2}}$ e sommo 4 volte

$$B = 4 \cdot \frac{1}{\sqrt{2}} \cdot \frac{\mu_0 I}{\sqrt{2} L \pi} = \frac{2\mu_0 I}{L \pi}$$

Flusso di B attraverso la spira ($r \rightarrow \phi$ quindi non ha flusso per $x(r)$):

$$\begin{aligned} \phi_B &= \pi r^2 B \cos(\omega_0 t) \quad \leftarrow \text{in quanto ruota} \\ &= \frac{2\mu_0 I r^2}{L} \cos(\omega_0 t) \end{aligned}$$

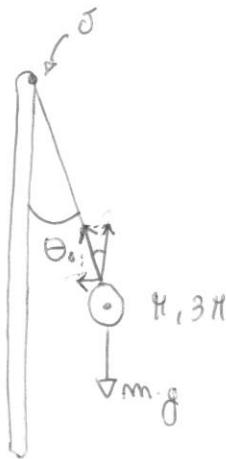
$$\text{F.e.m.} = \mathcal{E} = -\frac{d}{dt} \phi_B = \frac{2\mu_0 I r^2}{L} \omega_0 \sin(\omega_0 t)$$

$$P = \frac{\mathcal{E}^2}{R} = \frac{4\mu_0^2 I^2 r^4}{R} \cdot \omega_0^2 \cdot \sin(\omega_0 t)$$

$$\gamma = \frac{P}{\omega_0} = \frac{4\mu_0^2 I^2 r^4}{R} \cdot \omega_0 \cdot \sin(\omega_0 t)$$

23/02/2015

ES. 1



$$\begin{cases} T_y = T \cdot \cos \theta_0 = mg \\ T_x = T \cdot \sin \theta_0 = \frac{\sigma}{2\epsilon_0} \cdot q \end{cases}$$

$$E = \frac{\sigma}{2\epsilon_0} \quad ; \quad F_e = \frac{\sigma}{2\epsilon_0} \cdot q$$

$$\begin{cases} T = \frac{mg}{\cos \theta_0} \\ q = \frac{2\epsilon_0 \sigma}{\sigma} \cdot mg \frac{\sin \theta_0}{\cos \theta_0} \\ = \frac{2mg\epsilon_0 \cdot \tan \theta_0}{\sigma} \end{cases}$$

$$m = 1/3 M$$

$$Q_1 = \frac{2Mg\epsilon_0 + q\sigma}{\sigma}$$

$$Q_2 = \frac{6Mg\epsilon_0 + q\sigma}{\sigma}$$

Dopo che le sfere si toccano:

$$Q_1 = Q_2 = Q = \frac{4}{\sigma} Mg\epsilon_0 + q\sigma$$

$$\frac{Mg}{\cos \theta_1} \cdot \sin \theta_1 = \frac{\sigma}{2\epsilon_0} \cdot \frac{4}{\sigma} Mg\epsilon_0 + q\sigma$$

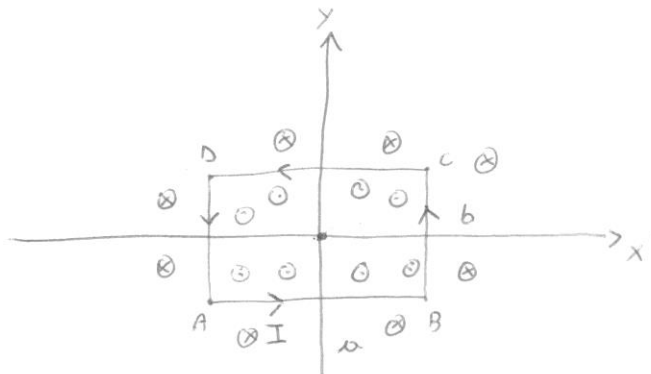
$$\tan \theta_1 = 2 + \tan \theta_0 \Rightarrow \theta_1 = \arctan(2 + \tan \theta_0)$$

$$\frac{3Mg}{\cos \theta_2} \cdot \sin \theta_2 = \frac{\sigma}{2\epsilon_0} \cdot \frac{4}{\sigma} Mg\epsilon_0 + q\sigma$$

$$\tan \theta_2 = \frac{2}{3} + \tan \theta_0 \Rightarrow \theta_2 = \arctan\left(\frac{2}{3} + \tan \theta_0\right)$$

23/02/2015

ES. 2



$$\overline{ABCD} = 2L$$

$$\overline{AB} = a$$

$$\overline{BC} = b$$

? Campo magnetico in O e dipendenza da $x = \frac{\overline{BC}}{\overline{AB}}$

$$x = \frac{b}{a} \Rightarrow a = x \cdot b > b$$

$$2(a+b) = 2L$$

$$2(x \cdot b + b) = 2L$$

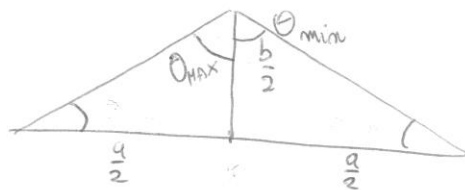
$$2(1+x)b = 2L \Rightarrow b = \frac{L}{1+x}; \quad a = \frac{xL}{1+x}$$

O dista $\frac{b}{2}$ da a e $\frac{a}{2}$ da b.

Legge di Biot-Savart:

Calcolo il campo nel centro dovuto al filo a:

$$B_a = \frac{\mu_0}{4\pi} \cdot I \cdot \frac{2 \sin \theta_{\max} - \sin \theta_{\min}}{b/2}$$



$$\begin{aligned} \sin \theta_{\max} &= \frac{\frac{b}{2}}{\sqrt{\frac{a^2}{4} + \frac{b^2}{4}}} \\ &= \frac{\frac{b}{2}}{\frac{1}{2} \sqrt{a^2 + b^2}} \\ &= \frac{b}{\sqrt{a^2 + b^2}} \end{aligned}$$

$$\sin \theta_{\max} = - \sin \theta_{\min}$$

$$\begin{aligned} B_a &= \frac{\mu_0 I}{4\pi} \cdot \frac{2 \sin \theta_{\max}}{\frac{b}{2}} \\ &= \frac{\mu_0 I}{4\pi} \cdot \frac{2 \cdot \frac{b}{\sqrt{a^2 + b^2}}}{\frac{b}{2}} = \frac{\mu_0 I}{\pi \sqrt{a^2 + b^2}} \cdot \frac{a}{b} \end{aligned}$$

I campi sono paralleli e concordi:

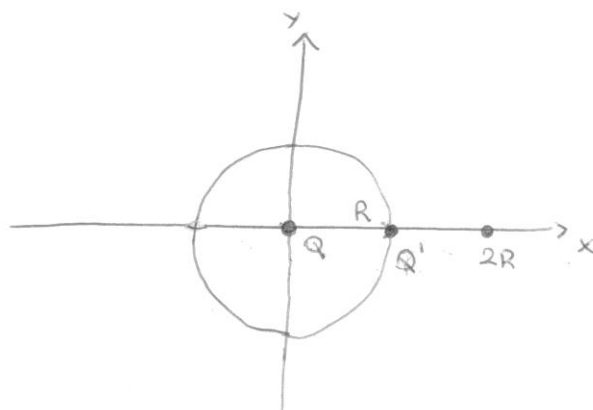
$$B_{TOT} = 2B_a + 2B_b$$

$$B_b = \frac{\mu_0 I}{\pi \sqrt{a^2 + b^2}}$$

$$B_{TOT}(x) = 2B_a(x) + 2B_b(x)$$

23/02/2015

ES. 3



Se guscio sferico è inizialmente scarico, si porta una carica Q' su di esso in modo da far dimezzare il campo in $2R$.
 ? Quanto varia l'energia elettrostatica del sistema.

$$E_Q(P) = k \cdot \frac{Q}{4R^2}$$

$$E_{Q'}(P) = k \cdot \frac{Q'}{4R^2}$$

$$\Rightarrow \frac{1}{2} k \cdot \frac{Q}{4R^2} = \frac{kQ}{4R^2} + \frac{kQ'}{4R^2}$$

$$\frac{k}{4R^2} \left(\frac{Q}{2} \right) = \frac{k}{4R^2} (Q + Q')$$

$$\frac{Q}{2} = Q + Q' \Rightarrow Q' = -\frac{Q}{2}$$

Immaginiamo di portare Q' , in parte infinitesime dq , da $+\infty$ a R , dove $V(R) = k \cdot \frac{Q+q}{R}$, essendo q la carica già presente su esso.

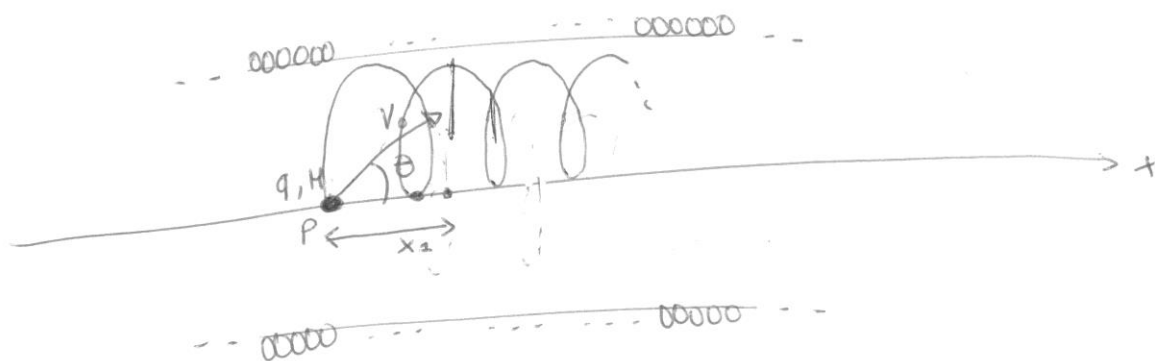
Se lavoro da compiere è:

$$\Delta U = \int_{\phi}^{Q'} k \cdot \frac{Q+q}{R} dq = k \int_{\phi}^{-Q/2} \frac{Q+q}{R} dq = \frac{k}{R} \left[-\frac{Q^2}{2} + \frac{Q^2}{8} \right]$$

$$= -\frac{3Q^2}{32\pi\epsilon_0 R}$$

: variazione di energia pot. del sistema.

ES. 4



? x_1, t_1

Moto di Circolazione

$x_1 = v_0 \cos \theta \cdot t_1$, quindi devo trovare t_1 !

$$T = \frac{2\pi}{\omega}, \quad \omega = \frac{v}{R}$$

$$= \frac{2\pi R}{v}$$

$$F = m \cdot a_c = m \cdot \frac{(v \cos \theta)^2}{R} = B \cdot q \cdot v \cos \theta$$

$$R = \frac{m v \cos \theta}{B q}$$

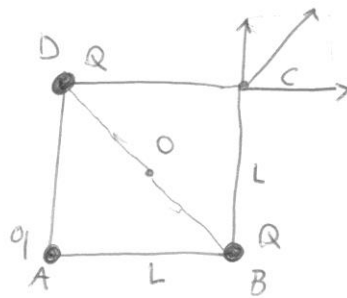
$$t_1 = \frac{2\pi R}{v \cos \theta} = \frac{2\pi m}{B q}$$

($R < 2$ Raggi
Ciclotrone)

con $B = \mu_0 N \cdot I$ in quanto si ha lunghezza infinita.

15/04/2015

ES. 1



$$\textcircled{*} \Delta V_{OC} \text{ prima } q = \frac{1}{2} \Delta V_{OC} \text{ dopo } q$$

? Valori di q per cui $\textcircled{*}$

$$\Delta V_{OC} \text{ prima } q = V_{O \text{ prima } q} - V_{C \text{ prima } q}$$

$$V_{O \text{ prima } q} = \frac{K \cdot Q}{L\sqrt{2}} \cdot 2 = \frac{4KQ}{L\sqrt{2}}$$

$$V_{C \text{ prima } q} = \frac{K \cdot Q}{L} \cdot 2 = \frac{2KQ}{L}$$

$$\Delta V_{OC} \text{ dopo } q = V_{O \text{ dopo } q} - V_{C \text{ dopo } q}$$

$$V_{O \text{ dopo } q} = V_{O \text{ prima } q} + \frac{Kq}{\frac{L\sqrt{2}}{2}} = V_{O \text{ prima } q} + \frac{2Kq}{L\sqrt{2}}$$

$$V_{C \text{ dopo } q} = V_{C \text{ prima } q} + \frac{Kq}{L\sqrt{2}}$$

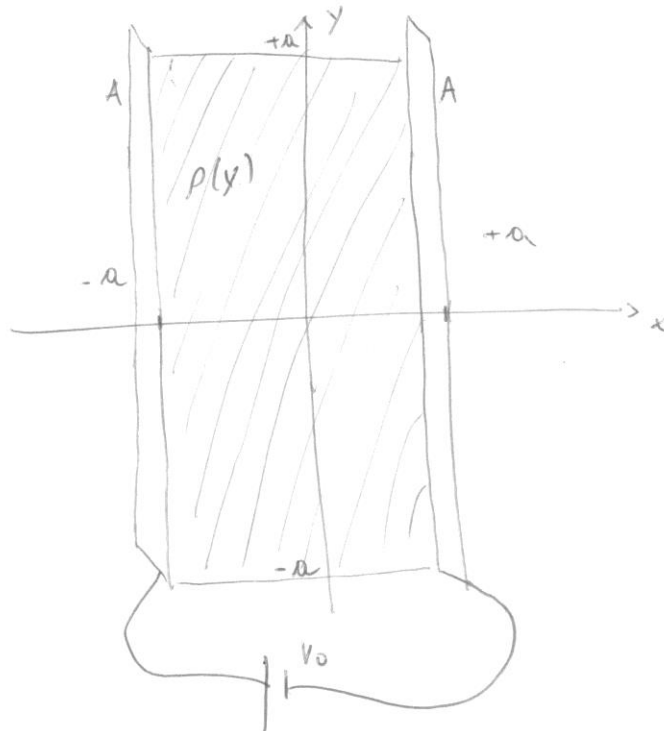
$$\Delta V_{OC} \text{ prima } q = \frac{1}{2} \Delta V_{OC} \text{ dopo } q$$

? Per i valori di cui determinate il campo E in O

$$E_e = \frac{K \cdot q}{(L\sqrt{2})^2} + \sqrt{\frac{K \cdot Q}{L^2} + \frac{K \cdot q}{L^2}} \quad \text{ovvero}$$

$$\begin{cases} E_x = \frac{KQ}{L^2} \cdot \frac{Kq}{L\sqrt{2}} \cdot \frac{\sqrt{2}}{2} \rightarrow \cos(45^\circ) \\ E_y = \frac{K \cdot Q}{L^2} \cdot \frac{K \cdot q}{L\sqrt{2}} \cdot \frac{\sqrt{2}}{2} \rightarrow \sin(45^\circ) \end{cases}$$

Es. 2



$$\rho(y) = \rho_0 \cdot \left(1 + \frac{|y|}{a} \right)$$

$\Rightarrow t = \infty$

$$J = J_{\infty}, \quad \text{div } J = 0$$

$$E(y) = \rho(y) \cdot J = \rho_0 \left(1 + \frac{|y|}{a} \right) \cdot J_{\infty}$$

$$V_0 = \int_{-a}^{+a} E(y) dy = \rho_0 J_{\infty} \int_{-a}^{+a} \left(1 + \frac{|y|}{a} \right) dy$$

$$= \rho_0 J_{\infty} \left[y + \frac{y^2}{2a} \right]_{-a}^a = \rho_0 J_{\infty} \left[a + \frac{a^2}{2a} + a - \frac{a^2}{2a} \right] = 2\rho_0 J_{\infty} a$$

$$J_{\infty} = \frac{V_0}{2\rho_0 \cdot a}$$

$$\frac{P}{\Delta V} = \rho(y) \cdot J_{\infty}^2$$

? $\vec{E}$ distribuita costante fra gli elettrodi a $t = \infty$

$$\vec{\nabla} \cdot \vec{E} \neq 0 \quad \text{a } t = \infty$$

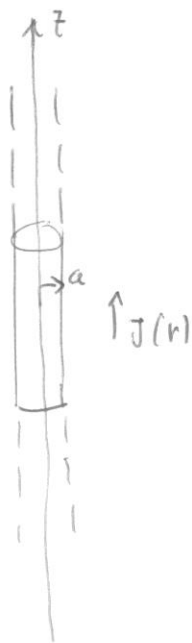
$$\frac{d}{dy} E(y) = \frac{d}{dy} \rho(y) \cdot J_{\infty} + \rho(y) \cdot \frac{d}{dy} J(y) = \frac{\lambda''}{\epsilon_0}$$

$$\lambda' = \frac{d}{dy} \rho(y) \cdot J_{\infty} \cdot \epsilon_0$$

Carica distribuita con densità λ .

$$I_{\infty} = J_{\infty} \cdot A$$

Es. 9



$$J(r) = J_0 \cdot \frac{r}{a}$$

? Intensità di B a distanza R .

