ELETTROTECNICA

[Fotocopie di Appunti]

A CURA DI ALESSANDRO PAGHI

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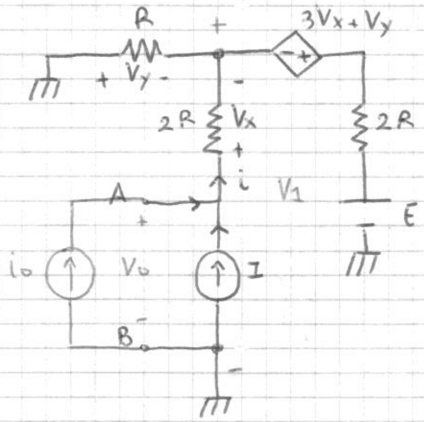
LINK AL CORSO:
http://www3.diism.unisi.it/FAC/index.php?bodyinc=didattica/inc.insegnamento.php&id=55109&aa=2015

FREQUENTAZIONE: Consigliata.

PARTE 1



? TR metodo rapido

$i = i_0 + I$

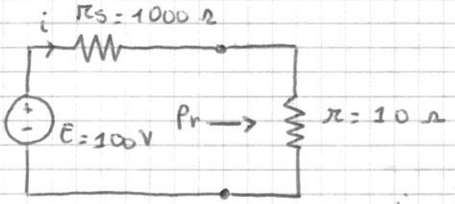
$V_x = 2R \cdot i$

$V_y = V_x - V_0 = 2Ri - V_0$

MILLMANN : $V_1 = \dfrac{i_0 + I + \dfrac{E - 3V_x - V_y}{2R}}{\dfrac{1}{R} + \dfrac{1}{2R}} = \dfrac{2Ri_0 + 2RI + E - 3V_x - V_y}{3}$

$V_0 = 2R(i_0 + I) + V_1 \leftarrow V_1 = V_0 - V_x$.

Trasformatore Adattatore di Resistenza



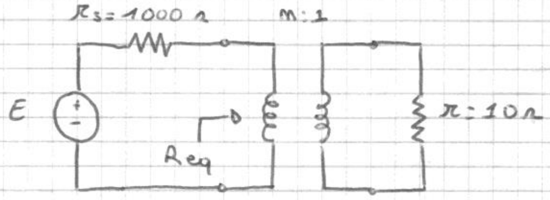
$r = r_s = 1000\ \Omega$

$P_{MAX} = \dfrac{1}{4} \dfrac{E^2}{r_s} = 2,5$ Watt

$i = \dfrac{100}{1010} \simeq 0,1\ A$

$P_r = 10 \cdot i^2 = 10 \cdot 0,1^2 = 0,1$ Watt

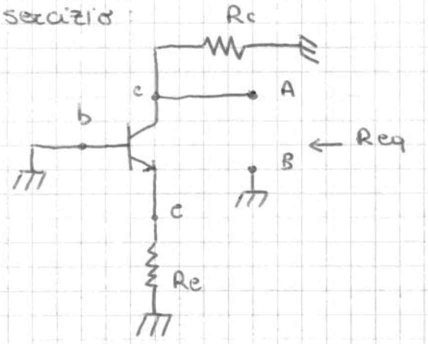
Butto fuori troppa poca potenza!



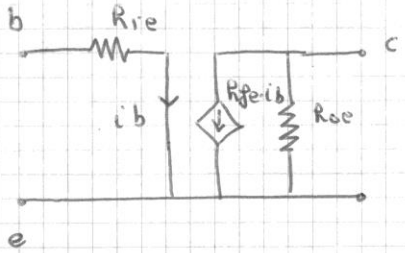
$r_s = R_{eq} = m^2 r \rightarrow m = \sqrt{\dfrac{r_s}{r}} = 10$.

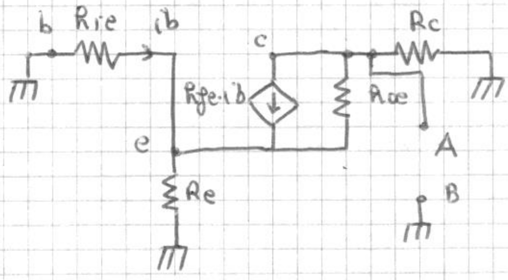
$P_r = 2,5$ Watt $= P_{MAX}$

Esercizio:

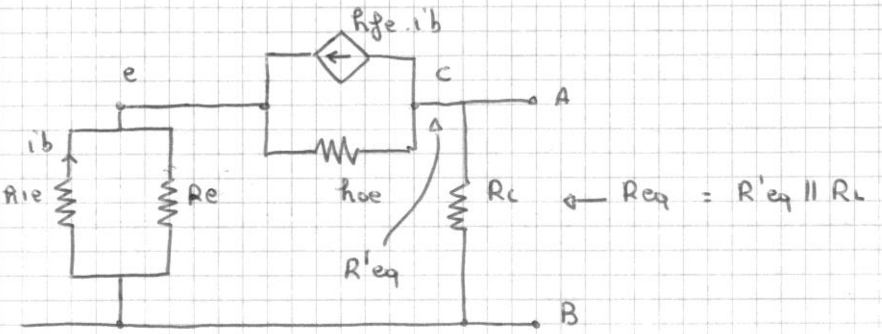


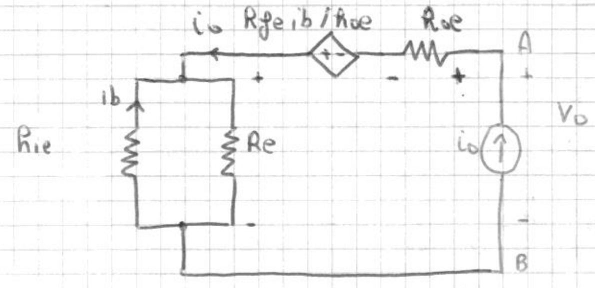
? Req utilizzando per il BJT il modello a parametri h con $h_{re} = \emptyset$





hoe conduttanza





$$i_b = -i_o \cdot \frac{\frac{1}{h_{ie}}}{\frac{1}{h_{ie}} + \frac{1}{R_e}} = -i_o \cdot \frac{R_e}{h_{ie}+R_e}$$

$$V_o = i_o \cdot \frac{1}{h_{oe}} - \frac{h_{fe}}{h_{oe}} \cdot i_b + (R_e \| h_{ie}) \cdot i_o = \frac{i_o}{h_{oe}} + \frac{h_{fe}}{h_{oe}} \cdot i_o \cdot \frac{R_e}{h_{ie}+R_e} + \frac{R_e h_{ie}\, i_o}{R_e + h_{ie}}$$

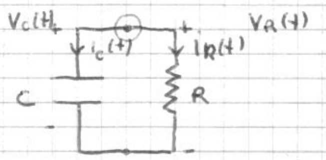
$$R'_{eq} = \frac{V_o}{i_o} = \frac{1}{h_{oe}} + \frac{h_{fe}}{h_{oe}} \cdot \frac{R_e}{h_{ie}+R_e} + \frac{R_e \cdot h_{ie}}{R_e + h_{ie}}$$

$R_{eq} = R'_{eq} \| R_L$.

24/04/2015

Circuiti del Primo ordine, analisi nel Dominio del Tempo.

Scarica di C su R



C inizialmente carico

$V_c(\emptyset) = V_\emptyset$

$V_c(t)$ variabile di Stato

$i_c(t) + i_r(t) = \emptyset$

$C \cdot \frac{d}{dt} V_c(t) + \frac{V_R(t)}{R} = \emptyset \quad , \quad t \geq \emptyset$

$V_R(t) = V_c(t)$

$$\begin{cases} \tau \cdot \frac{d}{dt} V_c(t) + V_c(t) = \emptyset \\ V_c(\emptyset) = V_0 \end{cases}$$

Eq. differenziale I° ordine omogenea.

$\tau = \tau_{RC} = RC$

Costante di tempo circuito RC 1° ordine

$[\tau] = \sec$

Problema di Cauchy → Unica Soluzione

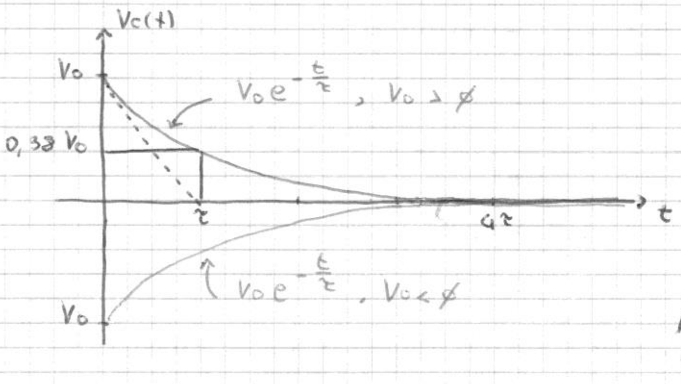
$\tau p + 1 = \emptyset \quad , \quad p = -\frac{1}{\tau} = -\frac{1}{RC}$ Radice Caratteristica

$V_c(t) = K e^{pt} = K e^{-\frac{t}{\tau}} = K e^{-\frac{t}{RC}} \quad , \quad t \geq \emptyset$

$K \in \mathbb{R}$

$V_c(\emptyset) = K = V_0$

$V_c(t) = V_0 \cdot e^{-\frac{t}{\tau}} = V_c(\emptyset) \cdot e^{-\frac{t}{\tau}} \quad , \quad t \geq \emptyset$ ← Scarica di C su R.



Risposta LIBERA, NATURALE, a INGRESSO NULLO

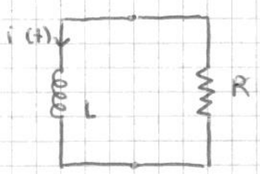
$p = -\frac{1}{RC} = -\frac{1}{\tau}$

Frequenza Naturale

Dopo $4 \div 5\,\tau$ → $V_c(t) = 0{,}02\,V_0$

Durata Transitorio di Scarica $4 \div 5\,\tau$

Scarica di L su R



Variabile di stato $i(t)$

$? \; i(t), \; t \geq 0$

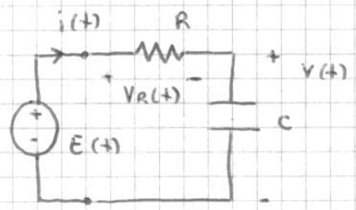
$i(0) = I_0 \qquad \tau = \frac{L}{R}$

$$\begin{cases} \tau \cdot \frac{d}{dt} i(t) + i(t) = 0 \\ i(0) = I_0 \end{cases}$$

Finire per casa!

Risposta Forzata ($v(t)$, qualora $v(0) = 0$)

↑ Risposta allo Stato Nullo



$V_R(t) + V_C(t) = E(t), \quad t \geq 0$

$R \cdot i(t) + v(t) = E(t)$

$R \cdot C \cdot \frac{d}{dt} v(t) + v(t) = E(t)$

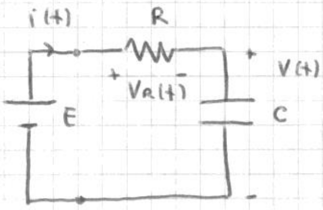
$$\begin{cases} \tau \cdot \frac{d}{dt} v(t) + v(t) = E(t), & t \geq 0 \\ v(0) = 0 \end{cases}$$

$$v(t) = V_{omog}(t) + V_{\substack{part. \\ affine}}(t) = K e^{-\frac{t}{\tau}} + V_{\substack{part. \\ affine}}(t)$$

$$V_{\substack{part. \\ affine}} = \int_0^t e^{-\frac{t-\sigma}{\tau}} \cdot E(\sigma) \, d\sigma$$

$V_{\substack{part. \\ affine}}$ ⟶ Eccitazioni Costanti

⟶ Eccitazioni Sinusoidali



$$\begin{cases} t \cdot \frac{d}{dt} \cdot v + v = E = cost. & t \geq 0 \\ v(0) = 0 \end{cases}$$

Eccitazione costante → integrale Particolare Costante

(della 'stessa forma' dell' eccitazione)

$V_{part}(t) = H$

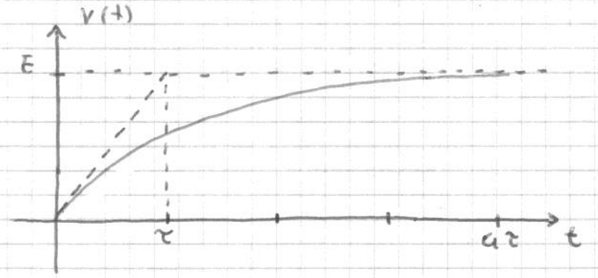
$H = E \quad , \quad t \geq 0$

$v(t) = K \cdot e^{-\frac{t}{\tau}} + E \quad , \quad t \geq 0$

$V(0) = 0 = K + E \quad \rightarrow \quad K = -E$

$v(t) = -E \cdot e^{-\frac{t}{\tau}} + E = E\left(1 - e^{-\frac{t}{\tau}}\right) , \quad t \geq 0$

Carica di C inizialmente scarico con Batteria



dopo $4\tau \rightarrow v(t) = E - 0{,}02\,E \simeq E$

Interpretazione Risposta Forzata

$v(t) = -E\, e^{-\frac{t}{\tau}} + E \, , \quad t \geq 0$

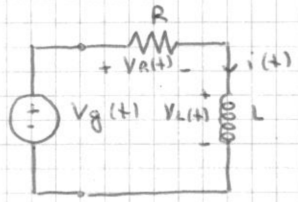
Parte Transitoria → Risposta in Regime Stazionario

Dopo $4\tau \rightarrow 0$

F = T + Regime Stazionario

Risposta in Regime Stazionario:
- int. part. affine;
- ciò che resta per $t \rightarrow +\infty$
- Della stessa Forma dell'Eccitazione;

Circuito RL con Eccitazione Sinusoidale



? $i(t)$, Risposta Forzata

$i(\emptyset) = \emptyset$: Stato nullo

$V_M \cdot \cos(\omega t) = V_R(t) + V_L(t)$, $t \geq \emptyset$

$L \cdot \frac{d}{dt} i(t) + R i(t) = V_M \cdot \cos(\omega t)$, $t \geq \emptyset$

$V_g(t) = V_M \cdot \cos(\omega t)$

$i(t) = i_{omog}(t) + i_{\substack{part. \\ affine}}(t) = K \cdot e^{-\frac{t}{\tau}} + I_M \cdot \cos(\omega t + \varphi_i)$

F = T + Regime Sinusoidale

$i(t) = i_{Trans}(t) + i_{\substack{regime \\ sim.}}(t)$

$i_{\substack{regime \\ sim}}(t) = \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} \cos\left(\omega t - \operatorname{arctg} \frac{\omega L}{R}\right)$

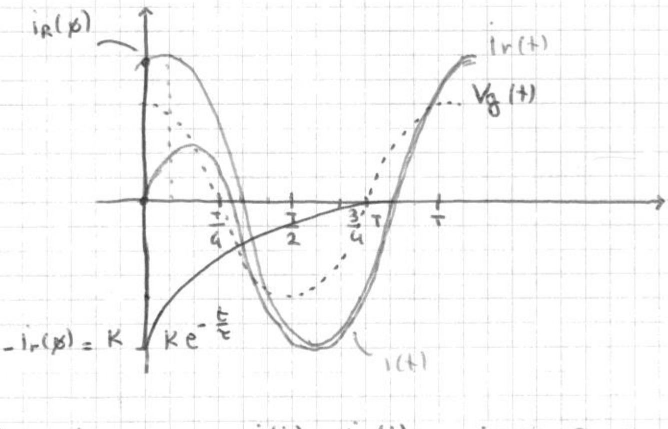
$\uparrow$

$\frac{V_M}{Z} \cdot \cos(\omega t + \varphi_V - \varphi_Z)$

$i(t) = K e^{-\frac{t}{\tau}} + \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} \cdot \cos\left(\omega t - \operatorname{arctg} \frac{\omega L}{R}\right)$

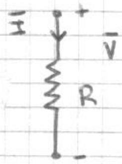
$i(\emptyset) = \emptyset = K + \frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} \cdot \cos\left(\operatorname{arctg} \frac{\omega L}{R}\right)$

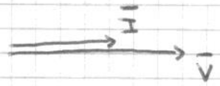
$$i(t) = \underbrace{-\frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} \cdot \cos\left(\operatorname{arctg} \frac{\omega L}{R}\right) \cdot e^{-\frac{t}{\tau}}}_{i_T(t)} + \underbrace{\frac{V_M}{\sqrt{R^2 + \omega^2 L^2}} \cdot \cos\left(\omega t - \operatorname{arctg} \frac{\omega L}{R}\right)}_{i_R(t)}$$



Con $t \to +\infty$: $i(t) = i_R(t)$, inizialmente $i(t) \neq i_R(t)$.

Espressione di P, Q per i bipoli ideali R, L, C.

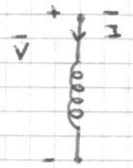




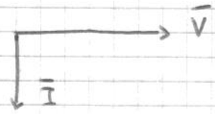
$$\bar P = \frac{1}{2}\bar V \bar I^* = \frac{1}{2} R \bar I \cdot \bar I^* = \frac{1}{2} R I_M^2 + j\emptyset$$

$\bar V = R \bar I$

$$\begin{cases} P_R = \frac{1}{2} R I_M^2 = \frac{1}{2}\frac{V_M^2}{R} \\ \quad = R I^2 = \frac{V^2}{R} \text{ Watt } \geq \emptyset \\ Q_R = \emptyset \text{ VAR} \end{cases}$$



$$\bar P = \frac{1}{2}\bar V \bar I^* = \frac{1}{2} j\omega L \cdot \bar I \cdot \bar I^* = j \cdot \frac{1}{2}\omega L I_M^2$$

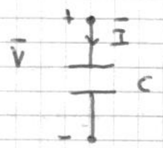


$\bar V = j\omega L \cdot \bar I$

$= jX_L \cdot \bar I$

$$\begin{cases} P_L = \emptyset \text{ Watt} \\ Q_L = \frac{1}{2} X_L I_M^2 = \frac{1}{2}\frac{V_M^2}{X_L} \end{cases}$$

$$= X_L I^2 = \frac{V^2}{X_L} \text{ VAR}$$



$$\bar P = \frac{1}{2}\bar V \cdot \bar I^* = \frac{1}{2}\bar V (j\omega C \bar V)^* = \frac{1}{2}\bar V \omega C (-j)\bar V^*$$

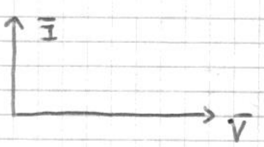
$$= \emptyset - j \cdot \frac{1}{2}\omega C V_M^2$$

$\bar I = j\omega C \bar V$

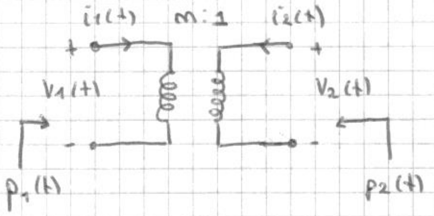
$= j\frac{\bar V}{X_c}$

$$\begin{cases} P_c = \emptyset \text{ Watt} \\ Q_c = -\frac{1}{2}\omega C \cdot V_M^2 = -\frac{1}{2}\frac{V_M^2}{X_c} = -\frac{1}{2} X_c I_M^2 \end{cases}$$

$$= -\frac{V^2}{X_c} = -X_c I^2 \text{ VAR}$$

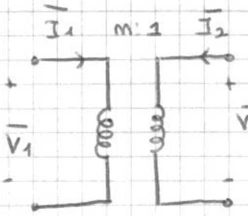


Trasformatore Ideale



$$\begin{cases} V_1(t) = m\,V_2(t) \\ i_1(t) = -\dfrac{1}{m}\, i_2(t) \end{cases}$$

$p_1(t) + p_2(t) =$
$= V_1(t)\, i_1(t) + V_2\, i_2(t)$
$= \phi$



$$\begin{cases} \bar{V}_1 = m\bar{V}_2 \\ \bar{I}_1 = -\dfrac{1}{m}\,\bar{I}_2 \end{cases}$$

$$\bar{P} = \bar{P}_1 + \bar{P}_2 = \frac{1}{2}\bar{V}_1 \bar{I}_1^{\,*} + \frac{1}{2}\bar{V}_2 \bar{I}_2^{\,*} = \frac{1}{2} m \bar{V}_2 \left(-\frac{1}{m}\right)\bar{I}_2^{\,*} + \frac{1}{2}\bar{V}_2\bar{I}_2^{\,*} = \phi$$

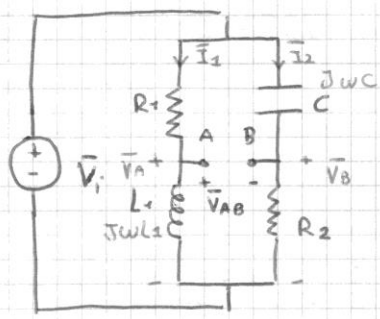
$$\bar{P} = (P_1 + P_2) + j(Q_1 + Q_2) = \phi .$$

$$\begin{cases} P_1 + P_2 = \phi & \text{è Trasparente a Potenza attiva} \\ Q_1 + Q_2 = \phi & \text{e Reattiva.} \end{cases}$$

Giratore Ideale (Per Casa)

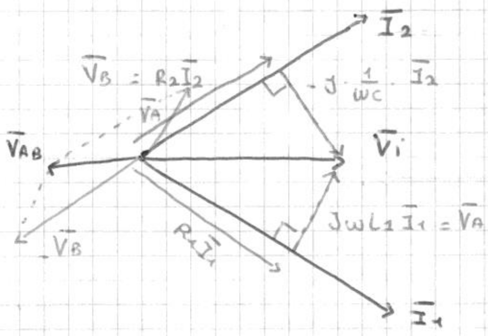
Trasparente a P ma non a Q.