Uso la legge di Ampere

$$B = \frac{\mu_0 I}{2\pi R}$$

$$\boxed{R < a}$$

$$B_i = \frac{\mu_0 I}{2\pi R}$$

$$NO: I = \int_p^a J(r) \cdot 2\pi r \cdot dr$$

$$= \frac{\mu_0 J(r) \cdot \pi R^2}{2\pi R}$$

$$= \frac{\mu_0 J(r) \cdot R}{2}$$

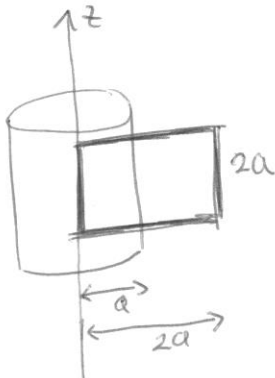
$$I = \iint J \cdot dS$$

$$\boxed{R > a}$$

$$B_e = \frac{\mu_0 I}{2\pi R} = \frac{\mu_0 J(r) \cdot \pi a^2}{2\pi R}$$

$$= \frac{\mu_0 J(r) \cdot a^2}{2R}$$

? Coefficiente mutua induzione



$$\phi(B) = \phi(B_i) + \phi(B_e)$$

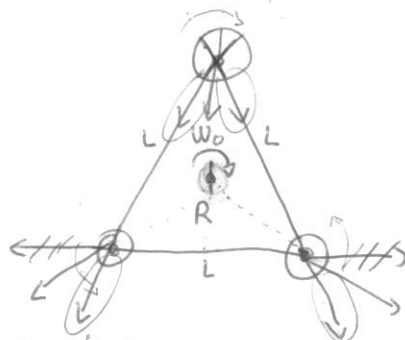
$$\phi(B_i) = \int_p^a B_i \cdot dS = \int_p^a B_i \cdot 2a \cdot dr$$

$$\phi(B_e) = \int_a^{2a} B_e \cdot dS = \int_a^{2a} B_e \cdot 2a \cdot dr$$

$$M = \frac{\phi(B)}{I}$$

ES. 4

$$r \ll L$$



? Potenza media dissipata dalla spira

Legge Ampere :

$$B = \frac{\mu_0 I}{2\pi \cdot \frac{L}{\sqrt{3}}} = \frac{\mu_0 I \sqrt{3}}{2\pi L}$$

Divido in componenti

$$B_{TOT} = B \cdot 4 \cdot \cos(30^\circ) = B \cdot 2\sqrt{3} = \frac{3\mu_0 I}{\pi L}$$

$$\phi(\vec{B})_{\text{attraverso spira in movimento}} = B_{TOT} \cdot \pi r^2 \cdot \cos(\omega_0 t) = \frac{3\mu_0 r^2 I}{L} \cos(\omega_0 t)$$

$$\mathcal{E} = - \frac{d}{dt} \phi(\vec{B}) = \frac{3\mu_0 r^2 I}{L} \cdot \omega_0 \cdot \sin(\omega_0 t)$$

$$P = \frac{\mathcal{E}^2}{R} = \frac{9\mu_0^2 r^4 I^2}{L^2} \cdot \omega_0^2 \cdot \sin^2(\omega_0 t) \cdot \frac{1}{R}$$

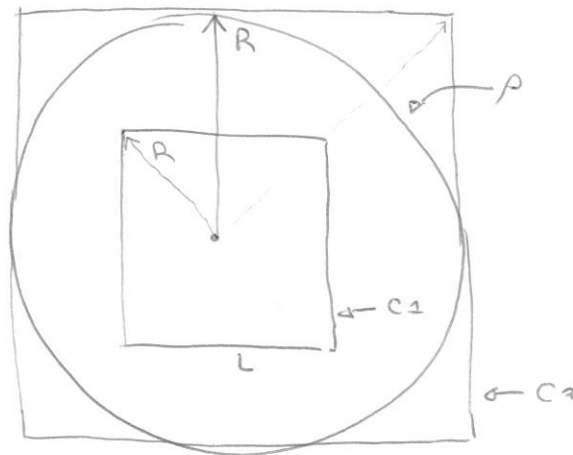
$$P_{media} = \left(\sin^2(\dots) = \frac{1}{2} \right) = \frac{9\mu_0^2 r^4 I^2 \omega_0^2}{2R L^2}$$

? Momento Torcente medio

$$\gamma = \frac{P}{\omega} \quad ; \quad \gamma_{media} = \frac{P_{media}}{\omega_0}$$

29/06/2015

ES. 1



$$2R = L\sqrt{3}$$

$$L = \frac{2}{\sqrt{3}} R$$

? $\phi(\vec{E})$ attraverso C_1 e attraverso C_2

$$\begin{aligned}\phi(\vec{E})_{C_1} &= \int \vec{E} \cdot d\vec{S} = \frac{Q_{\text{int}}}{\epsilon_0} ; Q_{\text{int}} = \rho \cdot L^3 \\ &= \frac{\rho L^3}{\epsilon_0} .\end{aligned}$$

$$\begin{aligned}\phi(\vec{E})_{C_2} &= \int \vec{E} \cdot d\vec{S} = \frac{Q_{\text{int}}}{\epsilon_0} ; Q_{\text{int}} = \rho \cdot \frac{4}{3} \pi R^3 \\ &= \frac{\rho 4\pi R^3}{3\epsilon_0} .\end{aligned}$$

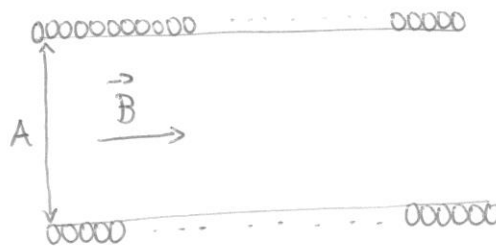
? ΔV vertice C_2 - vertice C_1

$$\Delta V = V_{C_2} - V_{C_1}$$

$$V_{C_2} = \frac{K \cdot Q}{r} = \frac{K \cdot Q}{R\sqrt{3}}$$

$$V_{C_1} = \frac{K \cdot Q}{r} = \frac{K \cdot Q}{R}$$

ES. 2



N spire

Filo con:

- resistenza ρ ;
- sezione di raggio r ;

? R

$$R = \rho \cdot \frac{l}{S} = \rho \cdot \frac{2\pi A \cdot N}{\pi r^2} = \frac{2\rho AN}{r^2}$$

? L (coefficiente di autoinduzione)

$$L = \frac{\Phi(\vec{B})}{I}$$

$$\vec{B} = \mu_0 \eta \cdot I \quad , \quad \eta = \frac{N}{R} = \frac{N}{2r \cdot N} = \frac{1}{2r}$$

$$= \mu_0 \frac{1}{2r} \cdot I$$

$$\Phi(\vec{B}) = \vec{B} \cdot S \cdot N = \mu_0 \frac{1}{2r} \cdot \pi A^2 \cdot N$$

$$L = \frac{\Phi(\vec{B})}{I} = \frac{\mu_0 \pi A^2 N}{2r}$$

? E che occorre applicare e la potenza dissipata per avere un assegnato campo magnetico $\vec{B}$.

$$B = \mu_0 \eta \cdot \frac{\mathcal{E}}{R} \Rightarrow \mathcal{E} = \frac{B \cdot R}{\mu_0 \eta}$$

? Collegando un condensatore di capacità C , per quali C la corrente che attraversa il solenoide si smorza senza compiere oscillazioni?

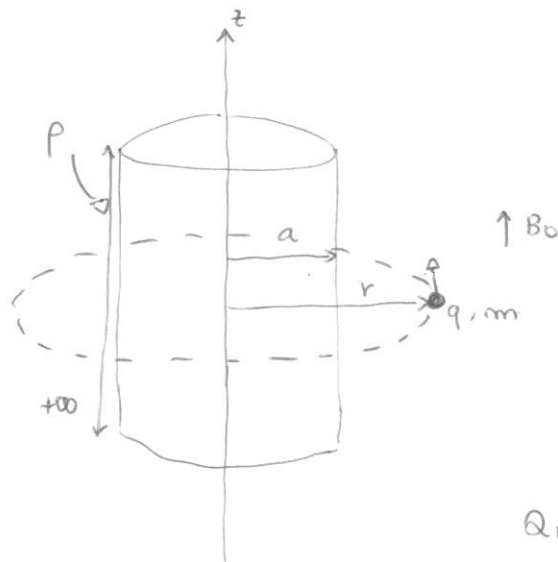
$$\omega_s \stackrel{!}{=} \omega_0$$

$$\frac{R}{2L} \stackrel{!}{=} \frac{1}{\sqrt{LC}}$$

$$\frac{2L}{R} \leq \sqrt{LC}$$

$$\frac{4L^2}{R^2} \leq C \Rightarrow \boxed{C \geq \frac{4L}{R^2}}$$

ES.3



$$Q_{int} = \pi r^2 \cdot h \cdot \rho$$

? T per $r < a$

$$E(r) \cdot \pi r h = \frac{Q_{int}}{\epsilon_0} \Rightarrow E(r) = \frac{r \rho}{2 \epsilon_0}$$

La densità di carica cilindrica produce un campo $\vec{E}$ che è uguale a quello prodotto da un filo infinito lungo.

$$\vec{E} = \frac{\rho}{2 \pi \epsilon_0 r}$$

$$E(r) = \frac{r \rho}{2 \epsilon_0}$$

$$F_{particella} = m \cdot a_{\perp} = F_e + F_L$$

$$F_e = \frac{\rho}{2 \pi \epsilon_0 r} \cdot q \cdot \frac{r \rho}{2 \epsilon_0} \cdot q$$

$$F_L = q v_{\perp} B_0$$

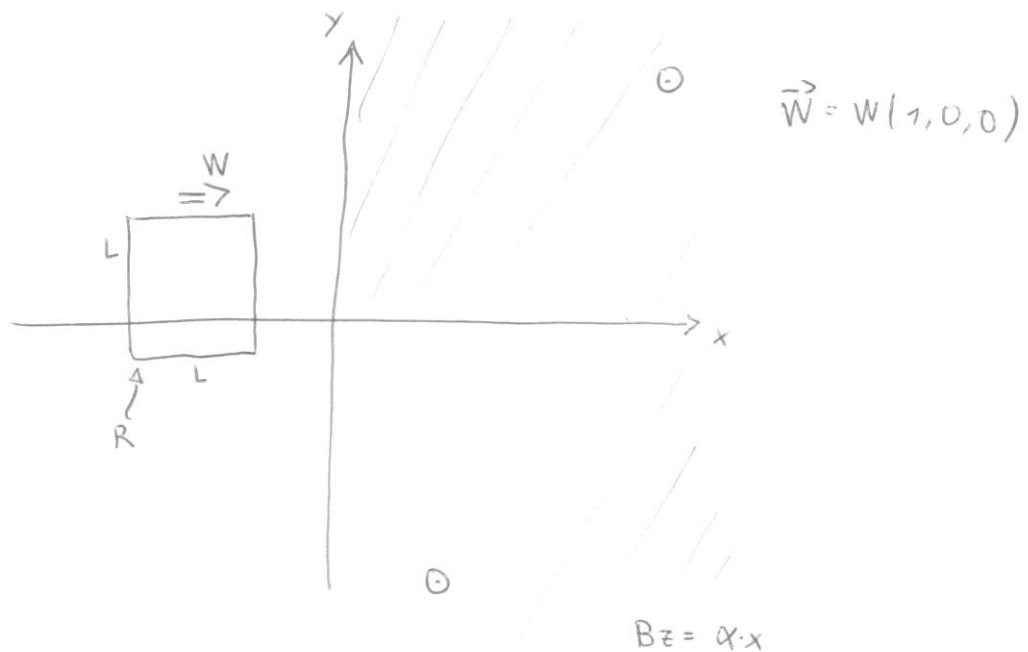
$$T_{\perp} = \frac{2 \pi r}{v_{\perp}} = \frac{2 \pi r}{\sqrt{(F_e + F_L) \cdot \frac{r}{m}}}$$

$$a_{\perp} = \frac{v_{\perp}^2}{r}$$

$$a_{\perp} = \frac{F_e + F_L}{m} \Rightarrow v_{\perp} = \sqrt{a_{\perp} \cdot r} = \sqrt{(F_e + F_L) \cdot \frac{r}{m}}$$

$$r > a \Rightarrow E(r) = \frac{a^2 \rho}{r \cdot 2 \epsilon_0}$$

ES. 4



? Calcolare f. e. m. indotta.

$$B_z = \alpha \cdot x = \alpha \cdot W \cdot t$$

$$x = W \cdot t$$

$$\phi(\vec{B}) = \int \vec{B} \cdot d\vec{S} =$$

Quando $x=L$ il flusso è MAX e vale:

$$\phi(\vec{B}) = \alpha W t \cdot L^2 \quad \text{per } x \geq L$$

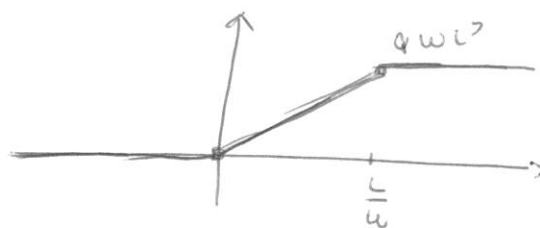
$$\phi(\vec{B}) = 0 \quad \text{per } x \leq 0$$

$$\phi(\vec{B}) = L \int_{x=0}^x B_z(x') dx' \quad \text{per } 0 < x < L$$

$$= L \alpha \frac{x^2}{2} = \frac{L}{2} \alpha W^2 t^2$$

$$\begin{cases} \phi(\vec{B}) = 0 & t \leq 0 \\ \phi(\vec{B}) = \frac{L}{2} \alpha W^2 t^2 & 0 < t < \frac{L}{W} \\ \phi(\vec{B}) = \alpha W t L^2 & t \geq \frac{L}{W} \end{cases}$$

$$\mathcal{E}(t) = - \frac{d}{dt} \phi(\vec{B})$$



? Potenza dissipata

$$P(t) = \frac{I(t)^2}{R}$$

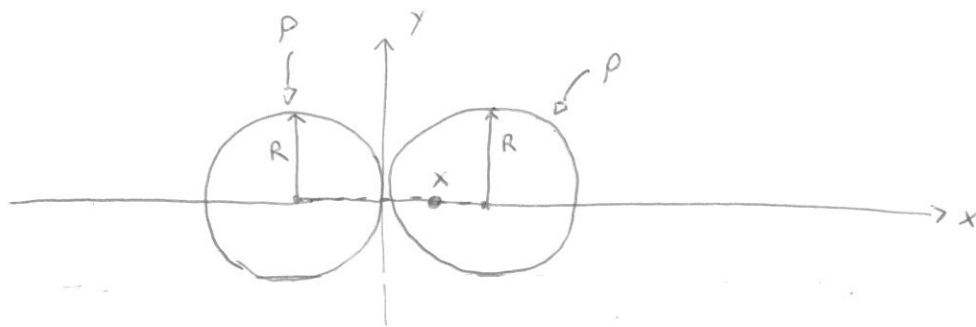
? Forza per mantenere costante W

$$P(t) = F_x(t) \cdot W$$

$$\Rightarrow F_x(t) = \frac{P(t)}{W}$$

24/07/2015

ES. 1



Carica ρ distribuita nel volume: sfere cariche

? Intensità e direzione di $\vec{E}$ nella congiungente dei centri

$$E_{\text{int}}(r) = \frac{\rho r}{3\epsilon_0}$$

↑
in generale

$$E_{\text{ext}}(r) = \frac{kQ}{r^2}$$

$$Q = \rho \cdot \frac{4}{3}\pi r^3$$

Posso anche calcolare $E_{\text{int}}(r)$:

$$\Phi(E) = \int E \cdot dS = \frac{Q_{\text{int}}}{\epsilon_0}$$

$$E(r) \cdot 4\pi r^2 = \frac{Q_{\text{int}}}{\epsilon_0} \Rightarrow E(r) = \frac{Q_{\text{int}}}{4\pi r^2 \epsilon_0} = \frac{\rho \cdot \frac{4}{3}\pi r^3}{4\pi r^2 \epsilon_0} = \frac{\rho r}{3\epsilon_0}$$

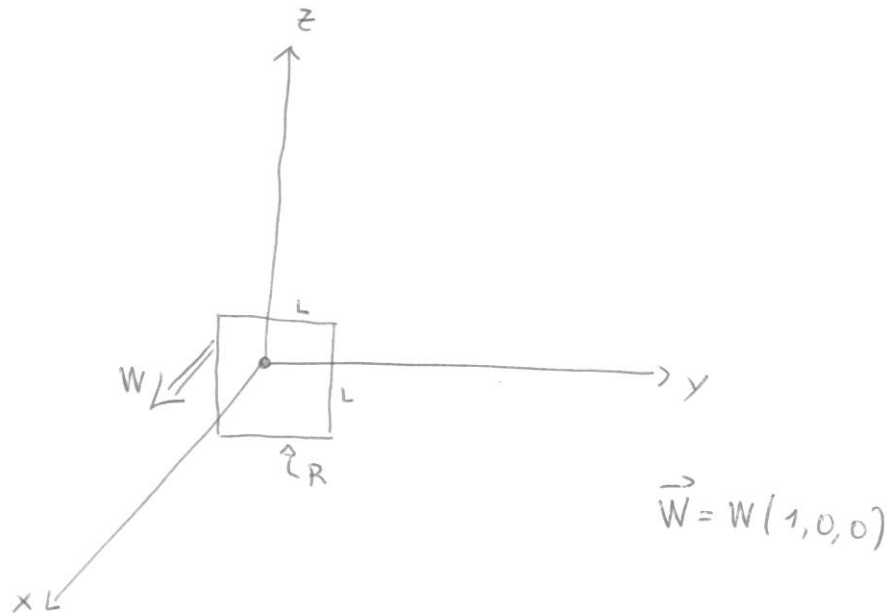
In conclusione:

$$E(x) = \frac{\rho(R-x)}{3\epsilon_0} - \frac{kQ}{(R+x)^2}, \quad Q = \rho \cdot \frac{4}{3}\pi R^3$$

? Punti in cui $E(x)$ è max

$E'(x) = 0$ e studio la funzione.