Esercizio



Verificare con metodo grafico dei Fasori che il ponte è bilanciato ($V_{AB} = \phi$) se e solo se: i rami in parallelo hanno stessa costante di tempo:

$$\tau_{LR} = \frac{L_1}{R_1} = R_2 C = \tau_{RC} .$$

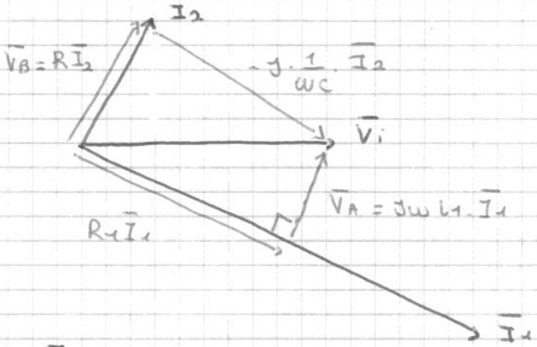


$$\bar{V}_{AB} = \bar{V}_A - \bar{V}_B$$
$$= j\omega L_1 \cdot \bar{I}_1 - R_2 \bar{I}_2$$

R_1, L_1, C fissi. Vario R_2

Bisogna che $\overline{V_A} \parallel \overline{V_B}$.

allora ho il valore di R_2



$\overline{V_A} = \overline{V_B}$

$\overline{V_A} - \overline{V_B} = \overline{V_{AB}} = \emptyset$

Ponte Bilanciato

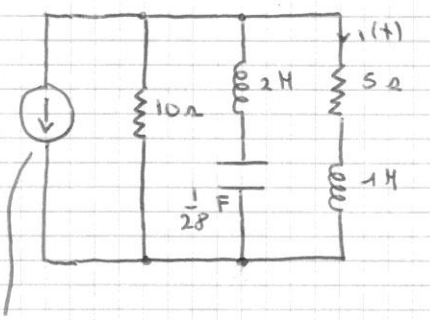
$$\begin{cases} R_2 I_{2M} = \omega L_1 I_{1M} \\ \frac{1}{\omega C} I_{2M} = R_1 I_{1M} \end{cases}$$

$$\frac{I_{2M}}{I_{1M}} = \frac{\cancel{\omega} L_1}{R_2} = R_1 \cancel{\omega} C \quad \rightarrow \quad \frac{L_1}{R_1} = C R_2$$

$\leftarrow$ Le costanti di Tempo sono uguali

Ho dimostrato!

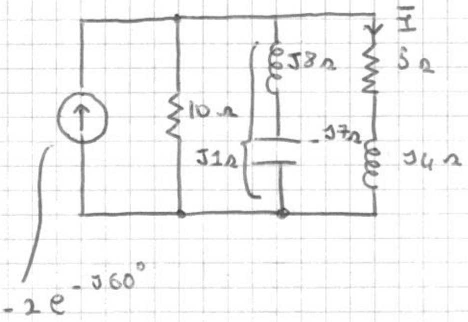
Esercizio

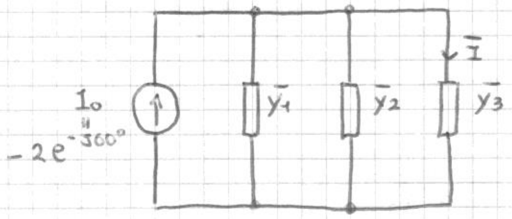


2 cos(4t - 60°) Amp.

? i(t) in regime sinusoidale

$\rightarrow$ metodo Fasori.





$$\bar{Y}_1 = \frac{1}{10}\,\Omega^{-1}$$

$$\bar{Y}_2 = -j\,\Omega^{-1}$$

$$\bar{Y}_3 = \frac{1}{5+j4}\,\Omega^{-1}$$

$$\bar{I} = \bar{I}_0 \cdot \frac{\bar{Y}_3}{\bar{Y}_1 + \bar{Y}_2 + \bar{Y}_3} = -2e^{-j60^\circ} \cdot \frac{\frac{1}{5+j4}}{\frac{1}{10} - j + \frac{1}{5+j4}}$$

$$= -2e^{-j60^\circ} \cdot \frac{1}{1 + \frac{5+j4}{10} - j(5+j4)} = -20e^{-j60^\circ} \cdot \frac{1}{55 - j46} \rightarrow \text{arctg}\left(-\frac{46}{55}\right)$$

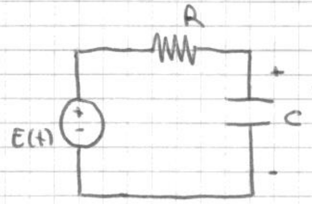
$$= \frac{20}{\sqrt{55^2+46^2}}\; e^{j180^\circ}\; e^{-j60^\circ}\; e^{j\,\text{arctg}\left(\frac{46}{55}\right) \simeq 40^\circ}$$

$$= \underbrace{0{,}279}_{I_M} \cdot e^{\overbrace{160^\circ}^{\varphi_i}} \text{ Amp.}$$

$$i(t) = 0{,}279 \cos(4t + 160^\circ) \text{ Amp.}$$

29/04/2015

Risposta Completa



$v(\phi) = V_0 \neq \phi$

$\frac{1}{2} c V_0^2$ Joule.

C = L + F ← (Principio di Sovrapposizione degli Effetti).

Completa, Libera, Forzata

DIMOSTRAZIONE:

$$\begin{cases} \tau \frac{d}{dt} V_{COM} + V_{COM} = E(t) \\ V_{COM}(\phi) = \phi \end{cases}$$

$$\begin{cases} \tau \cdot \frac{d}{dt} (V_{LIB} + V_{FOR}) + (V_{LIB} + V_{FOR}) = E(t) \\ (V_{LIB} + V_{FOR})(\phi) = V_0 \end{cases}$$

$$\begin{cases} \tau \cdot \frac{d}{dt} V_{LIB} + V_{LIB} = \phi \\ V_{LIB}(\phi) = V_0 \end{cases}$$

$$\begin{cases} \tau \cdot \frac{d}{dt} V_{FOR} + V_{FOR} = E(t) \\ V_{FOR}(\phi) = \phi \end{cases}$$

$E(t) = E =$ costante

C = L + F =

$$V_{COM}(t) = V_0 \cdot e^{-\frac{t}{\tau}} + E(1 - e^{-\frac{t}{\tau}})$$

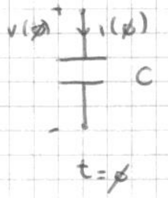
Scomposizione in Parti Significative

C = L + F = L + T + Regime

L + T = Transitorio Completo

C = TC + Regime.

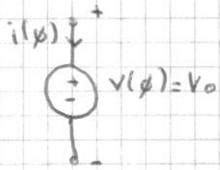
Comportamento a $t = \emptyset$ degli elementi reattivi L e C.



$V(\emptyset) = V_0$ conosco la variabile di stato.

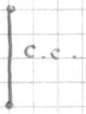
$\forall i(\emptyset)$.

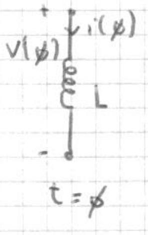
<=>



$V(\emptyset) = \emptyset$

$\forall i(\emptyset)$ <=>

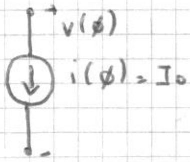




$i(\emptyset) = I_0$ conosco la variabile di stato

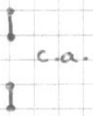
$\forall v(\emptyset)$

<=>



$i(\emptyset) = \emptyset$

$\forall v(\emptyset)$ <=>

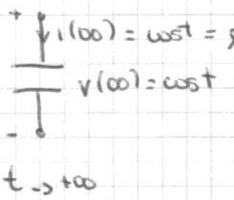


Comportamento per $t \to +\infty$

1) Circuito in Regime stazionario per $t \to +\infty$.

Generatori di V e i costanti.

Tutte le v e le i del circuito sono costanti per $t \to +\infty$.



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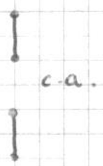

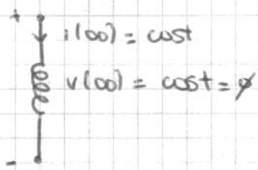

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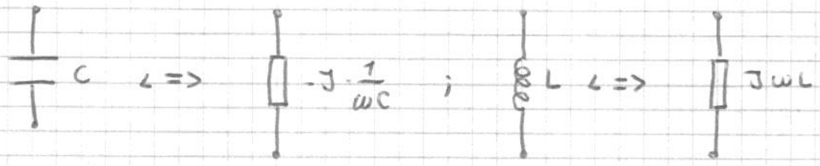
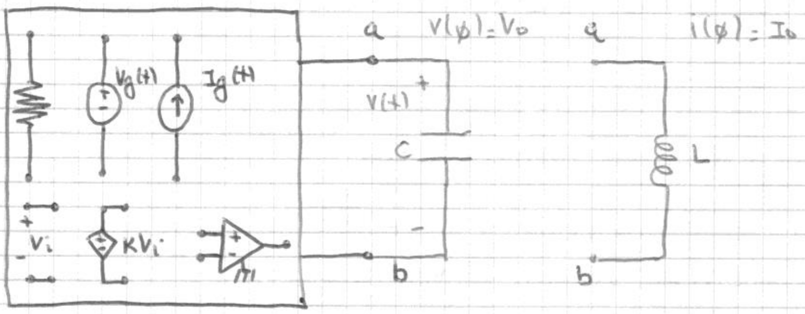
2) Circuito in Regime Sinusoidale per $t \to +\infty$

Generatori di V e i sinusoidali

Tutte le V e le i sono sinusoidali per $t \to +\infty$.

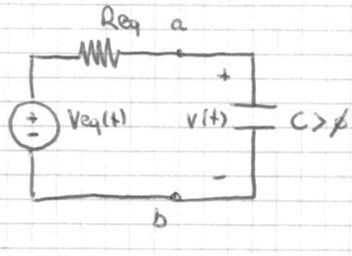


Circuiti del 1° Ordine



Rete senza memoria non omog.

Bipolo ab contr. in corrente



$v(0) = V_0$, $C, R_{eq} > 0$.

$$\begin{cases} \tau_{eq} \cdot \dfrac{d}{dt} \cdot v + v = V_{eq}(t) \quad , t \geq 0 \\ v(0) = V_0 \end{cases}$$

$\tau_{eq} = C \cdot R_{eq}$

C: $V_0 \neq 0$, $V_{eq}(t) \neq 0$

F: $V_0 = 0$, $V_{eq}(t) \neq 0$

L: $V_0 \neq 0$; $V_{eq}(t) = 0$

Eccitazioni interne ad esponenziali costanti

$V_g(t)$, $I_g(t)$ costanti (in continua)

$$V_{eq}(t) = V_{c.a}(t) = V_{eq} = \text{costante}$$

$$\begin{cases} \tau_{eq} \cdot \frac{d}{dt} v + v = V_{eq} = \text{costante} \\ V(\emptyset) = V_0 \end{cases}$$

$$C : v(t) = V_0 e^{-\frac{t}{\tau_{eq}}} + V_{eq} \cdot (1 - e^{-\frac{t}{\tau_{eq}}}) , \; t \geq \emptyset$$

$$F : v(t) = V_{eq} (1 - e^{-\frac{t}{\tau_{eq}}}) , \; t \geq \emptyset$$

$$L : v(t) = V_0 \cdot e^{-\frac{t}{\tau_{eq}}} , \; t \geq \emptyset$$

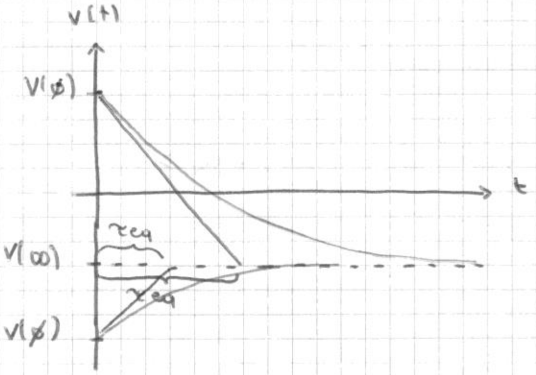
$V_0 = V(\emptyset)$

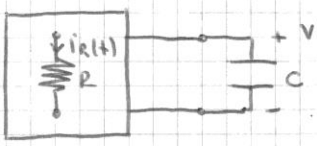
$V_{eq} = V(\infty)$ Regime Stazionario

$$v(t) = V(\emptyset) \cdot e^{-\frac{t}{\tau_{eq}}} + V(\infty) \cdot (1 - e^{-\frac{t}{\tau_{eq}}}), \; t \geq \emptyset.$$

$$= V(\infty) + [V(\emptyset) - V(\infty)] \cdot e^{-\frac{t}{\tau_{eq}}} , \; t \geq \emptyset$$

Per circuiti del primo ordine con eccitazioni costanti.