5.2



$\vec{B}$:

- $B_x = 0$ su yz ;
- $B_y = B_0 \cdot \frac{y}{L}$;
- $B_z = 0$;

? ϕ_B attraverso la spira

$$\text{div } \vec{B} = 0 \Rightarrow \frac{\partial}{\partial x} B_x + \frac{\partial}{\partial y} B_y + \frac{\partial}{\partial z} B_z = 0 \Rightarrow \frac{\partial}{\partial x} B_x = -\frac{B_0}{L}$$

$$\vec{B} = \begin{pmatrix} -\frac{B_0}{L} \cdot x \\ \frac{B_0}{L} \cdot y \\ 0 \end{pmatrix}$$

$$B_x = -\frac{B_0}{L} \cdot x$$

$$\phi(\vec{B}) = \int \vec{B} \cdot d\vec{S}$$

Siccome la spira ha velocità W : $x = W \cdot t$

$$\phi(\vec{B}) = -\frac{B_0}{L} \cdot W \cdot t \cdot L^2 = -B_0 W t L$$

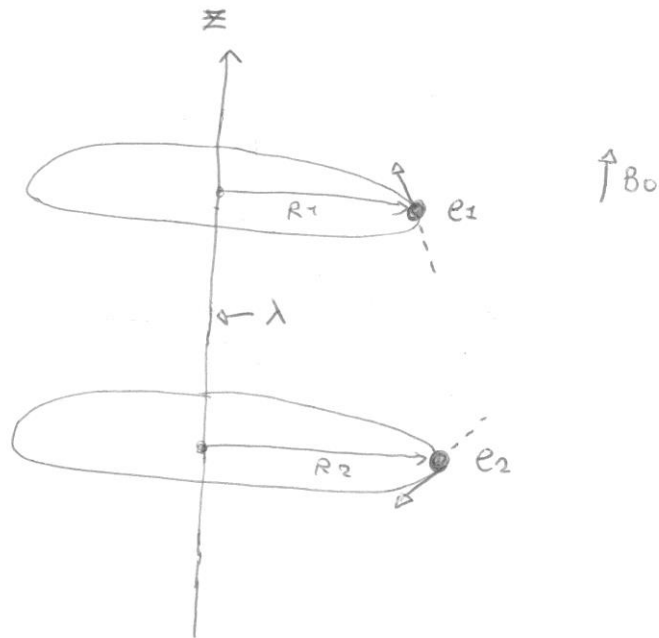
(+) indotta dalla Spira

$$\mathcal{E}(t) = - \frac{d}{dt} \phi(\vec{B}) = B_0 W L$$

? $P(t)$ per mantenere la Spira in movimento

$$P(t) = \frac{\mathcal{E}^2(t)}{R} = \frac{B_0^2 W^2 L^2}{R}$$

ES. 3



R moto

? $\frac{T_1}{T_2}$

N.B: se filo con densità di carica λ genera un campo E

$$F_{\text{elettrome } 1} = m \cdot a_1 = F_e + F_L$$

$$F_{\text{elettrome } 2} = m \cdot a_2 = F_e - F_L$$

$$a_1 = \frac{v_1^2}{R_1}$$

$$a_2 = \frac{v_2^2}{R_2}$$

$$R_1 = R_2$$

$$F_e = \frac{\lambda}{2\pi\epsilon_0 R} q$$

$$F_L = q v \cdot B_0$$

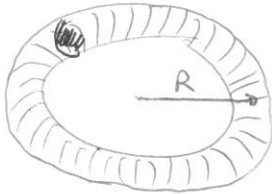
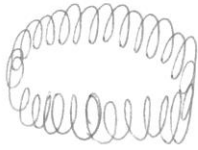
$$\frac{v_1^2}{R} = \frac{F_e + F_L}{m} ; \quad \frac{v_2^2}{R} = \frac{F_e - F_L}{m}$$

$$v_1 = \sqrt{(F_e + F_L) \frac{R}{m}} ; \quad v_2 = \sqrt{(F_e - F_L) \cdot \left(\frac{R}{m}\right)}$$

$$T_1 = \frac{2\pi R}{v_1} ; \quad T_2 = \frac{2\pi R}{v_2}$$

$$\frac{T_1}{T_2} = \frac{v_2}{v_1} = \sqrt{\frac{F_e - F_L}{F_e + F_L}}$$

19/02/2013



$$d = 0,2 \text{ mm}$$

$$a = 7,07 \text{ mm}$$

$$R = 31,6 \text{ cm}$$

$$N = 1234$$

? R, L

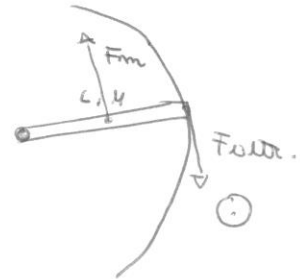
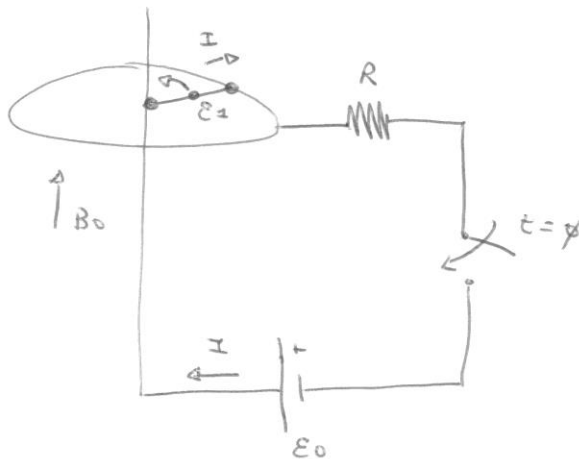
$$R = \frac{\rho \cdot \ell}{S} = \frac{\rho \cdot 2\pi a \cdot N}{\pi \left(\frac{d}{2}\right)^2}$$

$$L = \frac{\Phi(B)}{I}$$

$$\Phi(B) = \underset{\text{TOROIDE}}{B \cdot N \cdot S} = \frac{\mu_0 I N}{2\pi R} \cdot N \cdot \pi a^2$$

$$L = \frac{\mu_0 N^2 \cancel{\pi} a^2}{2\cancel{\pi} R} = \frac{\mu_0 N^2 a^2}{2R}$$

23/01/2012



$$F_m = L \cdot i \cdot B$$

$t = \infty$

$$I \cdot \dot{\omega} = 0 \Rightarrow \tau_A = L \cdot i \cdot B \cdot \frac{L}{2} \Rightarrow \boxed{i_{\infty}} = \frac{2 \tau_A}{B \cdot L^2}$$

$$P_{gen} = P_{ext} + P_{res}$$

$$\varepsilon_0 \cdot i = \tau_A \cdot \omega + R \cdot i^2$$

$$i_{\infty} = \frac{\varepsilon_0 + \varepsilon_{induced}}{R}$$



$$\Phi(\vec{B}) = B \cdot \frac{\theta}{2\pi} \cdot \pi L^2 = \frac{1}{2} B \theta L^2 = \frac{1}{2} L^2 B \omega \cdot t \Rightarrow \varepsilon_{ind} = -\frac{1}{2} L^2 B \omega$$

$$i_{\infty} = \frac{\varepsilon_0 - \frac{1}{2} L^2 B \omega_{\infty}}{R}$$

$$\frac{2 \tau_A}{B \cdot L^2} = \frac{\varepsilon_0}{R} - \frac{L^2 B \omega_{\infty}}{2R} \Rightarrow \left(\frac{2 \tau_A}{B L^2} + \frac{\varepsilon_0}{R} \right) \frac{2R}{L^2 B} = \boxed{\omega_{\infty}}$$

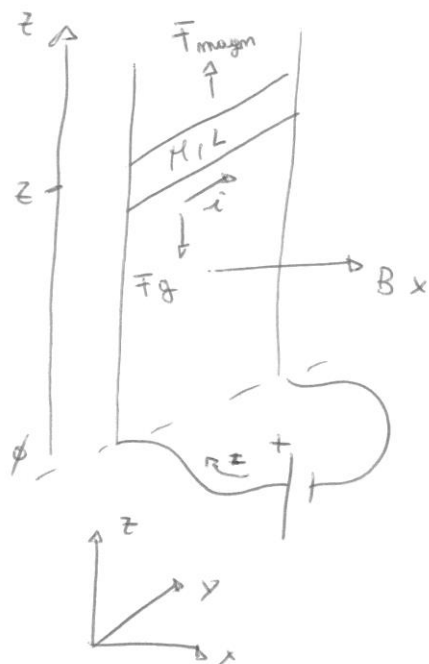
$$I \dot{\omega} = \frac{1}{3} H L^2 \cdot \dot{\omega} = \sum \tau = -\tau_A + \frac{1}{2} B L^2 i$$

$$\frac{1}{3} H L^2 \cdot \dot{\omega} = -\tau_A + \frac{\varepsilon_0 - \frac{1}{2} B L^2 \omega}{R}$$

$$\dot{\omega} + \frac{1}{2 R I_{mecc}} B L^2 \left(\frac{1}{2} B L^2 \omega - \varepsilon_0 \right) + \frac{\tau_A}{I_{mecc}} = 0$$

28/09/2012

$$B_0 = \begin{pmatrix} 1 \\ -1 \\ -1 \end{pmatrix}$$



$$F_{\text{magn}} = i \cdot L \cdot B_x$$

$$F_g = -M \cdot g$$

$$i \cdot L \cdot B = -Mg$$

$$i = \frac{\mathcal{E}_0}{R}$$

$$\begin{cases} \frac{\mathcal{E}_0}{R} \cdot L \cdot B = -Mg \\ R = \rho \cdot \frac{2\pi z}{S} \end{cases} \Rightarrow \boxed{\text{trova } z}$$

$$\boxed{P} = \frac{\mathcal{E}_0^2}{R(z)}$$

$$M \cdot a = -Mg + F_{\text{magn}}$$

$$F_{\text{magn}} = i \cdot L \cdot B = \frac{\mathcal{E}_0}{R(z)} \cdot L \cdot B = \frac{(\mathcal{E}_0 - B_0 L v)}{R(z)} \cdot L B_0$$

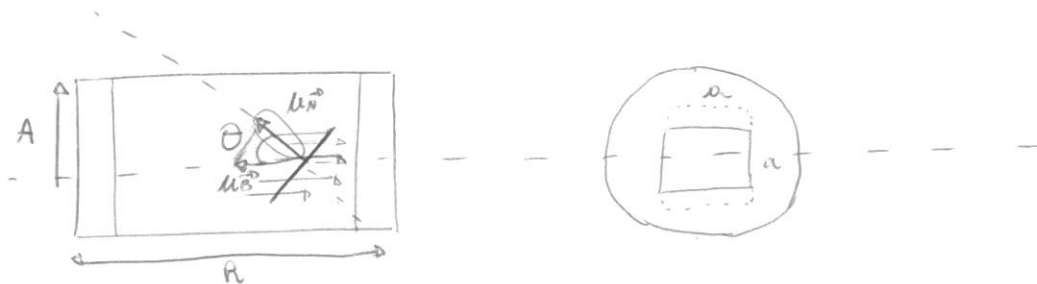
$$M \cdot \ddot{z} = -Mg + \frac{\mathcal{E}_0 - B_0 L \dot{z}}{R(z)} \cdot L B_0$$

$$\begin{cases} \phi(B_0) = B_0 L v \cdot t \\ -\mathcal{E}_{\text{ind}} = B_0 L v \end{cases}$$

$$\mathcal{E}_{\text{ind}} = \mathcal{E}_0 - \mathcal{E}_{\text{indotta}}$$

11/09/2012

ES. 3



N spire

$R \gg A$

? L

$$\Phi(B) = L \cdot I \Rightarrow L = \frac{\Phi(B)}{I}$$

$$B = \mu_0 n \cdot I = \mu_0 \frac{N}{R} \cdot I$$

$$\Phi(B) = \mu_0 \frac{N}{R} \cdot I \cdot A^2 \pi \cdot N \Rightarrow L = \mu_0 \frac{N}{R} \cdot A^2 \pi \cdot N$$

? $H(\theta)$ per solenoide e spira

$$\begin{aligned} H(\theta) &= \frac{\Phi(B)}{I} = \frac{B \cdot S \cdot \mu_B \cdot \mu_N}{I} \\ &= \frac{\mu_0 \frac{N}{R} \cdot I \cdot a^2 \cdot \cos(\theta)}{I} = \frac{\mu_0 N a^2 \cos(\theta)}{R} \end{aligned}$$

? Solenoide con corrente I_0 e spira a velocità ω

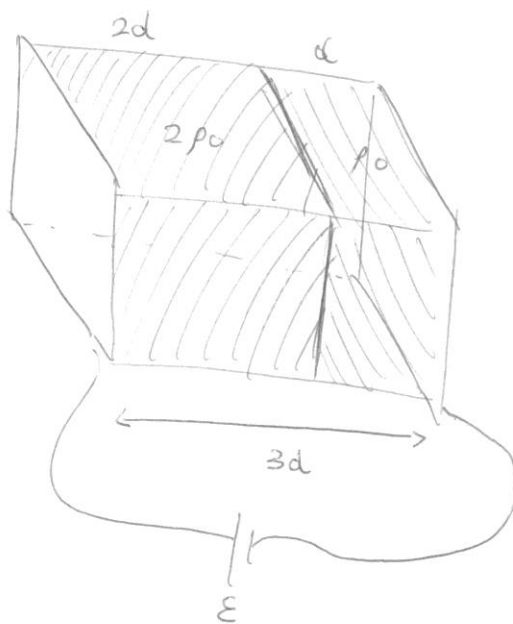
$$\theta = \theta(t) = \omega t$$

? $\mathcal{E}(t)$

$$\Phi(B) = B \cdot a^2 \cdot \cos \theta(t) = B \cdot a^2 \cos(\omega t)$$

$$\mathcal{E}(t) = -B a^2 \sin(\omega t) \cdot \omega = -\mu_0 \frac{N}{R} \cdot I_0 \cdot a^2 \cdot \sin(\omega t) \cdot \omega$$

23/01/2012



$t = \phi$

$$E(x) = \frac{\varepsilon}{3d} = \rho(x) \cdot J(x)$$

$$J(x) = \frac{E(x)}{\rho(x)} = \begin{cases} \frac{\varepsilon}{3d} \cdot \frac{1}{2\rho_0} & \text{in } 2\rho_0 \\ \frac{\varepsilon}{3d} \cdot \frac{1}{\rho_0} & \text{in } \rho_0 \end{cases} \quad t = \phi$$

$t = \infty$

$J = J_{\infty}$

$$E(x) = \rho(x) \cdot J(x)$$

$$\begin{cases} E_1 = \rho_1 \cdot J \\ E_2 = \rho_2 \cdot J \end{cases}$$

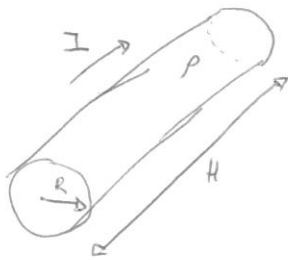
$$V = E_1 \cdot 2d + E_2 \cdot d = \rho_1 J 2d + \rho_2 J d$$

$$J = \frac{V}{5\rho_0 d}$$

$$\frac{P}{\Delta V} = \rho \cdot J^2 = \begin{cases} 2\rho_0 J^2 \\ \rho_0 J^2 \end{cases}$$

19/11/2012

ES.3



$$J(r) = J_0 \cdot e^{\frac{r-R}{A}} ; \phi < r, R$$

? I , P_{tot}

$$I = \iint J(r) dS = \int_0^R J_0 \cdot e^{\frac{r-R}{A}} dr \cdot 2\pi r$$

$$dS = 2\pi r \cdot dr$$

$$S = \pi R^2$$

$$= A J_0 2\pi \int_0^R \frac{r}{A} \cdot e^{\frac{r-R}{A}} dr = A J_0 2\pi \left(\frac{R}{A} - 1 \right) e^{R/A}$$

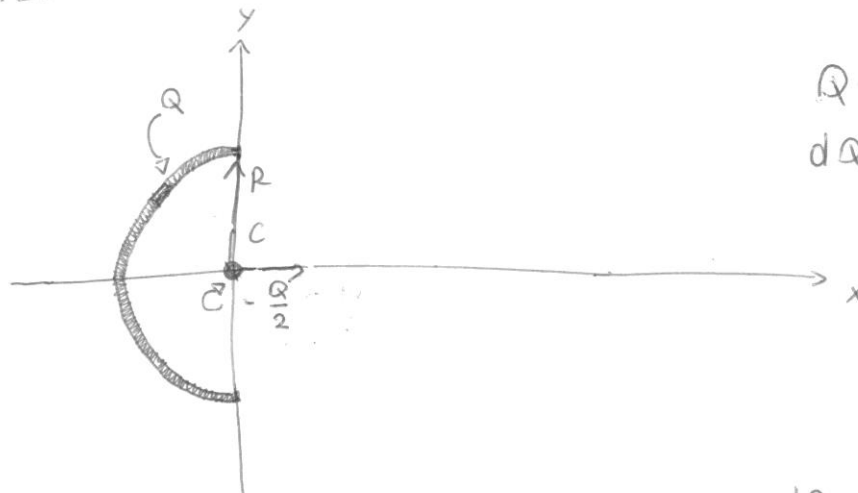
$$= A J_0 2\pi e^{-R/A} \left[\int_0^R \frac{r}{A} e^{\frac{r}{A}} dr \right] \quad \begin{matrix} t = \frac{r}{A} \\ dt = \frac{dr}{A} \end{matrix} \quad \boxed{A} \int t e^t \cdot dt$$

$$= A^2 J_0 2\pi e^{-R/A} \cdot \left(\frac{R}{A} - 1 \right) e^{R/A} = A^2 J_0 2\pi \left(\frac{R}{A} - 1 \right)$$

$$P = R \cdot I^2 = \rho \cdot \frac{L}{\pi R^2} \cdot I^2$$

23/01/2012

ES. 1



$$Q = \pi R \lambda$$

$$dQ = \pi \lambda dr$$

$$dr = R \cdot d\theta$$

$$dQ = \pi \lambda R d\theta$$

? modulo e direzione di $\vec{E}$ in C

$$E_y = 0$$

$$E_x =$$

$$dE = \frac{K \cdot dQ}{R^2} = \frac{K \pi \lambda R d\theta}{R^2} = \frac{K \lambda d\theta \pi}{R}$$

$$E_x = \int dE_x = \frac{\pi K \lambda}{R} \int_{-\pi/2}^{\pi/2} d\theta \cdot \cos\theta = \frac{\pi K \lambda}{R} 2 \sin \frac{\pi}{2} = \frac{2 \pi K \lambda}{R} \quad \text{(crossed out with a circle and slash)}$$

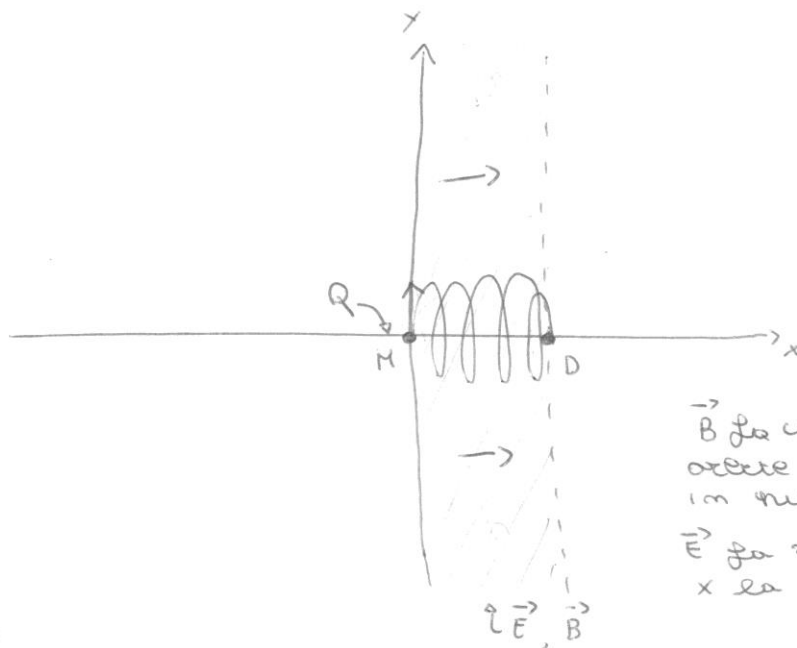
$$= \frac{2 \pi K \lambda}{R} = \frac{2 \cdot \frac{1}{2} \cdot \frac{1}{R} \cdot \cancel{\pi} \cdot \frac{Q}{\pi R}}{2 \pi \epsilon_0 R^2} = \frac{Q}{2 \pi \epsilon_0 R^2}$$

$$V_C = \frac{K Q}{R}$$

$$\vec{F}_e = \left(-\frac{Q}{2} \right) E_x = m \cdot a \quad \Rightarrow \quad a = \frac{\vec{F}_e}{m}$$

16/04/2012

ES. 1



$\vec{B}$ fa complete
orbite circolari
in quanto $Q\vec{v} \perp \vec{B}$
 $\vec{E}$ fa spostare lungo
x la particella

$$\vec{E} = Ax^2 \vec{u}_x$$

$$\vec{B} = B_0 \vec{u}_z$$

Il a $t=0$ si trova in $(0,0,0)$ con $\vec{v} = v_0(0,1,0)$

Il emerge dal piano in $(D,0,0)$ con $|v_{x-fin}| = 2|v_0|$

D, v_0 noti; A, B_0 da calcolare

? v_{x-fin}

Quando la carica emerge ha fatto un # intero di orbite
circolari intorno a B_0 , mentre ha accelerato in dir. x.
L'accelerazione dovuta ad $\vec{E}$ ha modificato l'energia
della carica:

$$\Delta U_e = Q \cdot \Delta V = Q \int \vec{E} \cdot d\vec{e} = Q \int_0^D E dx = A \frac{D^3}{3} \cdot Q$$

v_y e v_z sono le stesse che all'ingresso, ma v_x è cambiata!

$$\frac{1}{2} H v_0^2 + \underbrace{A \frac{D^3}{3} \cdot Q}_{\Delta U} = \frac{1}{2} H v_0^2 + \frac{1}{2} H v_{x-fin}^2$$

$$v_{x-fin} = \sqrt{\frac{2}{3} \frac{A}{H} D^3 Q}$$

$$E_{k-fin} = \frac{1}{2} H (v_{x-fin}^2 + v_0^2)$$

? A

$$|(V_{x-gim}, V_0, \phi)| = 2V_0$$

$$\sqrt{V_{x-gim}^2 + V_0^2 + \phi^2} = 2 \cdot V_0$$

$$\frac{4}{3} \frac{A}{R} \cdot D^3 Q + V_0^2 = 2V_0^2 \Rightarrow A = \frac{3 \cdot 3 V_0^2 R}{2 D^3 Q}$$

? B_0 in funzione del tempo di transito t_1 .

Bisogna compiere # intero di orbite di accelerazione

$$T = \frac{2\pi R}{Q B_0} \quad : \text{ periodo di accelerazione}$$

$$\downarrow$$
$$T = \frac{2\pi}{\omega}, \quad \omega = \frac{qB}{m}$$

$B_0 \Rightarrow$ i suoi valori sono quelli che soddisfanno

$$mT = t_1, \quad m \in \mathbb{N}$$

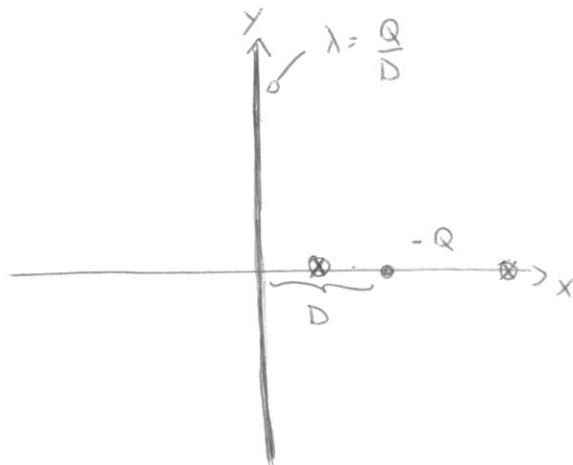
$$m \frac{2\pi R}{Q B_0} = t_1 \Rightarrow B_0 = \frac{m 2\pi R}{Q t_1}$$

Tale campo $\vec{E}$ non può esistere nel vuoto in quanto

$$\vec{\nabla} \cdot \vec{E} \neq 0$$

16/04/2012

ES. 2



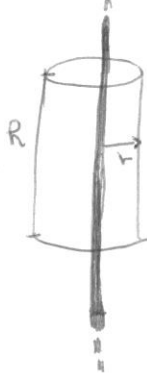
? in quanti punti dell'asse
x il campo è nullo

$$E = k \cdot \frac{Q}{r^2}$$

Supponiamo di porci sull'asse X a distanza x da $(0,0)$

$$E_p(x) = \frac{k \cdot (-Q)}{(x-D)^2} = -\frac{kQ}{(x-D)^2}$$

Per calcolare il campo del filo prendo una scatola cilindrica di raggio r .



Uso il T. di Gauss:

$$\phi(\vec{E}) = \vec{E} \cdot \vec{\Sigma} = E \cdot 2\pi r \cdot R \cdot E = \frac{q}{\epsilon_0}$$

$$q = \lambda \cdot R$$

$$\Rightarrow E_g = \frac{q}{2\pi r R \epsilon_0} = \frac{\lambda}{2\pi r \epsilon_0}$$

Per il nostro caso: $\lambda = \frac{Q}{D}$ e la distanza $r = x$

$$\Rightarrow E_g(x) = \frac{Q}{2\pi |x| \epsilon_0 \cdot D}$$

$$E_g(x) = E_p(x) \Rightarrow \frac{Q}{2\pi |x| \epsilon_0 \cdot D} = \frac{-kQ}{(x-D)^2} \cdot \frac{1}{\frac{1}{2\epsilon_0}}$$

$$\frac{1}{|x| \cdot D} = -\frac{1}{2(x-D)^2} \Rightarrow |x| \cdot D = -2(x-D)^2$$

$$x > \phi$$

$$DX = -2(x-D)^2$$

$$DX = -2x^2 - 2D^2 + 4xD$$

$$2x^2 + 2D^2 - 3DX = \phi \quad \Rightarrow \quad D < \phi, \text{ non ha soluzioni}$$

$$x < \phi$$

$$-DX = -2(x-D)^2$$

$$-DX = -2x^2 - 2D^2 + 4xD$$

$$2x^2 + 2D^2 - 5xD = \phi \quad \left\{ \begin{array}{l} x_1 = 2D \\ x_2 = \frac{D}{2} \end{array} \right.$$

? Calcolare la d.d.p. per x_1 e x_2

$$V(x) = \frac{K \cdot q}{x}$$

$$V(x) = V_p(x) + V_F(x)$$

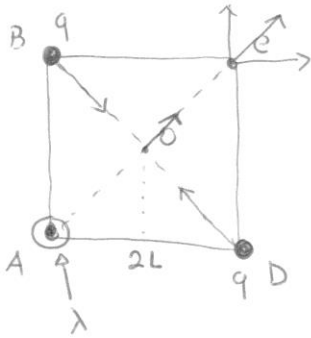
$$V_p(x) = \frac{-kQ}{x-D}$$

$$\begin{aligned} V_F(x) &= - \int E_F(x) dx = - \int \frac{Q}{2\pi|x| \epsilon_0 D} \cdot dx = - \frac{Q}{2\pi \epsilon_0 D} \int \frac{1}{|x|} dx \\ &= - \frac{Q}{2\pi \epsilon_0 D} \cdot \ln|x| \end{aligned}$$

$$V(x) = - \frac{kQ}{x-D} - \frac{Q}{2\pi \epsilon_0 D} \cdot \ln|x|$$

$$d.d.p. = V(2D) - V(D/2)$$

28/09/2012



? $|E_o| = |E_e|$, posizionare due estremità in B e D.

$$E(\phi, \phi)_{\text{free}} = \frac{\lambda}{2\pi\sqrt{2}L\epsilon_0} = \frac{\kappa 2\lambda}{\sqrt{2}L}$$

$$E_{c \text{ p.w.}} = \frac{\lambda}{2\pi \cdot 2L\sqrt{2} \epsilon_0} = \frac{\lambda}{4\sqrt{2} \pi L \epsilon_0}$$

$$E_{c, \text{free}} = \frac{1}{2} E_{\phi, \text{free}}$$

$$E(x)_{\text{LiCo}} = \frac{\lambda}{2\pi \times \epsilon_0}$$

In O, i contributi di B e D si eliconcono.

E_c deve essere moltiplicato per un fattore 2.

$$E_{D \rightarrow C} = \frac{Kq}{4L^2}$$

$$E_{B \rightarrow C} = \frac{Kq}{ql^2}$$

$$E_{\text{cp,ew}} = \frac{\lambda}{2\pi 2L\sqrt{2}\epsilon_0} = \frac{1}{4\pi\epsilon_0} \cdot \frac{\lambda}{L\sqrt{2}} = \frac{k\lambda}{L\sqrt{2}}$$

$$E_c = E_{\text{Ox}} + E_{D \rightarrow c} + E_{B \rightarrow c} = E_{\text{Ox}}$$

$$\frac{Kq}{4\epsilon^2} + \frac{Kq}{4\epsilon^2} + \frac{K\lambda}{\sqrt{2}} = \frac{K\lambda}{\sqrt{2}}$$

$$\frac{2a}{2\lambda L} = \frac{2\lambda}{\sqrt{2}} - \frac{\lambda}{\sqrt{2}}$$

$$\frac{q}{2L} = \frac{\lambda}{\sqrt{2}} \Rightarrow q = \frac{2\lambda L}{\sqrt{2}}$$

$$E_c = -E_o \text{ given}$$

$$\frac{K_9}{uL^2} + \frac{K_9}{uL^2} + \frac{K_1}{K\sqrt{2}} = -\frac{K_2\lambda}{\sqrt{2}K}, \quad \frac{2q}{2uL} = -\frac{2\lambda}{\sqrt{2}} - \frac{\lambda}{\sqrt{2}} \Rightarrow \frac{q}{2L} = -\frac{3}{\sqrt{2}}\lambda$$

$$\textcircled{9} = -\frac{6L}{\sqrt{2}} \lambda$$

? d.d.p. fra A e C.

$$\text{d.d.p. A-C} = - \int_{\phi}^{2L\sqrt{2}} E dx = - \int_{\phi}^{2L\sqrt{2}} \frac{\lambda}{2\pi x \epsilon_0} dx = - \frac{\lambda}{2\pi \epsilon_0} \int_{\phi}^{2L\sqrt{2}} \frac{1}{x} dx$$

= ... ?