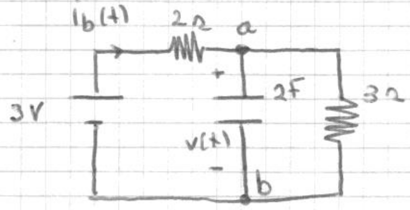




$$i_R(t) = i_R(\emptyset) e^{-\frac{t}{\tau_{eq}}} + i_R(\infty) \cdot (1 - e^{-\frac{t}{\tau_{eq}}})$$

τ_{eq} è sempre la stessa!

Esercizio:



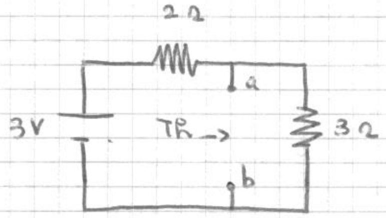
$v(0) = 4V$

? $i_b(t)$, $t \geq 0$, Risp. Completa.

$i_b(t)$ no var. di stato

1. Trovo v(t) (var. di stato) e poi $i_b(t)$:

$$i_b(t) = \frac{3 - v(t)}{2}.$$



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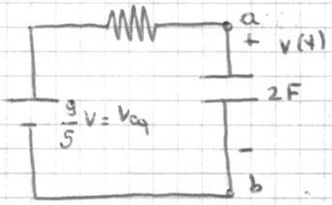


$$\tau_{eq} = C \cdot R_{eq} = \frac{12}{5} \text{ sec}$$

$$V_{com}(t) = 4 \cdot e^{-\frac{5}{12}t} + \frac{9}{5}\left(1 - e^{-\frac{5}{12}t}\right), \quad t \geq 0$$

$$i_b(t) = \frac{3 - V(t)}{2} = \dots = \frac{3}{5} - \frac{11}{10} \cdot e^{-\frac{5}{12}t}.$$

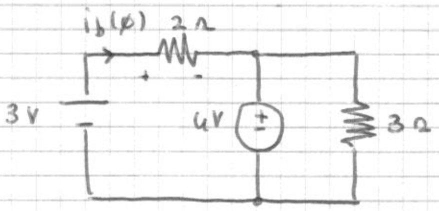
2. Trovo direttamente $i_b(t)$.

$$i_b(t) = i_b(0) \cdot e^{-\frac{t}{\tau_{eq}}} + i_b(\infty)\left(1 - e^{-\frac{t}{\tau_{eq}}}\right).$$

istantaneo a $t = 0$

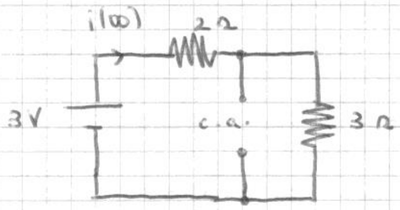
Regime stazionario $t \to \infty$

$t = 0$



$$i_b(0) = -\frac{1}{2} A$$

$t \to \infty$

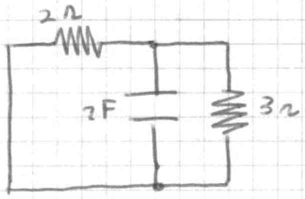


$i(\infty) = \frac{3}{5}$ A.

Determinazione di τ_{eq}.

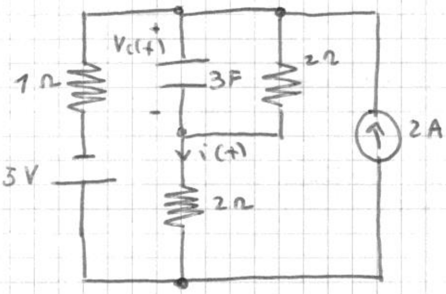
$K \cdot e^{-\frac{t}{\tau_{eq}}}$ → sol. omogenea

Evoluzione Libera → Generatori Spenti



$\tau = C \cdot R_{eq} = 2 \cdot \frac{6}{5} = \frac{12}{5}$ sec.

Esercizio Per casa:



$V_c(\emptyset) = 2V$

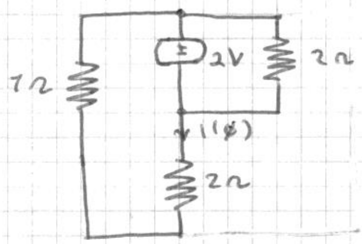
$?\, i(t),\ t \geq \emptyset$

L, F, T, Regime Stazionario

Circuito 1° Ordine con eccitazioni costanti

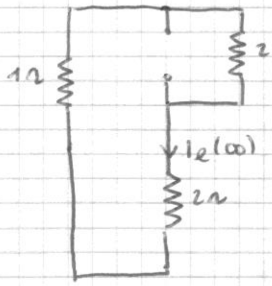
L:

$i_L(t) = i_L(\emptyset) \cdot e^{-\frac{t}{\tau_{eq}}} + i_L(\infty)\left(1 - e^{-\frac{t}{\tau_{eq}}}\right)$



$i(\emptyset) = -\frac{2}{3}$ A

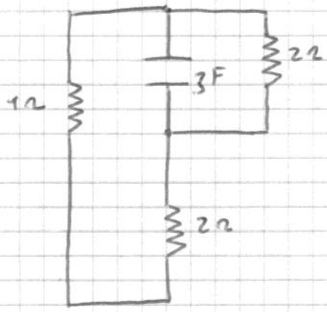
$t = \emptyset$



$I_e(\infty) = \emptyset A.$

$t \to \infty$

τ_{eq} :



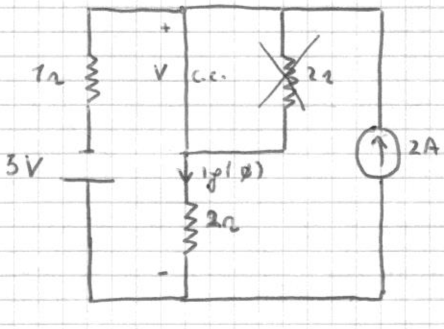
$\tau_{eq} = C \cdot R_{eq} = \frac{18}{5}$ sec.

$I_e(t) = -\frac{2}{3} \cdot e^{-\frac{5}{18}t}$, $t \geq \emptyset$.

F:

$i_f(t) = I_f(\emptyset) \cdot e^{-\frac{t}{\tau_{eq}}} + I_f(\infty)\left(1 - e^{-\frac{t}{\tau_{eq}}}\right).$

$t = \emptyset$

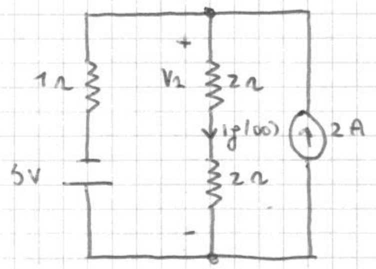


MILLMANN

$V = \frac{-5+2}{1+\frac{1}{2}} = -2V$

$i_f(\emptyset) = \frac{V}{2} = -1 A.$

$t \to \infty$



MILLMANN :

$$V_1 = \frac{-5+2}{1+\frac{1}{4}} = -\frac{12}{5}\ V$$

$$I_g(\infty) = \frac{V_1}{4} = -\frac{3}{5}\ A.$$

$$I_g(t) = -e^{-5/18\, t} - \frac{3}{5}\cdot\left(1 - e^{-5/18\, t}\right).$$

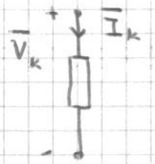
$F = T + R$

$$I_g(t) = -\frac{3}{5} - \frac{2}{5}\cdot e^{-5/18\, t}$$

$$R = -\frac{3}{5}\quad ,\quad T = -\frac{2}{5}\cdot e^{-5/18\, t}.$$

Teorema di Boucherot

Rete di ℓ bipoli in regime sinusoidale a pulsazione ω.



Per una rete di ℓ bipoli in regime sinusoidale a pulsazione ω, è nulla la somma (vettoriale) delle potenze complesse assorbite dai Bipoli:

$$\sum_{k=1}^{\ell} \frac{1}{2}\,\overline{V}_k\,\overline{I}_k^{*} = \sum_{k=1}^{\ell} \overline{P}_k = \emptyset$$

$$\sum_{k=1}^{\ell} (P_k + JQ_k) = \sum_{k=1}^{\ell} P_k + J\sum_{k=1}^{\ell} Q_k = \emptyset$$

$$\begin{cases} \sum_{k=1}^{\ell} P_k = \emptyset \text{ Watt.} \\ \sum_{k=1}^{\ell} Q_k = \emptyset \text{ VAR.} \end{cases}$$

Vale il principio di conservazione per potenza complessa attiva e reattiva.

Rete R, L, C e generatori indipendenti di tensione e corrente

$$\sum_{generatori} P_{gi} = \sum_{resistori} P_{Ri} = \sum_{resistori} R_i I_i^2 \text{ Watt.}$$

Potenza attiva erogata da gen. re i-esimo.

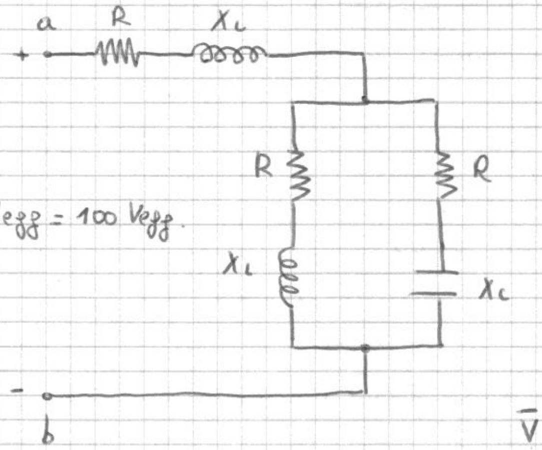
$$\sum_{generatori} Q_{gi} = \sum_{induttori} Q_{Li} + \sum_{condensatori} Q_{ci}$$

Potenza reattiva erogata da gen. re i-esimo

$$= \sum_{induttori} \omega L_i I_i^2 + \sum_{condensatori} (-\omega C_i V_i^2)$$

$$= 2\omega \left[\sum_{induttori} E_{Li}^{media} - \sum_{condensatori} E_{ci}^{media} \right] \text{ VAR}$$

Applicazione Teorema di Boucherot

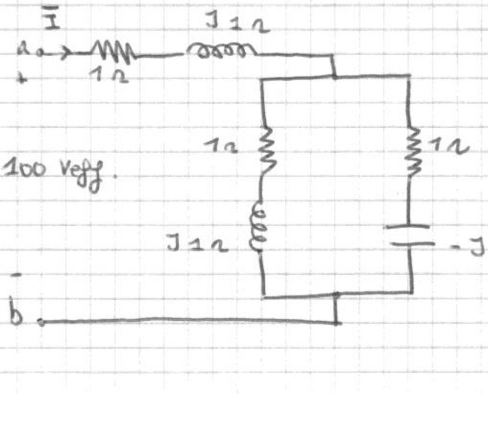


$V_{eff} = 100 \ V_{eff}$.