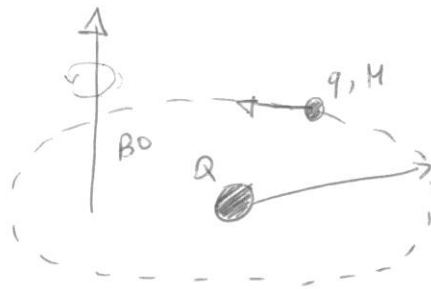
$$V(A) = \frac{Kq}{2L} + \frac{Kq}{2L} = \frac{2Kq}{2L} = \frac{Kq}{L}$$

$$V(C) = \frac{Kq}{2L} + \frac{Kq}{2L} - \frac{\lambda}{2\pi \epsilon_0} \ln |2L\sqrt{2}|$$

$$\text{d.d.p.} = V(C) - V(A) = - \frac{\lambda}{2\pi \epsilon_0} \ln |2L\sqrt{2}|$$

19/11/2012

ES. 1



T periodo di rivoluzione.

a)

R orbita, v_0 particella

$$F_e = \frac{q \cdot Q \cdot K}{R^2} = m \cdot a_c$$

$$\Rightarrow \frac{K Q q}{R^2} = M \frac{v_0^2}{R}$$

$$a_c = \frac{F_e}{m} = \omega^2 R = \frac{v_0^2}{R}$$

$$T = \frac{2\pi R}{v_0} \Rightarrow R = \frac{T v_0}{2\pi}$$

$$\omega = \frac{2\pi}{T} \Rightarrow v_0 = \omega R$$

$$\frac{K Q q 2\pi}{T v_0} = M v_0^2$$

$$v_0^3 = \frac{K Q q 2\pi}{T M}$$

$$\left\{ \begin{array}{l} v_0 = \sqrt[3]{\frac{K Q q 2\pi}{M T}} \\ R = \frac{T \cdot v_0}{2\pi} \end{array} \right.$$

b)

~~Moto elettrone:~~

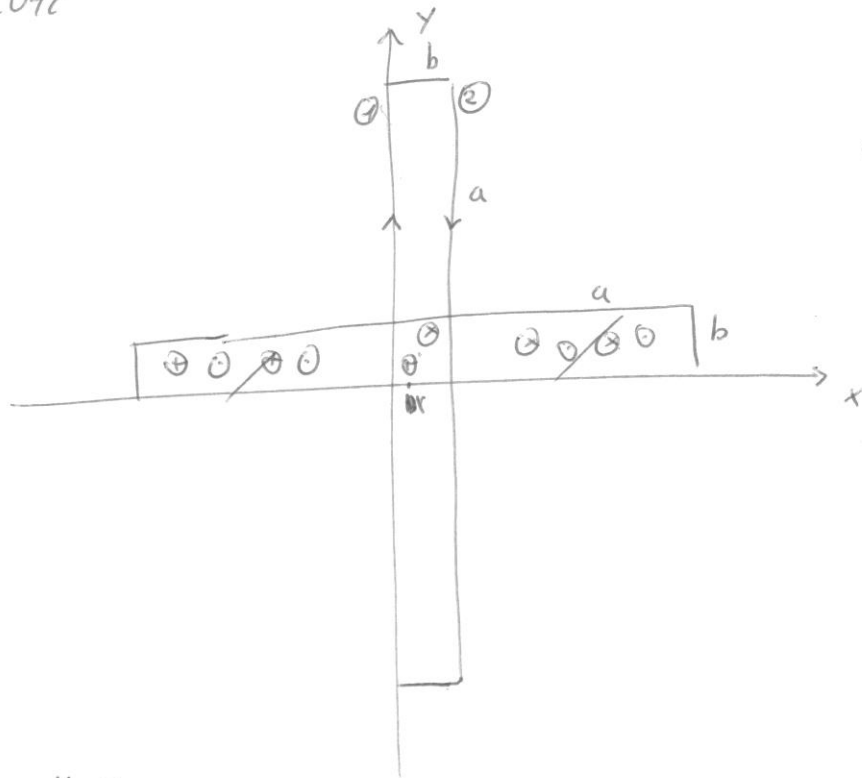
~~$$R' = \frac{m v_0}{q B_0}$$~~
~~$$a_c = \frac{F_L + F_e}{M} = \frac{\omega^2}{R'}$$~~
~~$$R = R'$$~~

$$F_L + F_e = m \cdot a_c = m \cdot \frac{v_{lim}^2}{R}$$

$$v_0 q B + \frac{K Q q}{R^2} = m \frac{v_{lim}^2}{R} \Rightarrow v_{lim} = \sqrt{\frac{R}{m} \left(v_0 q B + \frac{K Q q}{R^2} \right)}$$

19/11/2012

ES. 4



$a \gg b$

$$B_1(r) = \frac{\mu_0 I}{2\pi r}$$

$$B_{TOT} = B_1(r) + B_2(r)$$

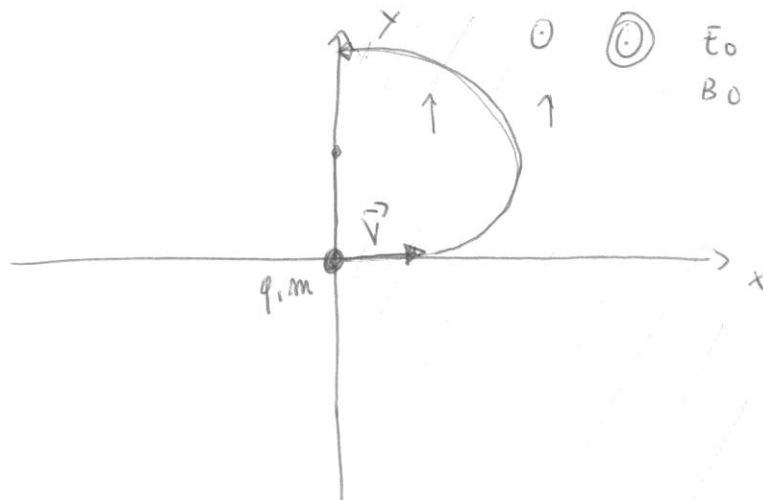
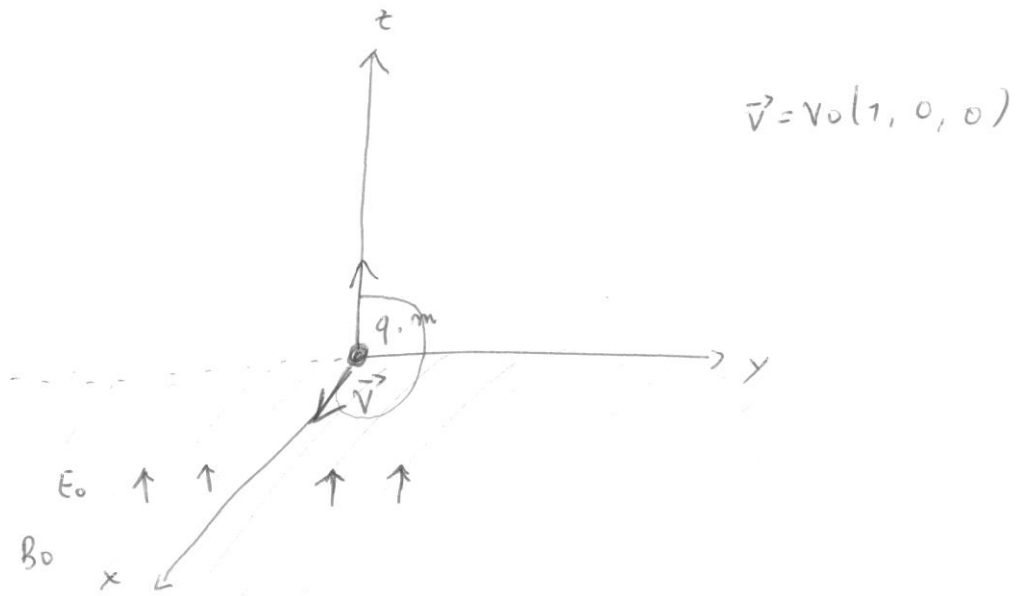
$$B_2(r) = - \frac{\mu_0 I}{2\pi(-r+b)}$$

Flusso solo su superficie centrale

$\phi(B) = \int B \cdot dS$, ma viene $\ln(\phi) = \text{IMPOSSIBILE}$

02/02/2011

ES. 1



$$R = \frac{v_0 m}{q B_0}$$

$$y = 2R$$

$$x = \phi$$

$$F_c = q \cdot B_0$$

$$m \cdot a = q \cdot E_0$$

$$a = \frac{q \cdot E_0}{m}$$

$$P = \left(\frac{2R}{\frac{q E_0}{m}} \cdot \left(\frac{1}{2} \right)^2 \right)$$

$$\text{time} = \frac{\pi m}{q B_0}$$

$$z(t) = \int \int \frac{q \cdot E_0}{m} \cdot dt^2 = \frac{q E_0}{m} \cdot \frac{t^2}{2}$$

$$\Rightarrow z = \frac{q E_0}{m} \cdot \frac{\left(\frac{T}{2} \right)^2}{2}$$

$$T = \frac{2\pi m}{q B_0}$$

$$\Rightarrow \frac{T}{2} = \frac{\pi m}{q B_0}$$

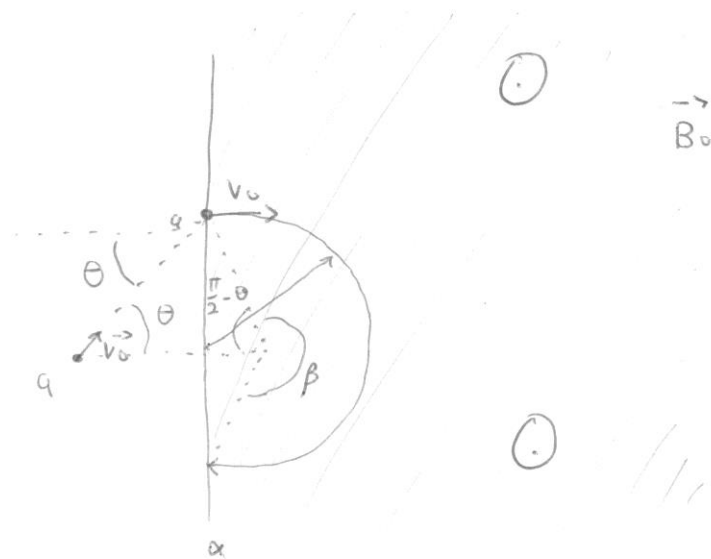
Velocità usata?

$$v(t) = \int a(t) = \frac{qE_0}{m} \cdot t$$

$$v_{im} = \begin{pmatrix} v_0 \\ 0 \\ 0 \end{pmatrix} \quad v_{fim} = \begin{pmatrix} v_0 \\ 0 \\ v_z \end{pmatrix}$$

$$v_z/t = \frac{qE_0}{m} \cdot t \Big|_{\frac{\tau}{2}} \quad v_{fim} = \begin{pmatrix} v_0 \\ 0 \\ \frac{qE_0 \tau}{m} \end{pmatrix}$$

09/02/2011



$$\vec{B}_0 \parallel \alpha$$

$$\vec{B}_0 \perp v_0$$

? Dopo quanto tempo q riemerge da $\vec{B}_0$

$$\beta = 2\pi - 2\left(\frac{\pi}{2} - \theta\right) = \pi + 2\theta$$

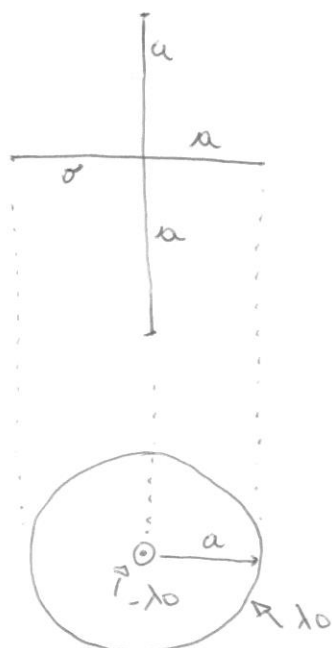
$$\Delta t : \beta = T : 2\pi$$

$$T = \frac{2\pi m}{q B_0}$$

$$\Delta t = \frac{\beta T}{2\pi} = \frac{(\pi + 2\theta) m}{q B_0}$$

14/07/2014

ES. 1

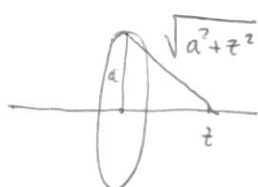


densità di carica anello: λ_0

densità di carica anello: $-\lambda_0$

Lo spazio si può muovere lungo l'asse dell'anello.

? Scrivere l'espressione della forza scambiata fra i due corpi in funzione della distanza x fra il centro della sfera e il centro dell'anello.



campo di un anello sul proprio asse.

Calcolo il campo elettrostatico:

$$E(z) = \int dE_z = \int_0^{2\pi a} \frac{k \cdot \lambda_0 d\ell}{a^2 + z^2} \cdot \frac{z}{\sqrt{a^2 + z^2}}$$

↑
componente lungo z .

$$= \frac{k \lambda_0 z}{(a^2 + z^2)^{3/2}} \int_0^{2\pi a} d\ell = \frac{k \lambda_0 z 2\pi a}{(a^2 + z^2)^{3/2}}$$

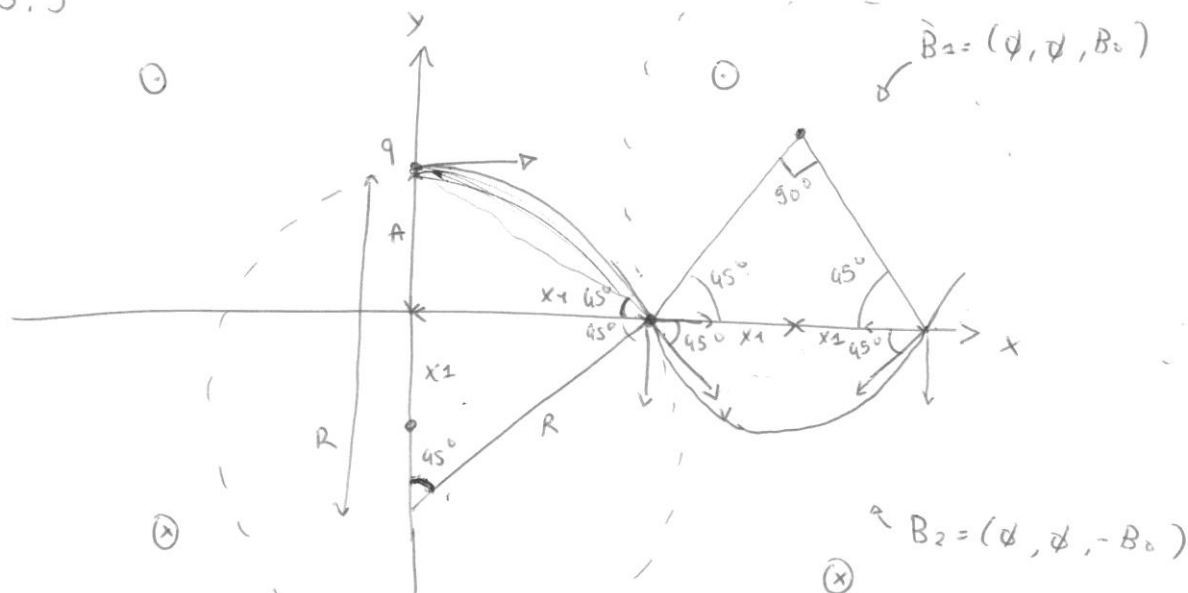
$$F(x) = \int E(z) \cdot dQ = \int_{x-a}^{x+a} E(z) \cdot (-\lambda_0) \cdot dz = \int_{x-a}^{x+a} \frac{k \lambda_0 2\pi a z}{(a^2 + z^2)^{3/2}} \cdot (-\lambda_0) \cdot dz$$

$$= -\lambda_0^2 \pi a k \int_{x-a}^{x+a} 2z (z^2 + a^2)^{-3/2} dz = -k \pi a \lambda_0^2 \cdot \left(\frac{z^2 + a^2}{-\frac{1}{2}} \right)^{-1/2} \Big|_{x-a}^{x+a}$$

$$= 2k \pi a \lambda_0^2 (x^2 + 2a^2 + 2ax)^{-1/2} - 2k \pi a \lambda_0^2 (x^2 - 2ax)^{-1/2}$$

14/07/2011

ES. 3



q in $t=0$ si trova in $(0, A, 0)$ si muove verso x

q passa per $y=0$ con $v = v_1 \left(\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}, 0 \right)$

? v_1, t_1, x_1 primo passaggio

$$\boxed{x_1} = R \cos 45^\circ = R \cdot \frac{\sqrt{2}}{2} = \dots$$

$$A = R - R \cos 45^\circ = R \left(1 - \frac{\sqrt{2}}{2} \right)$$

$$R = \frac{A}{1 - \frac{\sqrt{2}}{2}}$$

$$R = \frac{v_1 m}{q B_0} \Rightarrow \boxed{v_1} = \frac{R q B_0}{m} = \frac{A}{1 - \frac{\sqrt{2}}{2}} \cdot \frac{q B_0}{m}$$

$$\omega_1 = \frac{v_1}{R} : \text{V. angolare}$$

$$\boxed{t_1} = \frac{s_1}{\omega_1} = \frac{\pi/4}{\omega_1} = \dots$$

All'istante n-esimo

$$x_n = x_1 + 2(n-1)x_1$$

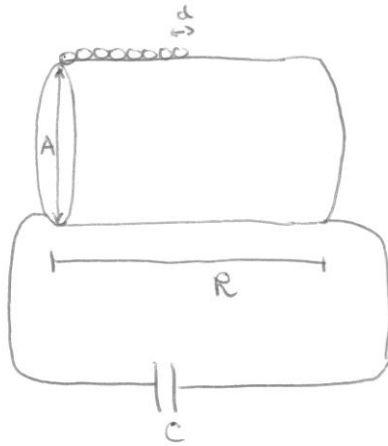
$$t_n = t_1 + 2(n-1)t_1$$

14/07/2011

ES. 4

filo con resistenza ρ e diametro

$A \gg d$



? Per quali valori di C il circuito supporta oscillazioni smorzate.

$$L = \frac{\Phi(B)}{I} \quad ; \quad R = \rho \cdot \frac{\ell}{S} = \rho \cdot \frac{2\pi N a}{\pi \left(\frac{d}{2}\right)^2} = \rho \frac{2\pi a N}{\frac{d^3 \pi}{4}} = \rho \frac{8 a N}{d^3}$$

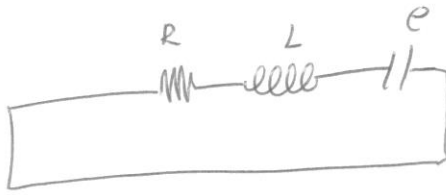
$$R = d \cdot N \Rightarrow N = \frac{R}{d}$$

$$\begin{aligned} \vec{B} &= \mu_0 n \cdot I, \quad n = \frac{N}{\ell} \\ &= \mu_0 \cdot \frac{N}{R} \cdot I \\ &= \frac{\mu_0 I}{d} \end{aligned}$$

$$\Phi(\vec{B}) = \frac{\mu_0 I}{d} \cdot \pi A^2 \cdot N = \frac{\mu_0 I \pi A^2 R}{d^2}$$

$$L = \frac{\Phi(B)}{I} = \frac{\mu_0 \pi A^2 R}{d^2}$$

RLe



$$V_R(t) + V_L(t) + V_E(t) = 0$$

$$R \cdot i(t) + L \cdot \dot{i}(t) + \frac{1}{C} \int_{-\infty}^t i(t') dt' = 0$$

$$i(t) + \frac{R}{L} i(t) + \frac{1}{LC} \int_{-\infty}^t i(t') dt' = 0$$

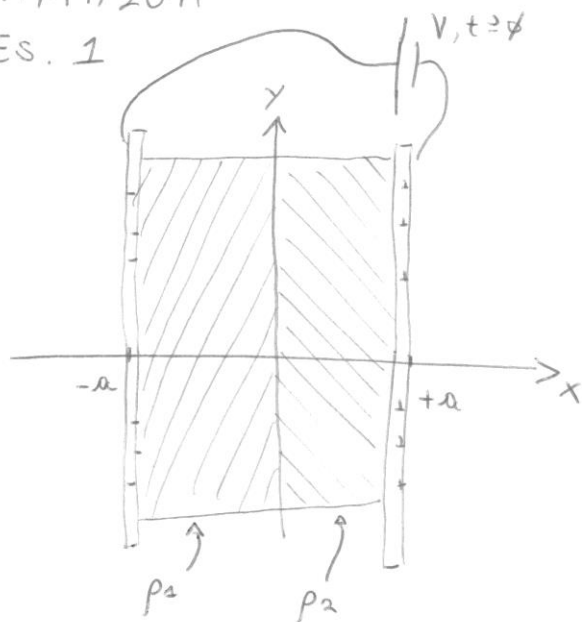
$$\left\{ \begin{array}{l} \alpha = \frac{R}{2L} \\ \omega_0 = \frac{1}{\sqrt{LC}} \end{array} \right.$$

$\boxed{\alpha < \omega_0}$ OSCILLAZIONI SMORZATE

$$\frac{R}{2L} < \frac{1}{\sqrt{LC}} \Rightarrow \boxed{C < \frac{4L}{R^2}}$$

17/11/2011

Es. 1



? $E(x)$ a $t=\phi$, $t=\infty$.

$$E(x) \text{ a } t=\phi = \frac{V}{2a}$$

$$\text{div} E = \phi, \quad t=\phi$$

E uniforme

$t=\infty \Rightarrow J$ deve essere uniforme $\Rightarrow J = J_\infty = \text{costante}$

$$E_1 = \rho_1 J; \quad E_2 = \rho_2 J$$

$$V = \int E_1 dx + \int E_2 dx = E_1 a + E_2 a$$

$$\begin{cases} E_1 = \rho_1 J \\ E_2 = \rho_2 J \end{cases}$$

$$V = E_1 a + E_2 a = \rho_1 a J + \rho_2 a J \Rightarrow J = \frac{V}{\rho_1 a + \rho_2 a}$$

$$E = E_2 - E_1 = \frac{V (\rho_2 - \rho_1)}{a (\rho_1 + \rho_2)} \text{ a } t=\infty.$$

? a $t=\infty$ è presente carica distribuita nel volume.

Si ha carica all'interfaccia fra i due conduttori

$$\text{disc } E = \frac{\sigma}{\epsilon_0} \Rightarrow \sigma = \text{disc } E \cdot \epsilon_0 = (E_2 - E_1) \cdot \epsilon_0$$

? Densità di potenza in funzione di x a $t=\phi$; $t=\infty$.

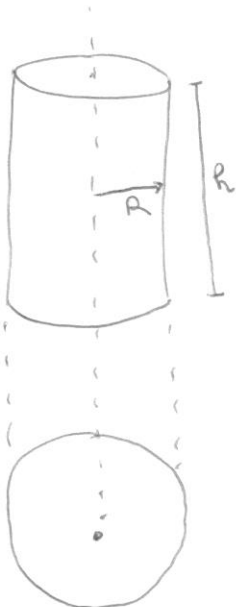
$$\begin{cases} u_p(x) |_{t=\phi} = \frac{E^2}{\rho} \\ u_p(x) |_{t=\infty} = \rho J^2 \end{cases}$$

17/11/2014

ES. 3

$$\rho(r) = \rho_0 e^{-\frac{r^2}{R^2}}$$

, r distanza da asse di simmetria



? Espressione del campo $\vec{E}$

- componente radiale; ✓
- " tangenziale; X
- " assiale; X

$$Q = \int_0^R \rho(r) \cdot 2\pi r R dr$$

$$= \int_0^R \rho_0 e^{-\frac{r^2}{R^2}} \cdot 2\pi r R dr$$

$$x = \frac{r^2}{R^2} ; dx = \frac{2r}{R^2} dr \quad = \int_0^{\frac{r^2}{R^2}} \rho_0 \pi R e^{-x} R^2 dx$$

$$= \rho_0 \pi R R^2 \int_0^{\frac{r^2}{R^2}} e^{-x} dx = \rho_0 \pi R R^2 \left(1 - e^{-\frac{r^2}{R^2}} \right)$$

Tramite T. di Gauss:

$$\Phi_E = E(4)2\pi r R = \frac{Q}{\epsilon_0} \Rightarrow E(r) = \frac{Q}{2\pi r R \epsilon_0} = \frac{\rho_0 R^2 (1 - e^{-\frac{r^2}{R^2}})}{2r \epsilon_0}$$

? Flusso di E attraverso un cilindro di raggio R e altezza R

$$\Phi_E = \frac{Q}{\epsilon_0} = \frac{\rho_0 \pi R \cdot R^2 (1 - e^{-\frac{R^2}{R^2}})}{\epsilon_0} = \frac{\rho_0 \pi R^3 (1 - 1/e)}{\epsilon_0}$$

? Divergenza di E in un punto sulla superficie di tale cilindro

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho(r)}{\epsilon_0} \Big|_{r=R} = \frac{\rho_0 \cdot e^{-R^2/R^2}}{\epsilon_0} = \frac{\rho_0}{e \cdot \epsilon_0}$$

17/11/2011

Es. 4

È possibile produrre in una regione di spazio vuoto un campo magnetostatico e elettrostatico così descritto:

$$\vec{E}(x, y, z) = \vec{E}(x) = E_0 (\phi, k_E x, \phi)$$

$$\vec{B}(x, y, z) = \vec{B}(x, z) = B_0 (k_H x, \phi, k_H z)$$

Campo Elettrostatico:

- IRROTAZIONALE

$$\text{rot } \vec{E} = \vec{0}$$

$$\begin{aligned} \text{rot } \vec{E} = \vec{\nabla} \wedge \vec{E}(x) &= \vec{u}_x \left(\frac{\partial}{\partial y} E_z - \frac{\partial}{\partial z} E_y \right) + \vec{u}_y \left(\frac{\partial}{\partial z} E_x - \frac{\partial}{\partial x} E_z \right) + \vec{u}_z \left(\frac{\partial}{\partial x} E_y - \frac{\partial}{\partial y} E_x \right) \\ &= E_0 k_E \cdot \vec{u}_z = (\phi, \phi, E_0 k_E) \neq \vec{0}, \text{ non può esistere} \end{aligned}$$

- CONSERVATIVO

Campo Magnetostatico:

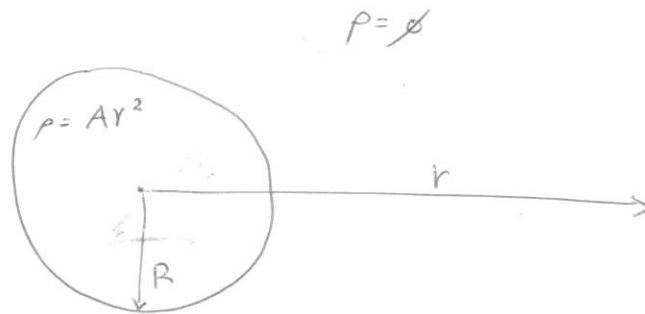
$$\text{rot } \vec{B} \neq \vec{0}, \text{ div } \vec{B} = \vec{0}$$

$$\text{rot } \vec{B} = \vec{0}$$

, non può esistere.

21/06/2010

ES. 1



$$\rho(r) = Ar^2, \quad r < R$$

$$\rho(r) = \emptyset, \quad r \geq R$$

$$q_{int} = \int_0^r \rho(r) \cdot 4\pi r^2 dr$$

$$\phi(E) = 4\pi r^2 E(r) \frac{q_{int}}{\epsilon_0}$$

~~$$E(r) = \frac{q}{4\pi r^2 \epsilon_0} = \frac{\rho(r) \cdot \frac{4\pi}{3} r^3}{4\pi r^2 \epsilon_0} = \frac{\rho(r) r}{3 \epsilon_0}$$~~

$$Q = \int_0^R \rho(r) \cdot 4\pi r^2 dr = \int_0^R Ar^2 \cdot 4\pi dr$$

superficie gauss

$$= A 4\pi \left. \frac{r^5}{5} \right|_0^R = A \frac{4}{5} \pi R^5$$

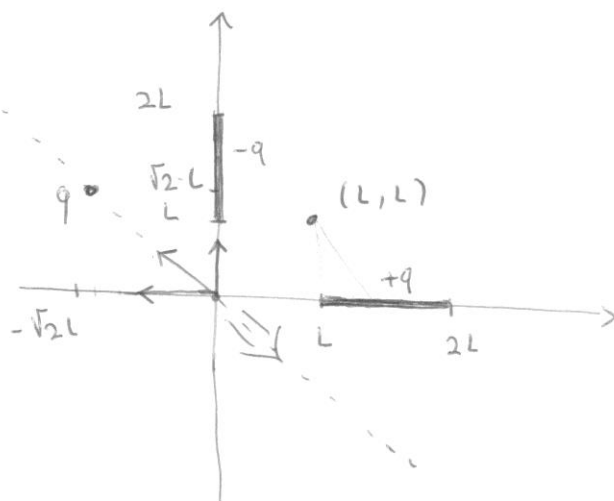
$$\Rightarrow A = \frac{5Q}{4\pi R^5} \left[\frac{C}{m^3} \right]$$

~~$$V(R) = \frac{k q}{R} = \frac{1}{4\pi \epsilon_0} \cdot \frac{1}{R} \cdot \rho(R) \cdot \frac{4\pi}{3} R^3 = \frac{\rho(R) R^2}{3 \epsilon_0}$$~~

$$\Delta V = \int_0^R E(r) dr$$

17/09/2010

ES. 1



? modulo, direzione e verso di E in (φ, φ)

$\vec{E}$ è diretto lungo la diagonale del II° e IV° quadrante, ovvero verso nel senso del II° quadrante

$$E_x(\varphi) = \int dE = \int \frac{k dQ}{r^2} = \int_L^{2L} \frac{k \frac{q}{L} dx}{x^2} = k \frac{q}{L} \int_L^{2L} x^{-2} dx$$

$$dQ = \lambda dx = \frac{q}{L} dx$$

$$= -k \frac{q}{L} \frac{1}{x} \Big|_L^{2L} = -k \frac{q}{L} \frac{1}{2L} + k \frac{q}{L} \frac{1}{L} = \frac{kq}{L^2} - \frac{kq}{2L^2} = \frac{kq}{2L^2}$$

$$E_y(\varphi) = \int dE = \int \frac{k dQ}{r^2} = \int_L^{2L} \frac{-k \frac{q}{L} dy}{y^2} = -k \frac{q}{L} \int_L^{2L} y^{-2} dy$$

$$= \frac{kq}{L} \frac{1}{y} \Big|_L^{2L} = \frac{kq}{L} \frac{1}{2L} - \frac{kq}{L} \frac{1}{L} = \frac{kq}{2L^2} - \frac{kq}{L^2} = -\frac{kq}{2L^2}$$

$$|E(0,0)| = \sqrt{E_x(\varphi)^2 + E_y(\varphi)^2} = \sqrt{\frac{k^2 q^2}{4L^4} + \frac{k^2 q^2}{4L^4}} = \frac{kq}{\sqrt{2} L^2}$$

? Disporre sul piano una carica q in modo da annullare il campo in (ϕ, ϕ) .

$$E_x(\phi) = E_{qx}(\phi)$$

$$\frac{Kq}{2L^2} = \frac{Kq}{x^2} \Rightarrow x = \pm \sqrt{2} \cdot L$$

$$E_y(\phi) = E_{q,y}(\phi)$$

$$(-\sqrt{2}L, \sqrt{2}L)$$

$$-\frac{Kq}{2L^2} = \frac{Kq}{y^2} \Rightarrow y = \pm \sqrt{2} \cdot L$$

? d.d.p. fra (L, L) e (ϕ, ϕ)

PRIMA di q

$$V(\phi) = \int \frac{K dQ}{d} + \int \frac{K dQ}{d} = \int_L^{2L} \frac{K \cdot \frac{q}{L} dx}{L+x} + \int_L^{2L} \frac{K \cdot \left(-\frac{q}{L}\right) dy}{L+y}$$

$\uparrow$ $\uparrow$
 Striscia x Striscia y

$$= K \cdot \frac{q}{L} \ln\left(\frac{3}{2}\right) - K \cdot \frac{q}{L} \ln\left(\frac{3}{2}\right) = \phi$$

$$V(L, L) = \int \frac{K dQ}{d} + \int \frac{K dQ}{d} = \int_L^{2L} \frac{K \cdot \frac{q}{L} dx}{\sqrt{L^2 + (L+x)^2}} + \int_L^{2L} \frac{K \cdot \left(-\frac{q}{L}\right) dy}{\sqrt{L^2 + (L+y)^2}} = \phi$$

d.d.p. prima = ϕ

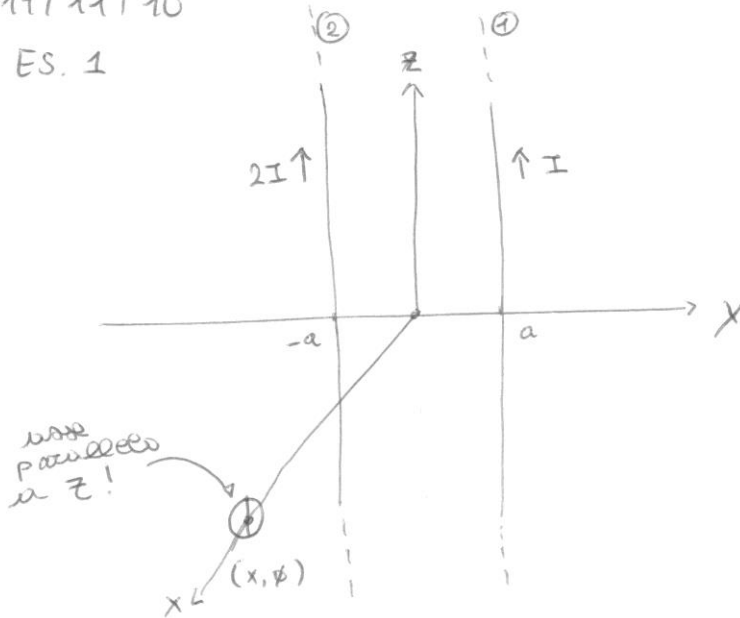
DOPO di q

$$V(\phi) = V(\phi)_{\text{prima}} + \frac{Kq}{2L}$$

$$V(L, L) = V(L)_{\text{prima}} + \frac{Kq}{\sqrt{6}L^2}$$

11/11/10

ES. 1



Una spira circolare di raggio $A \ll a$ e resistenza R ruota intorno al suo diametro a v. angolare ω_0 .