$R = X_L = X_C = 1 \ \Omega$

? P, Q assorbita da ab e verificare T. di Boucherot.

$\bar{V} = V_M e^{j\varphi_v}$ Volt



$\bar{V} = 100 \ V_{eff}$.

$\bar{V} = V_{eff} \ e^{j\varphi_v} \ V_{eff}$.

$\bar{I} = I_{eff} \ e^{j\varphi_i} \ A_{eff}$.

La potenza assorbita è indipendente dalla Fase di $\bar{V}$.

$$\bar{P} = \bar{V}\,\bar{I}^*$$

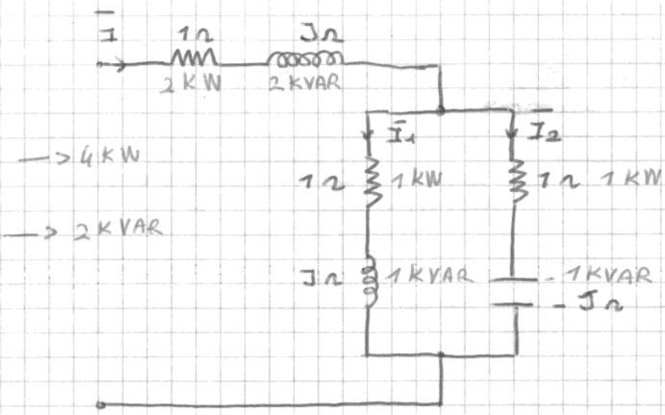
$$\bar{Z} = 1+j + \frac{1}{\frac{1}{1+j} + \frac{1}{1-j}} = 1+j + \frac{(1+j)(1-j)}{1+j+1-j} = 1+j+\frac{2}{2} = 2+j\ \Omega$$

$$\bar{I} = \frac{\bar{V}}{\bar{Z}} = \frac{100}{2+j} = \frac{100}{5}(2-j) = 40 - j20\ A_{eff}. \qquad I^2 = 2000\ A_{eff}^2$$

$$\bar{P} = 100 \cdot (40+j20) = 4000 + j2000\ VA.$$

$P = 4\ KW.$

$Q = 2\ KVAR.$



$$P_R = R \cdot I^2_{eff} =$$

$$\bar{I}_1 = \bar{I} \cdot \frac{\frac{1}{1+j}}{\frac{1}{1+j} + \frac{1}{1-j}} = (40-j20) \cdot \frac{1-j}{1+j+1-j}$$

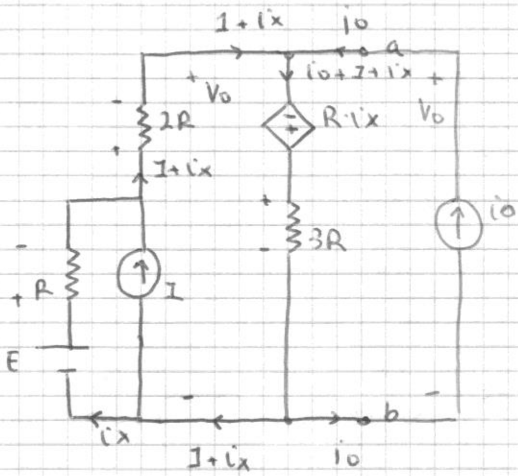
$$= (40-j20) \cdot \frac{1-j}{2} = (20-j10)(1-j) = 10 - j30\ A_{eff}$$

$$\bar{I}_2 = \bar{I} - \bar{I}_1 = 30 + j10\ A_{eff}.$$

$$I_1^2 = I_2^2 = 1000\ A_{eff}^2$$

Esercizi Parte 1.

? TR con metodo rapido



$$V_0 = -2R(i_x + I) - R i_x + E \quad = -3R i_x - 2RI + E$$

$$i_x = \frac{E - 2RI - V_0}{3R}$$

$$V_0 = 3R(i_0 + I + i_x) - R \cdot i_x$$

$$= 3R\left(i_0 + I + \frac{E - 2RI - V_0}{3\cancel{R}}\right) - \cancel{R} \cdot \frac{E - 2RI - V_0}{3\cancel{R}}$$

$$= 3Ri_0 + 3RI + E - 2RI - V_0 - \frac{E}{3} + \frac{2}{3}RI + \frac{V_0}{3}$$

$$= 3Ri_0 + \frac{5}{3}RI + \frac{2}{3}E - \frac{2}{3}V_0$$

$$\frac{5}{3}V_0 = 3Ri_0 + \frac{5}{3}RI + \frac{2}{3}E$$

$$V_0 = \frac{9}{5}Ri_0 + RI + \frac{2}{5}E$$

$$R_{eq} = \frac{9}{5}R \quad ; \quad V_{eq} = RI + \frac{2}{5}E$$

06/05/2015

C ideale oppure L ideale

$i_c(t)$ limitata in $[t_a, t_b]$

$\rightarrow V_c(t)$ è continua in (t_a, t_b)

$V_L(t)$ limitata in $[t_a, t_b]$

$\rightarrow i_L(t)$ è continua in (t_a, t_b)

DIM:

$t_0 \in (t_a, t_b)$

$V_c(t_0^+) \neq V_c(t_0^-)$

$$|V_c(t_0+\varepsilon) - V_c(t_0-\varepsilon)| = \left|\frac{1}{C}\int_{t_0-\varepsilon}^{t_0+\varepsilon} i_c(t)\,dt\right| \leq \frac{1}{C}\int_{t_0-\varepsilon}^{t_0+\varepsilon} |i_c(t)|\,dt \leq M < +\infty$$

$$\frac{1}{C}\int_{t_0-\varepsilon}^{t_0+\varepsilon} |i_c(t)|\,dt \leq \frac{1}{C}\cdot M\cdot 2\varepsilon$$

$$\lim_{\varepsilon\to\emptyset} \frac{1}{C} M\, 2\varepsilon = \emptyset \qquad \text{ovvero } V_c(t_0^+) = V_c(t_0^-)$$

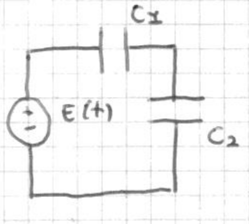
Assurdo!

Quindi se $i_c(t)$ è limitata $\rightarrow V_c(t)$ è continua!

Evento Critico: Chiusura o Apertura di un interruttore oppure discontinuità della V o I impressa da un generatore indipendente.

Istante Critico: Istante in cui avviene l'evento critico. $(t = \emptyset)$

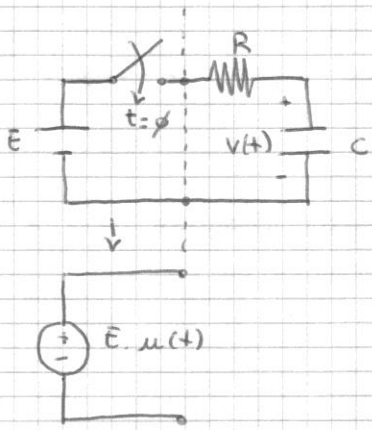
Rete Degenere RLC: Rete che contiene una maglia di soli C e/o generatori indipendenti di V, oppure, per dualità, un taglio di soli L e/o generatori indipendenti di I.



Continuità delle Variabili di Stato:

in una rete RLC non degenere le variabili di stato (V_{Ci}, i_{Li}) sono continue in presenza di eventi critici.

... Deriva dal fatto che i_{Ci} e V_{Li} sono limitate in corrispondenza degli istanti critici.

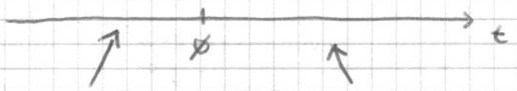


C scarico per $t < 0$.

? $v(t)$, $t > 0$

$$\begin{cases} R \cdot C \cdot \frac{d}{dt} v + v = E & , t > 0 \\ v(0^+) = V(0^-) = 0 \end{cases}$$

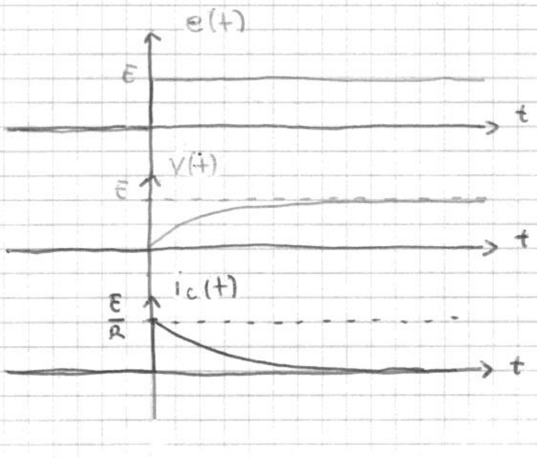
↑ continuità var. di stato



Conosco lo stato per $t < 0$
$V(0^-) = 0$.

Per risolvere equazione differenziale ho bisogno dello stato a $t = 0^+$.

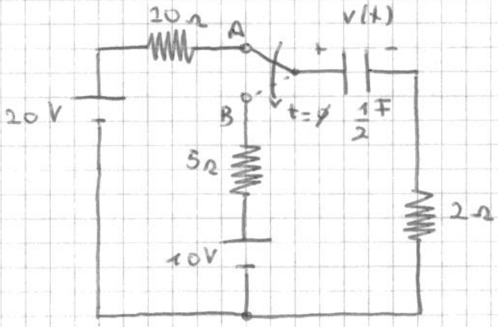
$V(t) = E(1 - e^{-\frac{t}{\tau}})$, $t \geq 0$.



$i_c(t) = \frac{E}{R} e^{-\frac{t}{\tau}}$, $t > 0$

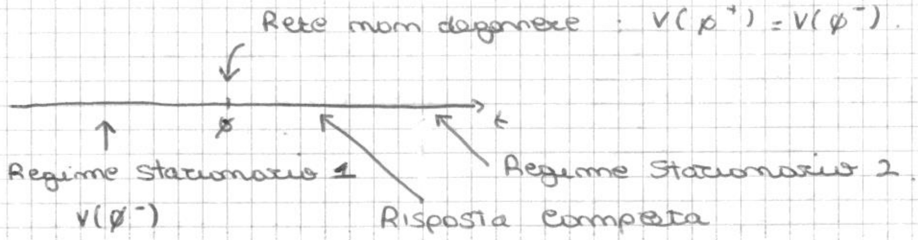
$i_c(t)$ non è var. di stato

-> è discontinua

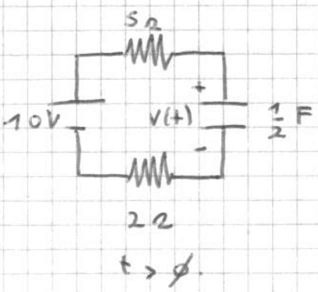


? $v(t)$, $t > \varnothing$

il commutatore, da lungo tempo nella posizione A, viene portato a $t = \varnothing$, nella posizione B.