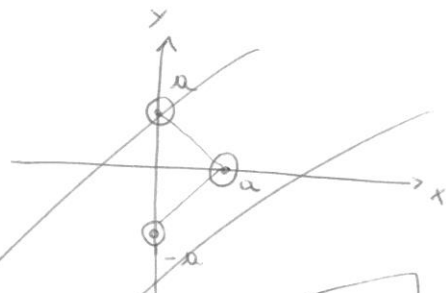
asse di rotazione parallelo a z interseca il piano xy in (x, y)

? Potenza media dissipata dalla spira in $x=a$.

Legge di Ampere, campo magnetico generato da un filo a distanza r : $B = \frac{\mu_0 I}{2\pi r}$, diretto perpendicolarmente al piano del filo.

$$B_1 = \frac{\mu_0 I}{2\pi \sqrt{x^2 + a^2}}, \quad B_2 = \frac{2\mu_0 I}{2\pi \sqrt{x^2 + a^2}}$$

Il campo B ha solo componenti x, y ; z è nulla.



$$B_2(a) = \frac{2\mu_0 I}{2\pi \sqrt{2}a}, \quad B_1(a) = \frac{\mu_0 I}{2\pi \sqrt{2}a}$$

$$B_{2x}(a) = \frac{2\mu_0 I}{2\pi \sqrt{2}a} \cdot \frac{1}{\sqrt{2}}$$

$$B_{2y}(a) = \frac{2\mu_0 I}{2\pi \sqrt{2}a} \cdot \frac{1}{\sqrt{2}}$$

$$B_{1x}(a) = -\frac{\mu_0 I}{2\pi \sqrt{2}a} \cdot \frac{1}{\sqrt{2}}$$

$$B_{1y}(a) = \frac{\mu_0 I}{2\pi \sqrt{2}a} \cdot \frac{1}{\sqrt{2}}$$

$$\frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$B_{tot}^2 = (B_{1x} + B_{2x})^2 + (B_{1y} + B_{2y})^2$$

$$B_{tot}(a) = \frac{\sqrt{5}}{2\sqrt{2}} \frac{\mu_0 I}{\pi a}$$

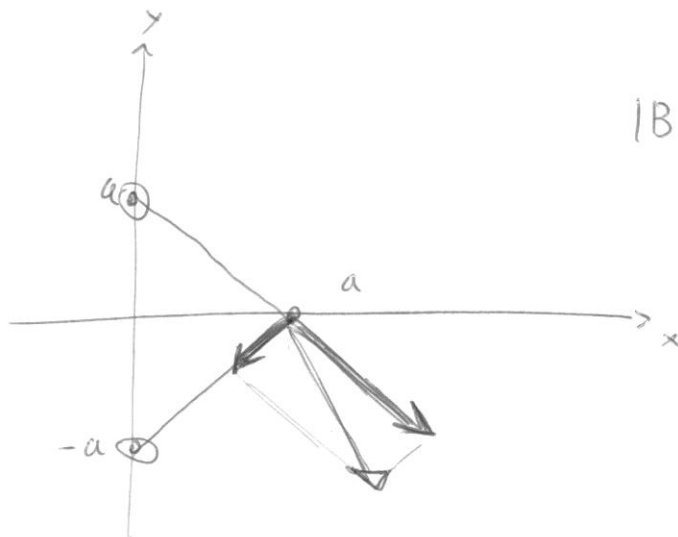
$$\phi_B = \pi A^2 \cdot B \cdot \cos(\omega_0 t)$$

$$S(t) = \pi A^2 \cos(\omega_0 t)$$

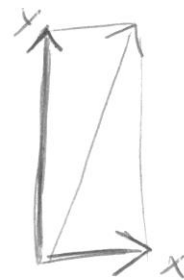
$$\mathcal{E} = - \frac{d}{dt} \phi_B = \frac{\sqrt{5}}{2\sqrt{2}} A^2 \omega_0 \sin(\omega_0 t)$$

$$P(t) = \frac{\mathcal{E}^2}{R} = \frac{5}{8} \cdot \frac{A^4 \omega_0^2 \sin^2(\omega_0 t)}{R}$$

$$P_{\text{media}} = \frac{2\pi}{\omega_0} \int_0^{2\pi/\omega_0} p(t) dt = \dots \text{media di } \sin^2(\omega_0 t) \cdot \frac{1}{2} = \dots = \frac{5}{8} \cdot \frac{\omega_0^2 \cdot 7/2}{R}$$



$$|B| = \frac{\mu_0 i}{2\pi\sqrt{2} \cdot a}$$



$$B_1 = |B| \vec{u}_x$$

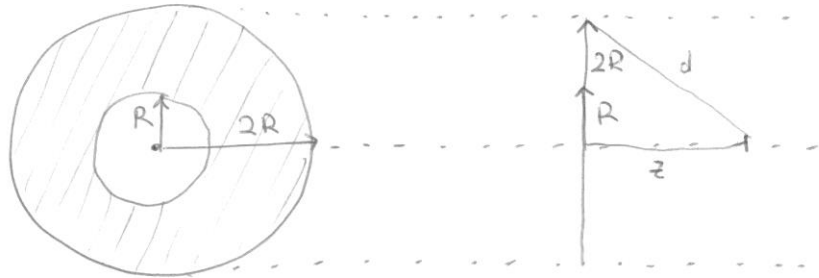
$$B_2 = |B| \cdot \vec{u}_y$$

$$B_{\text{TOT}} \begin{pmatrix} B \\ 2B \end{pmatrix}; B_{\text{TOT}} = \sqrt{B^2 + 4B^2} = \sqrt{5} \cdot B = \frac{\sqrt{5} \mu_0 i}{2\pi\sqrt{2} \cdot a}$$

11/11/2010

ES. 3

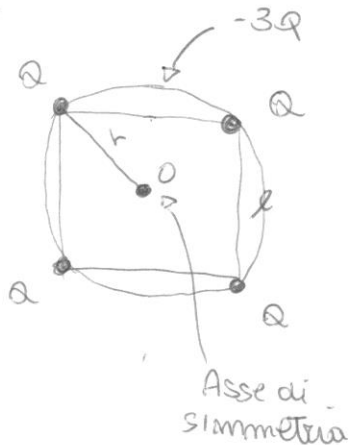
7. Potenziale elettrostatico lungo l'asse di un disco forato su cui è unig. distrib. carica superficiale con densità σ .



$$\begin{aligned} V(z) &= \int_R^{2R} \frac{K dq}{\sqrt{r^2 + z^2}}, \quad dq = 2\pi r dr \cdot \sigma \\ &= \int_R^{2R} \frac{K 2\pi r dr \sigma}{\sqrt{r^2 + z^2}} = K\pi\sigma \int_R^{2R} \frac{2r (r^2 + z^2)^{-1/2}}{dr} \\ &= 2K\pi\sigma (r^2 + z^2)^{-1/2} \Big|_R^{2R} = 2K\pi\sigma \left\{ (4R^2 + z^2)^{\frac{1}{2}} - (R^2 + z^2)^{\frac{1}{2}} \right\} \end{aligned}$$

14/01/09

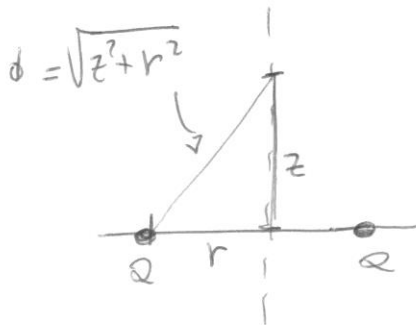
ES. 1



$$r = \frac{l\sqrt{2}}{2}$$

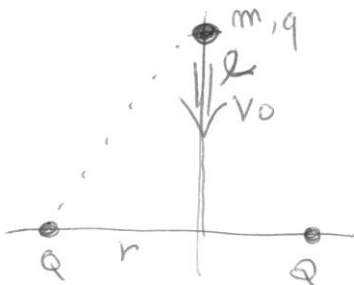
? Calcolare il Potenziale elettrostatico sull'asse di simmetria del sistema

Calcolando prospettiva



$$\begin{aligned} V(z) &= \frac{kQ}{d} \cdot 4 + \frac{k(-3Q)}{d} \\ &= \frac{4kQ}{d} - \frac{3kQ}{d} = \frac{kQ}{d} = \frac{kQ}{\sqrt{z^2 + r^2}} \\ &= \frac{k \cdot Q}{\sqrt{z^2 + \frac{l^2}{2}}} \end{aligned}$$

? Una particella di massa m e carica q si muove lungo l'asse; essa a distanza l ha velocità V_0 . Per quale valore di q essa arriva in $(0,0)$ con $V=0$?



L'energia meccanica totale di m si conserva!

$$E_t = E_k + U_e = \frac{1}{2} m V_0^2 + q \cdot V(l) = \frac{1}{2} m V_0^2 + q \cdot \frac{kQ}{\sqrt{l^2 + \frac{l^2}{2}}}$$

$\uparrow$ energia cinetica $\uparrow$ energia potenziale $\uparrow$ U_e di q nel campo della figura sopra

Per ipotesi $V = \emptyset$ in $z = \emptyset$

$$\Rightarrow E_{\emptyset} = \cancel{E_k} + U_e = q \cdot V(\emptyset) = \frac{q \cdot K \cdot Q}{\sqrt{\frac{l^2}{2}}}$$

$\swarrow$
= $\emptyset$ in quanto
 $V = \emptyset$.

Per la conservazione:

$$E_c = E_{\emptyset}$$

$$\frac{1}{2} m V_0^2 + \frac{K q Q}{\sqrt{\frac{3}{2}} \cdot l} = \frac{K q Q}{\sqrt{\frac{1}{2}} \cdot l}$$

$$\frac{K q Q}{\sqrt{\frac{1}{2}} \cdot l} - \frac{K q Q}{\sqrt{\frac{3}{2}} \cdot l} = \frac{1}{2} m V_0^2$$

$$q \cdot \frac{K Q}{l} \left\{ \frac{1}{\sqrt{\frac{1}{2}}} - \frac{1}{\sqrt{\frac{3}{2}}} \right\} = \frac{1}{2} m V_0^2$$

$$\textcircled{q} = \frac{1}{2} m V_0^2 \cdot \frac{l}{K Q} \cdot \frac{1}{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}} = \frac{1}{2} m V_0^2 \cdot \frac{l}{K \cdot Q} \cdot \frac{1}{\frac{1}{\sqrt{2}} - \frac{1}{\sqrt{3}}}$$

q e Q sono correlati, quindi la monotonia di $V(z)$ garantisce che la forza sia repulsiva $\forall z$, quindi q avrà segno infinito

$$E_{\infty} = E_k + \cancel{U_e} = \frac{1}{2} m V_0^2$$

Dalla conservazione di E_c :

$$E_{\infty} = E_{\emptyset}$$

$$V_{00} = \sqrt{\frac{2}{m} \cdot \frac{K \cdot q Q}{\frac{l}{\sqrt{2}}}}$$

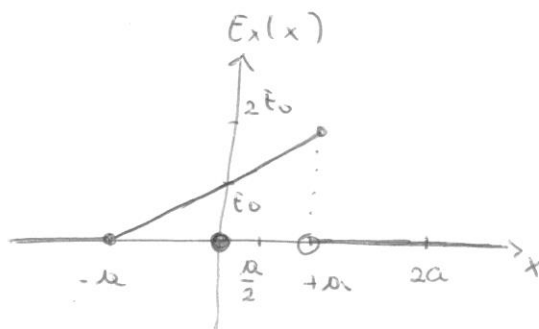
09/06/2009

ES. 1

$$\vec{E} = E_x(x) \vec{u}_x$$

$$E_x(x) = E_0 \left(1 + \frac{x}{a}\right), \quad x \in [-a, +a]$$

$$E_x(x) = 0, \quad x < -a, \quad x > a$$



? d.d.p. fra A ($x_a = \frac{a}{2}$) e B ($x_b = 2a$)

$$\Delta V = - \int_{\frac{a}{2}}^a E_x(x) dx = - \int_{\frac{a}{2}}^a \left(E_0 + \frac{E_0}{a} x \right) dx = - E_0 \int_{\frac{a}{2}}^a \left(1 + \frac{x}{a} \right) dx$$

$$= - E_0 \left[x + \frac{x^2}{2a} \right]_{\frac{a}{2}}^a = - E_0 \left[a + \frac{a^2}{2a} - \left(\frac{a}{2} + \frac{a^2}{8a} \right) \right] =$$

$$- E_0 \left[a + \frac{a}{2} - \frac{a}{2} - \frac{a}{8} \right] = - \frac{7}{8} E_0 a = V_A - V_B$$

$$V_B - V_A = \frac{7}{8} E_0 a.$$

? densità superficiale di carica sui piani $x=a$ e
rotore e divergenza di $\vec{E}$ sui piani $x = \frac{a}{2}$ e $x = 2a$ e
densità di carica sui tali piani.

La discontinuità di $E_x(a) = E_x(a^+) - E_x(a^-) = -2E_0$.

Dal T. di Gauss:

$$\sigma = -2E_0 \cdot \epsilon_0.$$

$$\text{div } \vec{E} = \frac{dE_x}{dx} = \frac{E_0}{a}, \quad x < \frac{a}{2} \quad ; \quad \text{div } \vec{E} = 0, \quad \text{altrove.}$$

$$\text{div } \vec{E}\left(\frac{a}{2}\right) = \frac{E_0}{a} \quad \sigma_{\text{in } \frac{a}{2}} = \frac{E_0}{a} \cdot \epsilon_0 \quad \text{e} \quad \sigma = \vec{\nabla} \cdot \vec{E} \cdot \epsilon_0.$$

$$\text{div } \vec{E}(2a) = 0 \quad \sigma_{\text{in } 2a} = 0$$

$\text{rot } \vec{E} = 0$ e campo elettrostatico!

? Particella di massa m e carica q ($qE_0 > 0$) si
trova in $x=0$ con $v=0$.

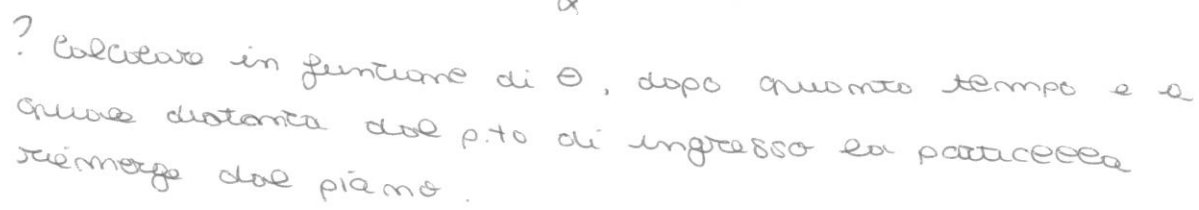
Con quale v uscirà dalla destra di carica Q dove.

$$\Delta V_{0A} = \dots = \int_A^0 E_x(x) dx = \dots = -3E_0 \frac{a}{2}$$

$$\frac{1}{2} m v^2 = q \cdot \Delta V_{0A} \Rightarrow v = \sqrt{\frac{2}{m} q \Delta V_{0A}} = \sqrt{3 \frac{q}{m} a E_0} \text{ in } x=a.$$

Campo elettrico con $Q \Rightarrow q$ esce da $x=+a$.

ES. 4



Se il campo fosse nullo la traiettoria sarebbe
 circolare di raggio $R = \frac{mv_0}{qB_0}$ con velocità in modulo
 costante v_0 e tempo: $\frac{2\pi R}{v_0} = T$.