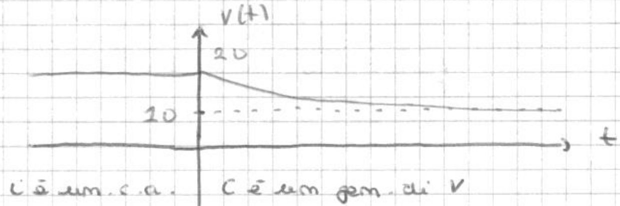
Reg. Stat. 1 , $V(\varnothing^-) = 20\,V = V(\varnothing^+)$

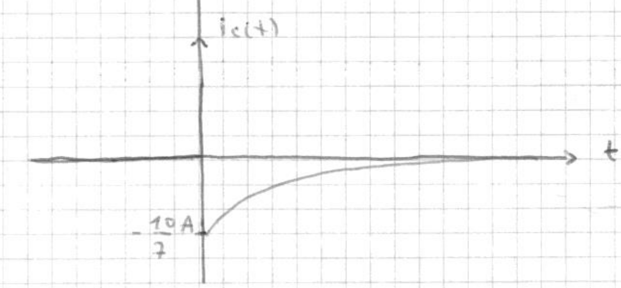


$$V_{COM}(t) = V_{LIB}(t) + V_{FOR}(t)$$

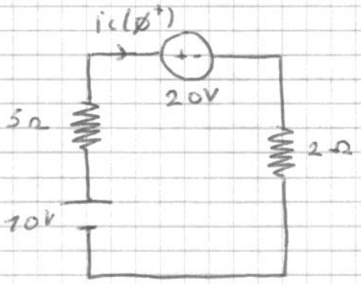
$$\tau = RC = \frac{7}{2} \text{ sec}$$

$$V_{COM}(t) = V(\varnothing^+)\, e^{-\frac{t}{\tau}} + 10\left(1 - e^{-\frac{t}{\tau}}\right)$$
$$= 20\, e^{-\frac{2}{7}t} + 10\left(1 - e^{-\frac{2}{7}t}\right), \quad t \geq \varnothing.$$



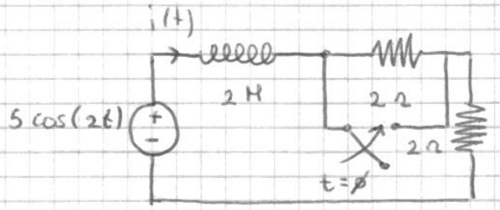


$t = 0$



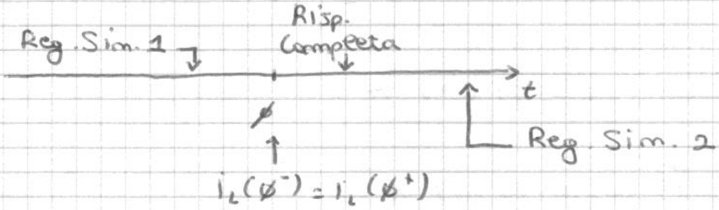
$i_L(0^+) = -\frac{10}{7}\,A$.

17/02/1997

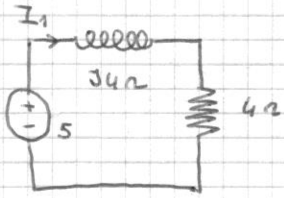


L'interruttore aperto da lungo tempo si chiude a $t = 0$

? $i(t)$, $t > 0$.



Reg. Sim 1, $t = 0^-$



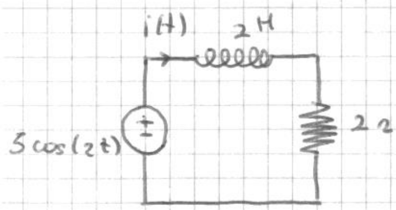
$$\bar{I}_1 = \frac{5}{4+J4} = \frac{5}{4}\cdot\frac{1}{1+J} = \frac{5}{4}\cdot\frac{1-J}{2}$$

$$= \frac{5}{8} - J\frac{5}{8}$$

$$i_1(t) = \mathrm{Re}\left[\bar{I}_1 e^{J2t}\right] \rightarrow i_1(0) = \mathrm{Re}\left[\bar{I}_1\right]$$

$$i_L(0^-) = \frac{5}{8}\,A = i_L(0^+).$$

$t > \emptyset$, evoluzione continua per $t > \emptyset$.



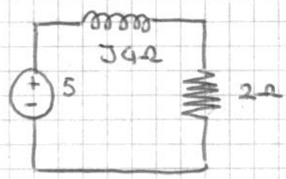
$$C = L + F$$
$$= L + T + \text{Regime Sin. } 2$$
$$= TC + \text{Regime Sin. } 2$$

$$i(t) = K \cdot e^{-\frac{t}{\tau}} + I_{M2} \cos(2t + \varphi_{I2})$$

$$\tau = \frac{L}{R} = 1 \text{ sec}$$

K si calcola come ultima cosa!

Reg. Sin. 2



$$\bar{I}_2 = \frac{5}{2 + j4} = \frac{5}{2} \cdot \frac{1}{1 + j2} = \frac{5}{2} \cdot \frac{e^{-j\,\mathrm{arctg}\, 2}}{\sqrt{5}}$$
$$= e^{-j63,4°} \cdot \frac{\sqrt{5}}{2}$$

$$i(t) = K \cdot e^{-t} + \frac{\sqrt{5}}{2} \cos(2t - 63,4°) \text{ A}$$

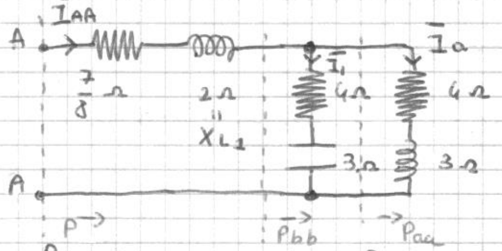
$$i(\emptyset^+) = \frac{5}{8} \text{ A} = K + \frac{\sqrt{5}}{2} \cos(-63,4°) \text{ A}$$

$$\rightarrow K = 0,124$$

$$i(t) = 0,124 \cdot e^{-t} + \frac{\sqrt{5}}{2} \cos(2t - 63,4°) \text{ A}, \quad t \geq \emptyset$$

Il transitorio permette di Raccordare le Variabili di Stato!

Esercizio



$I_a = 20\ A_{eff}$

? P_{AA}, Q_{AA}, V_{AA}, I_{AA} usando T. di Boucherot.

Bipolo in regime Sin. a pulsazione ω

$P_{aa} = 4 I_a^2 = 1600\ W$

$Q_{aa} = 3 I_a^2 = 1200\ VAR$

$A_{aa} = \sqrt{P_{aa}^2 + Q_{aa}^2} = 2000\ VA = V_a I_a \rightarrow V_a = 100\ V_{eff}$

$I' = \frac{V_{aa}}{Z} = \frac{100}{\sqrt{3^2+4^2}} = \frac{100}{5} = 20\ A_{eff}.$

$P_{bb} = 1600 + 4 \cdot 20^2 = 3200\ W$

$Q_{bb} = 1200 - (3 \cdot 20^2) = \emptyset\ VAR$

$A_{bb} = 3200\ VA = V_{aa} \cdot I_{AA} \rightarrow I_{AA} = 32\ A_{eff}$

$\bar{I}_{AA} = \bar{I}' + \bar{I}_a$ $\leftarrow$ N.B.

$I_{AA} \neq I' + I_a$

Non si sommano i Valori Efficaci!

$P_{AA} = 3200 + \frac{7}{8} \cdot I_{AA}^2 = 4096\ W \simeq 4100\ W$

$Q_{AA} = \emptyset + 2 \cdot I_{AA}^2 = 2050\ VAR$

$A_{AA} = \sqrt{P_{AA}^2 + Q_{AA}^2} = 4580\ VA = V_{AA} \cdot I_{AA} \rightarrow V_{AA} = 143\ V_{eff}$

$P = V \cdot I \cos\varphi_z = V \cdot I \cdot \cos(\varphi_V - \varphi_z)$ Watt

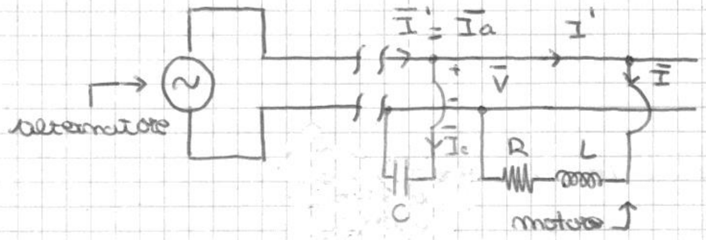
$Q = V \cdot I \, \text{sen}\varphi_z = V \cdot I \cdot \text{sen}(\varphi_V - \varphi_z)$ VAR

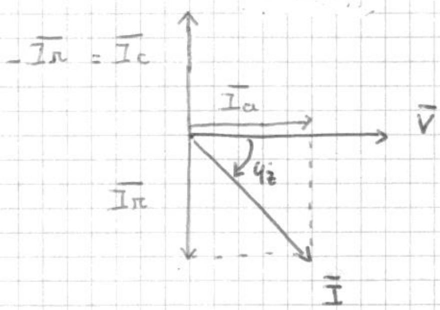
$\cos\varphi_z = \cos(\varphi_V - \varphi_z)$: Fattore di Potenza (Attiva)

$\cos\varphi_z = 1 \rightarrow P_{MAX}, Q_{MIN}$

$\cos\varphi_z = \emptyset \rightarrow P_{MIN}, Q_{MAX}$

RIFASAMENTO:





Solo $\bar{I}_a$ trasporta lavoro!

$\rightarrow \bar{I}_a = \bar{I}'$ dopo il rifasamento

La linea ora trasporta solo Potenza Attiva

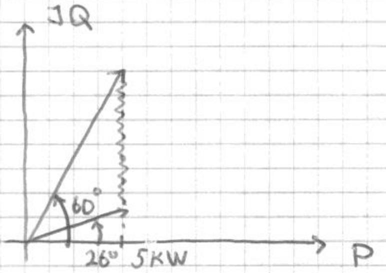
La Potenza Scambiata fa crescere la corrente di linea la quale non trasporta lavoro!
La elimino col rifasamento.

Motore assorbe 5 kWatt con fattore di potenza 0,5 induttivo.
Effettuare un rifasamento a $\cos\varphi_{rif} = 0,9$ (induttivo);

$\omega = 314$ rad/sec ; $V = 160$ Veff.

$\cos\varphi_m = 0,5 \rightarrow \varphi_m = 60°$

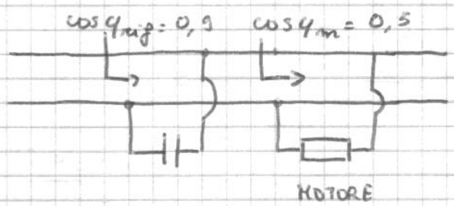
$\cos\varphi_{rif} = 0,9 \rightarrow \varphi_{rif} \simeq 26°$



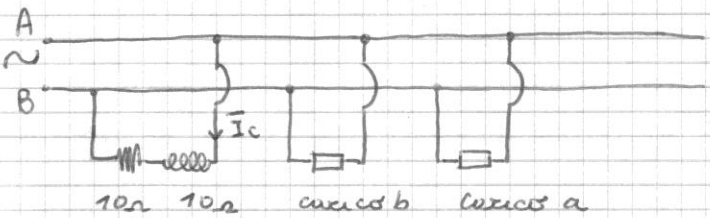
$|Q_C| = P\,tg\,\varphi_m - P\,tg\,\varphi_{rif}$

$\omega C V^2 = P(tg\,\varphi_m - tg\,\varphi_{rif})$

$C = \frac{P}{\omega V^2}(tg\,\varphi_m - tg\,\varphi_{rif}) = \frac{5000(tg\,60° - tg\,26°)}{314 \cdot 160^2} = 773\ \mu F.$



Esercizio:



$\sim$: 100 V_{eff}

$f = 50\ Hz$

carico a : $A_a = 1000\ VA$

$\cos\varphi_a = 0,8$ (induttivo)

carico b : corrente attiva : 5 A_{eff}

corrente reattiva induttiva : 5 A_{eff}.

Rifasamento a $\cos\varphi_{rif} = 0,9$ (induttivo)

a) $P_a = A_a \cos\varphi_a = 800\ W$

$Q_a = A_a \,sen\,\varphi_a = 600\ VAR$

b) $P_b = V_{eff} \cdot I_{att} = 300\ W$

$Q_b = V_{eff} \cdot I_{reatt} = 500\ VAR$

carico RL:

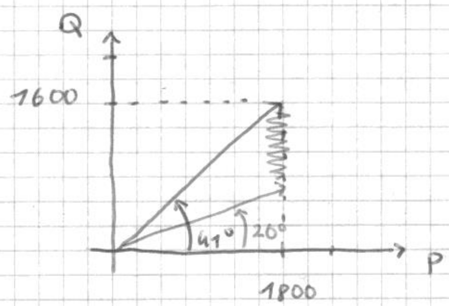
$$I_c = \frac{100}{\sqrt{10^2+10^2}} = 7,07\ A_{eff}$$

$$P_c = 10 \cdot 7,07^2 = 500\ W$$

$$Q_c = 10 \cdot 7,07^2 = 500\ VAR$$

$$P_{AB} = 1800\ W.$$

$$Q_{AB} = 1600\ VAR.$$



$$\omega C V^2 = Q_{AB} - P_{AB}\, tg\, 26° = \frac{1600 - 1800\, tg\, 26°}{314 \cdot 100^2} = 232\ \mu F$$

$$\omega = 2\pi f.$$

? Valutare la diminuzione di corrente assorbita dal bipolo AB in valore efficace, ottenuta col rifasamento.

I_{AB} prima:

$$A_{AB\ prima} = \sqrt{P_{AB}^2 + Q_{AB}^2} = 2400\ VA = V I_{AB\ prima}$$

$$I_{AB\ prima} = 24\ A_{eff}$$

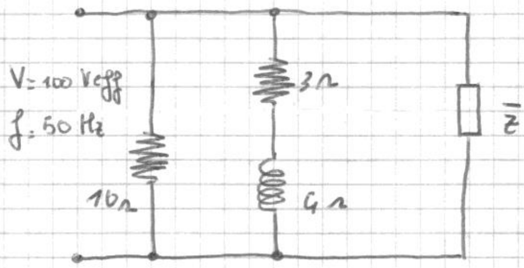
I_{AB} dopo:

$$A_{AB\ dopo} = \sqrt{P_{AB}^2 + (P_{AB}\, tg\, rif)^2} = 2000\ VA$$

$$I_{AB\ dopo} = 20\ A_{eff}.$$

Diminuzione: $24 - 20 = 4\ A_{eff}$.

Esercizio per casa:



$\bar{Z}$: 5 Aeff attiva
10 Aeff reattiva induttiva

1. Rifasamento a $\cos\varphi_{rif} = 0,9$ (induttivo)

   [C = 555 μF]

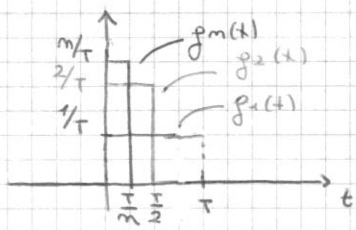
2. Corrente Fasore assorbita dal bipolo.

$$\begin{cases} I_{prima} = 37 - J54 \\ I_{dopo} = 37 - J17,2 \end{cases}$$

PARTE 3

08/05/2015

Richiami su Funzioni Impulsive

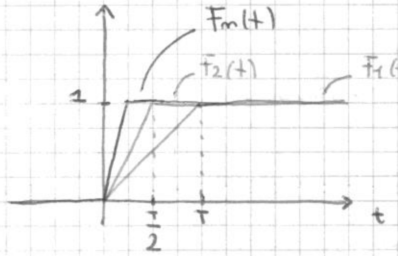


$$\delta(t) = \lim_{n \to \infty} f_n(t) = \begin{cases} \emptyset, & t \neq \emptyset \\ \infty, & t = 0 \end{cases}$$

con area sottesa unitaria

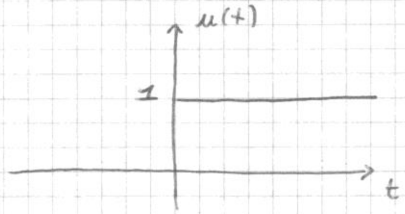
$$\int_{-\infty}^{+\infty} \delta(t)\,dt = \int_{-\varepsilon}^{+\varepsilon} \delta(t)\,dt = 1 \quad , \forall \varepsilon > \emptyset .$$

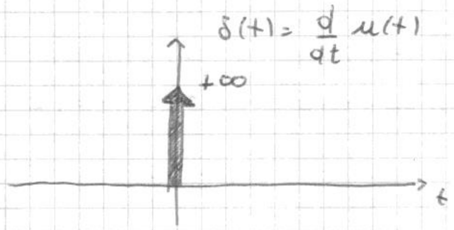
Da non confondere con la $\delta(k)$ in tempo discreto che $\delta(\emptyset) = 1$!



$$f_n(t) = \frac{d}{dt} F_n(t)$$

$$\lim_{n \to \infty} f_n(t) = \delta(t) = \frac{d}{dt}\Big(\underbrace{\lim_{n \to \infty} F_n(t)}_{u(t)}\Big) = \frac{d}{dt} u(t).$$





$\delta(t) = \frac{d}{dt} u(t)$, derivata nel senso delle funzioni impulsive.

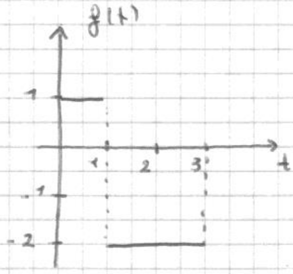
Sia $f(t)$ derivabile nel senso delle funzioni ordinarie, eccetto che in un numero finito di istanti t_i, $i = 1, \dots, n$ dove esistono discontinuità di Prima Specie ($\exists$ finiti $f(t_i^+)$, $f(t_i^-)$).

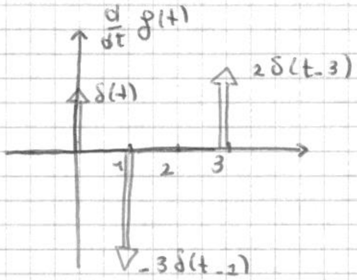
allora:

$$\frac{d}{dt} f(t) = f'(t) + \sum_{i=1}^{n} [\underbrace{f(t_i^+) - f(t_i^-)}_{\sigma_i \,:\, \text{salto di } f \text{ in } t_i}]\, \delta(t - t_i)$$

derivata nel senso delle funzioni impulsive

derivata nel senso delle funzioni ordinarie

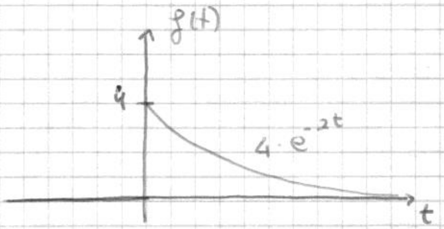




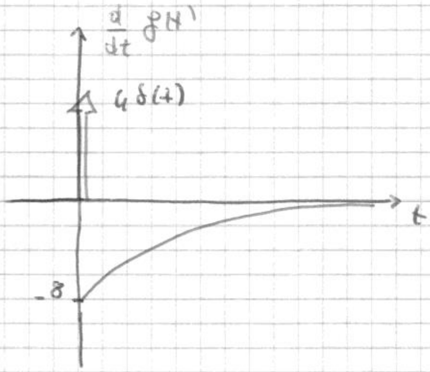
$+3\,\delta(t)$ 

unità di misura di 3 : V·sec oppure A·sec , [3] = V·sec

$f(t) = 4\,e^{-2t} \cdot u(t)$? $\frac{d}{dt} f(t)$



$$\frac{d}{dt} f(t) = \underbrace{-8\, e^{-2t} \cdot u(t)}_{f'(t)} + 4\,\delta(t - 0)$$



Richiami su Teoria di Laplace

$f(t) : \mathbb{R} \to \mathbb{C}$

$L[f] = F(s) \quad , \quad s = \sigma + j\omega$

$L[f] = F(s) = \int_{0}^{+\infty} f(t)\, e^{-st}\, dt$

$F(s) : \mathbb{C} \to \mathbb{C}$

Funzioni Ordinarie Trasformabili secondo Laplace

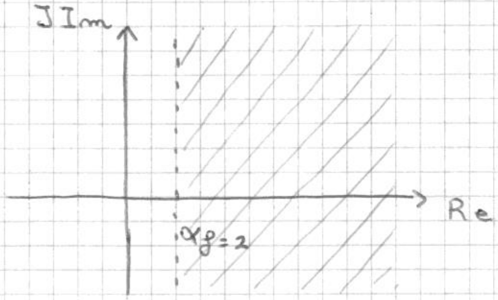
•) f continua a tratti; ha se più discontinuità di prima specie ($\exists$ finiti $f(t^+)$, $f(t^-)$)

••) f di ordine esponenziale

$|f(t)| \leq M \cdot e^{\beta t} \quad , \quad t \geq 0 \qquad M, \beta > 0 .$

$\to \alpha_f \geq 0$ ascissa di convergenza

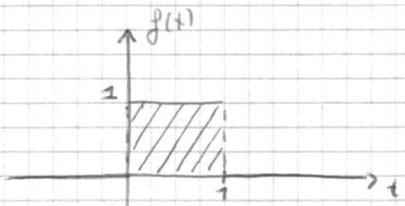
$F(s)$ è ben definita per $Re[s] = \sigma > \alpha_f$.