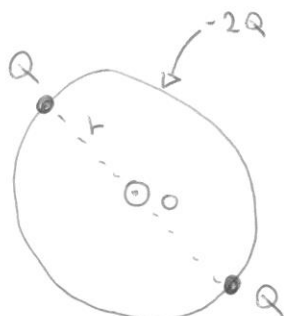
$$\beta = 2\pi - 2\left(\frac{\pi}{2} - \theta\right) = \pi + 2\theta$$

$$\Delta t = \frac{\beta T}{2\pi} = \frac{(\pi + 2\theta)H}{Q B_0} \quad \angle = \Delta t : \beta = T : 2\pi$$

$$\Delta x = 2R \sin\left(\frac{\pi}{2} - \theta\right) = 2R \cos(\theta) = \frac{2KV_0 \cos \theta}{Q B_0} \quad \therefore \frac{\Delta x}{2} = R \cdot \sin\left(\frac{\pi}{2} - \theta\right).$$

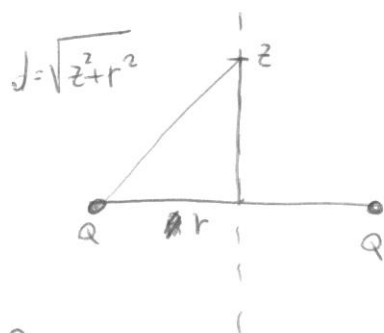
14/09/09

ES. 1



? Potenziale Elettrostatico del sistema sull'asse di simmetria

$$V(z) = \frac{Kq}{z}$$



$$V(z) = K \cdot \frac{Q}{d} \cdot 2 + K \cdot \frac{(-2Q)}{d} = 0$$

Il potenziale sull'asse risulta essere nullo.

? Una particella di massa m e carica q si muove lungo l'asse; essa a distanza r ha velocità v_0 . Per quale valore di q essa arriva in (ϕ, ϕ) con $v=0$?

$$E = E_k + U_e$$

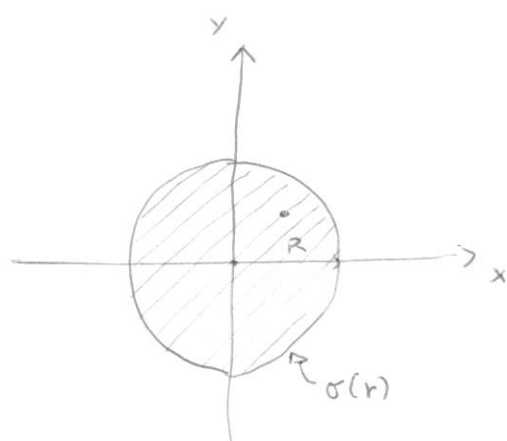
$$E_r = \frac{1}{2} m v_0^2 + q \cdot V(r) = \frac{1}{2} m v_0^2$$

$$E_0 = 0$$

↑
a occhio per nessun
valore...

08/07/2008

ES. 1



$$\sigma(r) = \sigma_0 \frac{r}{R}, \quad r = \sqrt{x^2 + y^2}$$

? Quantità di carica presente sul disco, espressioni
 analitiche di $E_x(x)$ per $x \gg R$ sui punti dell'asse x .

$$Q = \iint \sigma dS = \int_0^R \sigma(r) 2\pi r dr = \int_0^R \sigma_0 \frac{r}{R} 2\pi r dr = \frac{2\pi\sigma_0}{R} \int_0^R r^2 dr$$

$$= \frac{2\pi\sigma_0}{R} \left[\frac{r^3}{3} \right]_0^R = \frac{2\pi\sigma_0}{R} \cdot \frac{R^3}{3} = \frac{2}{3} \pi \sigma_0 R^2$$

A grande distanza, dal disco, di carica apparirà come
 una carica puntiforme e formerà un campo elettrico
 descritto dalla legge di Coulomb:
 $x \gg R$

$$E_x(x) = K \cdot \frac{Q}{x^2}$$

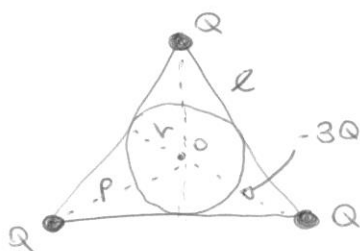
? $V(z)$ in tutti i punti di z

$$V(z) = \int K \cdot \frac{\sigma(r) \cdot dS}{\sqrt{r^2 + z^2}} = K \int_0^R \frac{\sigma(r) \cdot 2\pi r dr}{\sqrt{r^2 + z^2}} = \frac{K 2\pi \sigma_0}{R} \int_0^R \frac{r^2}{\sqrt{r^2 + z^2}} dr$$

primitiva di $\frac{x^2}{(a+x^2)^{3/2}}$ = $\frac{x}{2} \left[a+x^2 \right]^{-\frac{1}{2}} - \frac{a}{2} \ln \left[x + \left(a+x^2 \right)^{\frac{1}{2}} \right]$

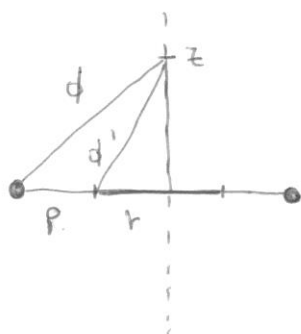
$$= \frac{K 2\pi \sigma_0}{R} \cdot \left\{ \frac{R}{2} \left[z^2 + R^2 \right]^{-\frac{1}{2}} - \frac{z^2}{2} \ln \left[R + \left(z^2 + R^2 \right)^{\frac{1}{2}} \right] + \frac{z^2}{2} \ln \left[|z| \right] \right\}$$

ES. 1



Raggio

$$\begin{aligned} \text{Circumferenza} &= 2 \cdot \frac{\text{area triangolo}}{\text{perim. triangolo}} = \frac{2 \cdot \sqrt{l^2 - \frac{l^2}{4}} \cdot l \cdot \frac{1}{2}}{3l} \\ &= \frac{\sqrt{l^2 - l^2/4}}{3} = \frac{\sqrt{3}l^2}{6} = \frac{l\sqrt{3}}{6} \end{aligned}$$



$$d = \sqrt{p^2 + z^2}$$
$$d' = \sqrt{r^2 + z^2}$$

$$P = \sqrt{r^2 + \frac{l^2}{4}} = \sqrt{\frac{l^2 \cancel{3}^1}{\cancel{36}^1} + \frac{l^2}{4}} = \frac{l}{\sqrt{3}}$$

$$V(z) = \frac{K \cdot Q}{d} \cdot 3 + \frac{K \cdot (-3Q)}{d'} = \frac{3KQ}{\sqrt{L^2 + z^2}} - \frac{3KQ}{\sqrt{\frac{L^2}{12} + z^2}}$$

$$E(z) = \frac{K \cdot Q}{d^2} \cdot 3 + \frac{K \cdot (-3Q)}{d^2} = \frac{3KQ}{\frac{Q^2+z^2}{3} + z^2} - \frac{3KQ}{\frac{Q^2+z^2}{12} + z^2}$$

? Partecella di massa m

? Particella di massa m e carica Q si muove lungo z .
 la distanza e la velocità v_0 .
 essa attraversa il piano? Sotto quali condizioni?

$$E_e = E_k + U_e = \frac{1}{2} m v_0^2 - Q \cdot V(l)$$

$$\bar{t}_0 \neq -\varphi \cdot v(\varphi).$$

Supponiamo: $t_0 = -Q \cdot V(p)$

$$\bar{E}_2 = \bar{E}_0$$

$$\frac{1}{2} m v_0^2 - Q V(l) = -Q \cdot V(0)$$

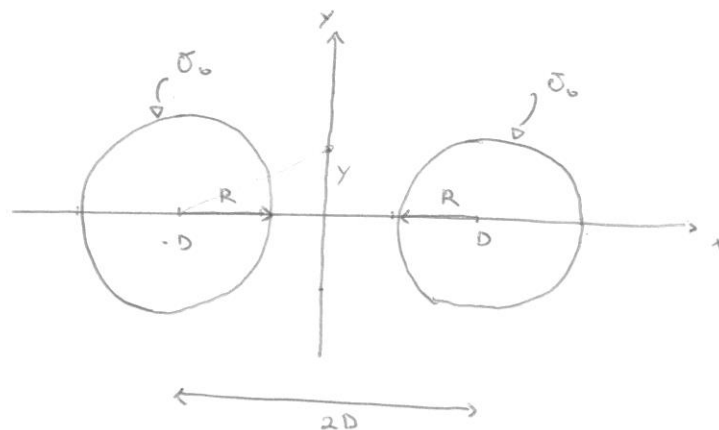
$$V_0^2 = \frac{2}{m} \cdot \{ QV(\ell) - QV(\varnothing) \} \Rightarrow V_0 = \sqrt{\frac{2}{m} \cdot \{ QV(\ell) - QV(\varnothing) \}}$$

Quindi per non avere velocità nulla in φ .

$$V_0 > \sqrt{\frac{2}{m} \cdot \{QV(x) - QV(\varphi)\}}.$$

12/12/2006

ES. 1



Carga distribuita sulla superficie

? Potenziale al centro di una delle due sfere ($V(0) = ?$).

Il campo fuori delle distribuzioni è lo stesso che si avrebbe per cariche puntiformi. ^{il comp}
 all'interno di ciascuna sfera V ha solo il contributo della carica sull'altra sfera.

$$V(z) = K \cdot \frac{Q}{R} + K \cdot \frac{Q}{2D} = K Q \left(\frac{1}{R} + \frac{1}{2D} \right) = K \cdot 4\pi R^2 \sigma_0 \cdot \frac{2D+R}{2RD}$$

$\uparrow$ $\uparrow$
 densità della carica a Coulomb
 superficie dell'altra sfera

$$= \frac{1}{4\pi\epsilon_0} \cdot 4\pi R^2 \sigma_0 \cdot \frac{2D+R}{2RD} = \frac{R\sigma_0}{\epsilon_0} \cdot \frac{2D+R}{2D}$$

? Potenziale sull'asse y

$$V(y) = K \frac{Q}{d} = K \cdot \frac{4\pi R^2 \sigma_0}{\sqrt{D^2+y^2}} = \frac{R^2 \epsilon_0}{\epsilon_0} \cdot \frac{1}{\sqrt{D^2+y^2}} \cdot 2$$

! ? Densità carica moltiplicata $\times 2$.

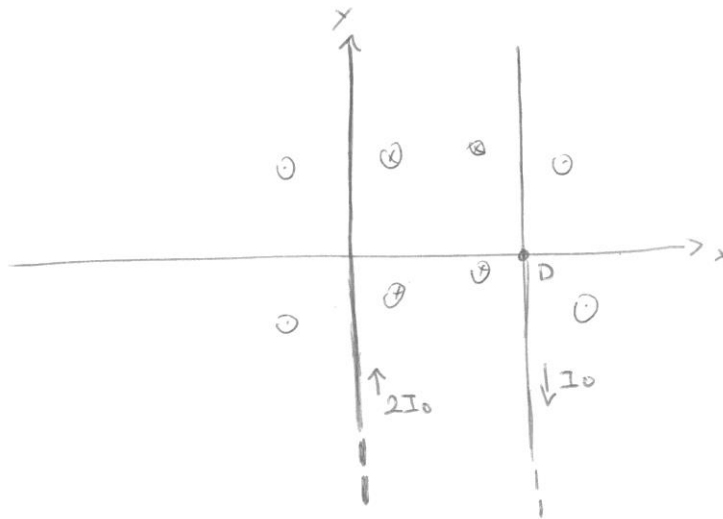
? Campo su base y e dove ha modulo max.

$$E(y) = - \frac{d}{dy} V(y)$$

$$\text{p.ti stazionari : } - \frac{d^2}{dy^2} V(y) = 0$$

12/12/06

ES. 2



? Individuare e' ensemble dei punti dell'asse x in cui è nullo il campo magnetico e dire come è diretto negli altri punti.

Per la legge di Ampere: $B = \frac{\mu_0 I}{2\pi r}$

$$B_{TOT} = B_1 + B_2 = \frac{\mu_0 2I_0}{2\pi x} + \frac{\mu_0 I_0}{2\pi (D-x)} = \frac{\mu_0 I_0}{2\pi} \left(\frac{2}{x} + \frac{1}{D-x} \right)$$

$$= \frac{\mu_0 I_0}{2\pi} \left(\frac{2D - 2x + x}{x(D-x)} \right) = \frac{\mu_0 I_0}{2\pi} \left(\frac{2D - x}{x(D-x)} \right)$$

$B_{TOT} = 0$ se $x = 2D$.

? $\frac{\partial B_z}{\partial x}$ in $x = 2D$

$$\frac{\partial B_z}{\partial x} = \frac{\mu_0 I_0}{2\pi} \left(-\frac{2}{x^2} + \frac{1}{(D-x)^2} \right) \Big|_{x=2D} = \frac{\mu_0 I_0}{2\pi} \left(-\frac{2}{4D^2} + \frac{1}{(D-2D)^2} \right)$$

$$= \frac{\mu_0 I_0}{2\pi} \left(-\frac{1}{2D^2} + \frac{1}{D^2} \right) = \frac{\mu_0 I_0}{2\pi} \cdot \frac{1}{2D^2}$$

? $\text{div } \vec{B}$ e $\text{rot } \vec{B}$ in $x = 2D$

$\vec{\nabla} \cdot \vec{B} = 0$, $\forall x$ sempre vero!

$\text{rot } \vec{B} = \vec{\nabla} \wedge \vec{B} \Big|_{x=2D} = \mu_0 \vec{J} = 0$, in quanto non c'è alcuna corrente.

$$? \quad \frac{\partial}{\partial z} B_x \quad \text{in } x=2D$$

$$\vec{\nabla} \wedge \vec{B} = \emptyset$$

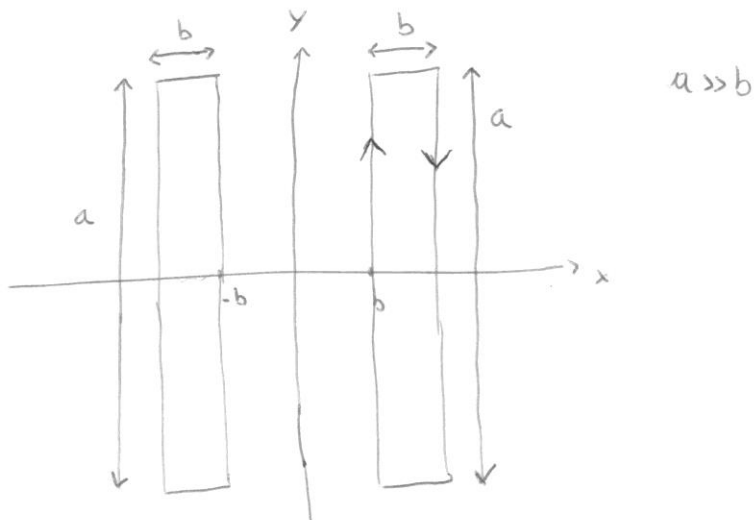
$$\left(\frac{\partial}{\partial y} B_z - \frac{\partial}{\partial z} B_y, \frac{\partial}{\partial z} B_x - \frac{\partial}{\partial x} B_z, \frac{\partial}{\partial x} B_y - \frac{\partial}{\partial y} B_x \right) = (\emptyset, \emptyset, \emptyset)$$

$$\frac{\partial}{\partial z} B_x - \frac{\partial}{\partial x} B_z = \emptyset$$

$$\frac{\partial}{\partial z} B_x = \frac{\partial}{\partial x} B_z = \emptyset !$$

12/12/2006

ES. 3



? Coefficiente di mutua induzione

$a \gg b$: Considero il campo come generato da una coppia di fili infiniti

$$B = \frac{\mu_0 I}{2\pi(x+b)} - \frac{\mu_0 I}{2\pi(x+2b)} \quad \text{Campo di una spira}$$

$$\begin{aligned} \phi_B &= \int_S B \cdot dS = \int_0^a \int_b^{2b} B \cdot dx dy = a \cdot \int_b^{2b} B \cdot dx = a \cdot \frac{\mu_0 I}{2\pi} \left(\ln(x+b) - \ln(x+2b) \right) \Big|_b^{2b} \\ &= a \cdot \frac{\mu_0 I}{2\pi} \left(\ln\left(\frac{3}{2}\right) - \ln\left(\frac{4}{3}\right) \right) = \frac{\mu_0 I a}{2\pi} \ln \frac{9}{8} \end{aligned}$$

$$M I = \phi_B \quad \Rightarrow \quad M = \frac{\phi_B}{I} = \frac{\mu_0 a}{2\pi} \ln \frac{9}{8}$$