$e^{2t} \cdot u(t) \to \frac{1}{s-2} \quad , \quad \alpha_f = 2 .$

Trasformate di Laplace di Funzioni Elementari

$f(t)$	$F(s)$	
$u(t)$	$\frac{1}{s}$	($\alpha_u = \emptyset$)
$e^{s_0 t} \cdot u(t)$, $s_0 \in \mathbb{C}$	$\frac{1}{s - s_0}$	
$\text{sen}(\omega_0 t) \cdot u(t)$, $\omega_0 \in \mathbb{R}$	$\frac{\omega_0}{s^2 + \omega_0^2}$	($\alpha = \emptyset$)
$\cos(\omega_0 t)\, u(t)$, $\omega_0 \in \mathbb{R}$	$\frac{s}{s^2 + \omega_0^2}$	($\alpha = \emptyset$)
$e^{-\alpha t} \text{sen}(\beta t) \cdot u(t)$	$\frac{\beta}{(s+\alpha)^2 + \beta^2}$	
$e^{-\alpha t} \cos(\beta t) \cdot u(t)$	$\frac{s+\alpha}{(s+\alpha)^2 + \beta^2}$	
$t^m \cdot u(t)$	$\frac{m!}{s^{m+1}}$	
$t \cdot u(t)$	$\frac{1}{s^2}$	
$e^{-\alpha t} \cdot t^m \cdot u(t)$	$\frac{m!}{(s+\alpha)^{m+1}}$	



$$f(t) = u(t) - u(t-1)$$

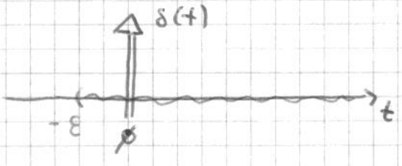
$$F(s) = \frac{1}{s} - \frac{1}{s} \cdot e^{-s}$$

$$= \frac{1 - e^{-s}}{s} \xrightarrow[s = \emptyset]{} 1$$ area sottesa dalla funzione

$$F(s) = \int_{\emptyset}^{+\infty} f(t)\, e^{-st}\, dt$$

Trasformata di Laplace di Funzioni Impulsive

$$\delta(t) = \begin{cases} 0 & t \neq 0 \\ \infty & t = 0 \\ \text{area unitaria} \end{cases}$$



$$L[\delta(t)] = \int_0^{+\infty} \delta(t)\, e^{-st}\, dt$$

$$L_{0^-}[\delta(t)] = \lim_{\varepsilon \to 0^+} \int_{-\varepsilon}^{+\infty} \delta(t)\, e^{-st}\, dt = \left[e^{-st} \right]_{t=0} = 1 .$$

Teorema di derivazione (allo 0^-).

$$L_{0^-}\left[\frac{d}{dt} f(t) \right] = s \cdot L_{0^-}[f(t)] - f(0^-) .$$

Trasformata allo zero meno

Trasformata nel senso delle f. impulsive

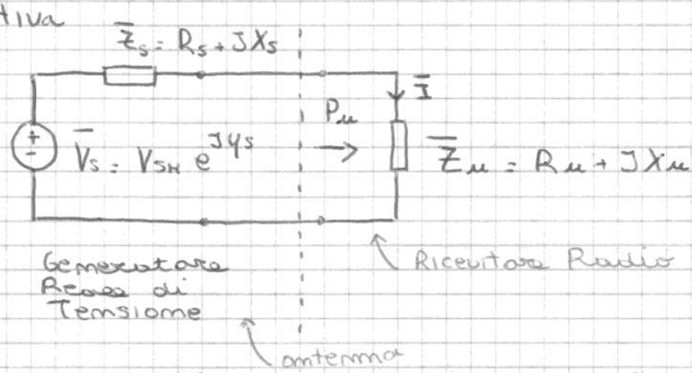
Limite sinistro di f in $t = 0$.

$$\delta(t) = \frac{d}{dt} u(t)$$

$$L_{0^-}[\delta(t)] = s \cdot L_{0^-}[u(t)] - u(0^-) = s \cdot \frac{1}{s} - 0 = 1 .$$

PARTE 2

Teorema del massimo Trasferimento di Potenza Attiva



Il carico che assorbe la massima potenza attiva è:

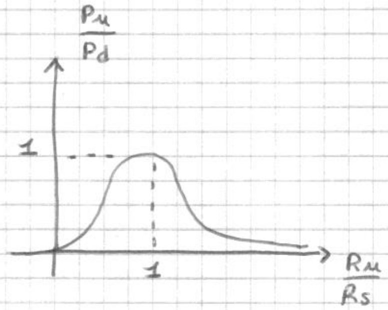
$$\overline{Z}_u = \overline{Z}_s^{*} \quad \rightarrow \quad \begin{cases} R_u = R_s \\ X_u = -X_s \end{cases} \rightarrow$$

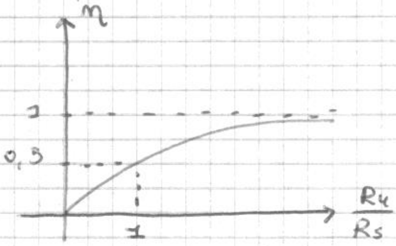
Impedenza adattata per massimo assorbimento di Potenza.

Potenza massima vale:

$$P_{u,MAX} = \frac{1}{8} \cdot \frac{V_{sM}^2}{R_s} = \frac{1}{4} \cdot \frac{V_{s\,eff}^2}{R_s} = P_d$$

Potenza disponibile del generatore.





$X_u = -X_s$

$$\eta = \frac{P_u}{P_u + P_s} = \frac{R_u}{R_u + R_s}$$

Nel caso di una rete elettrica: η MAX $\rightarrow R_u \gg R_s$

$P_{RICEVUTA}$ MIN.

Nel caso di una ricezione di segnali: $P_{RICEVUTA}$ MAX, $\eta = 0,5$.

$$P_u = \frac{1}{2} Re\left[\bar{Z}_u\right] I_M^2 = \frac{1}{2} R_u I_M^2 = \frac{1}{2} \cdot R_u \cdot \frac{V_{SM}^2}{(R_u+R_s)^2 + (X_u+X_s)^2}$$

$$F_u(R_u, X_u) = \frac{(R_u+R_s)^2 + (X_u+X_s)^2}{R_u}$$

$$\frac{\partial}{\partial X_u} F = \frac{2(X_u+X_s)}{R_u} = \emptyset \rightarrow X_u = -X_s$$

$$\frac{\partial}{\partial R_u} F = \left.\frac{2(R_u+R_s)R_u - (R_u+R_s)^2 - \cancel{(X_u+X_s)^2}}{R_u^2}\right|_{X_u=-X_s}$$

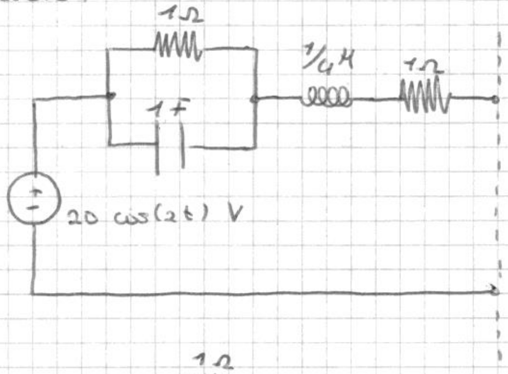
$$= \frac{2R_u^2 + 2R_s\cancel{R_u} - R_u^2 - R_s^2 - 2\cancel{R_u}R_s}{R_u^2} = \frac{R_u^2 - R_s^2}{R_u^2} = \frac{(R_u+R_s)(R_u-R_s)}{R_u^2} = \emptyset$$

$$\begin{cases} X_u = -X_s \\ R_u = R_s \end{cases}$$

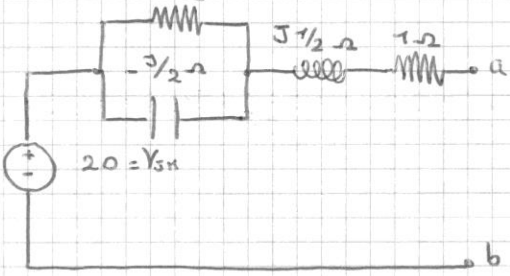
$$P_u = \frac{1}{2} R_u \cdot \frac{V_{SM}^2}{(R_u+R_s)^2 + (X_u+X_s)^2}$$

$$P_{u,MAX} = \frac{1}{2} R_s \cdot \frac{V_{SM}^2}{(2R_s)^2} = \frac{1}{8} \cdot \frac{V_{SM}^2}{R_s}$$

Esercizio:



? Potenza disponibile generatore Reale.



$$\bar{Z}_s = \frac{-\frac{j}{2}}{1 - \frac{j}{2}} + \frac{j}{2} + 1 = \frac{-j}{2-j} + \frac{j}{2} + 1 = -j \cdot \frac{2+j}{5} + \frac{j}{2} + 1$$

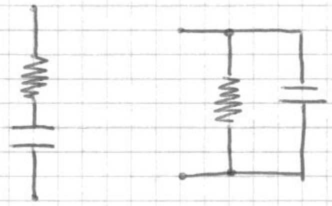
$= \frac{1}{5} + 1 - j \cdot \frac{2}{5} + \frac{j}{2} \ \Omega$

$\overline{Z}_s = \frac{6}{5} + j \cdot \frac{1}{10} \ \Omega$

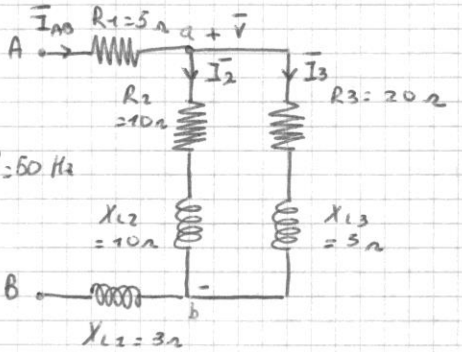
$P_d = \frac{1}{8} \cdot \frac{V_{SH}^2}{R_s} = \frac{1}{8} \cdot \frac{20^2}{\frac{6}{5}} = 41{,}7 \ W$

$\overline{Z}_u = \overline{Z}_s^* = \frac{6}{5} - j \cdot \frac{1}{10} \ \Omega$ (ORmico Capacitivo)

$\overline{Z}_u$:



24/06/2014



$f = 50$ Hz

0,9 induttivo dopo il rifasamento

Diminuzione I_{eff}

$P_{R2} = 10$ kW.

$\begin{cases} P_{AB} = 32635 \ W \\ Q_{AB} = 20290 \ VAR \end{cases}$

$P_{R2} = I_{2eff}^2 \cdot R_2 \Rightarrow \quad I_{2eff} = 31{,}62 \ A_{eff}$

$V_{eff} = 31{,}62 \cdot \sqrt{200} = 447 \ V_{eff}$

$I_{3eff} = \frac{V_{eff}}{\sqrt{425}} = 21{,}68 \ A_{eff}$

$P_{R3} = I_{3eff}^2 \cdot R_3 = (21{,}68)^2 \cdot 20 = 9400 \ W$

$Q_{L3} = I_{3eff}^2 \cdot X_{L3} = 2350 \ VAR$

$P_{ab} = P_{R2} + P_{R3} = 19{,}4 \ kW$

$Q_{ab} = Q_{L2} + Q_{L3} = 12350 \ VAR$

$A_{ab} = \sqrt{P_{ab}^2 + Q_{ab}^2} = 23 \ kVA = I_{AB} \cdot V_{ab}$, $I_{AB} = \frac{A_{ab}}{V_{ab}} = 51{,}45 \ A_{eff}$.

$P_{R1} = I_{AB}^2 \cdot 5 = 13235 \ W$, $Q_{L1} = I_{AB}^2 \cdot 3 = 7941 \ VAR$

$A_{AB} = 38428 \ VA$

$= V_{AB} \cdot I_{AB}$

$\rightarrow V_{AB} = 746 \ V_{eff}$

$C = \frac{20290 - P_{AB} \, tg \, \phi_{rif}}{\omega V_{AB}^2}$

$= 25 \ \mu F$.

$\Rightarrow I_{AB\,rif} = 48{,}5 \ A_{eff}$.

7. Energia magnetica media su un periodo T per i 3 induttori

$E_{L1} = 12,6\ J$

$E_{L2} = 15,0\ J$

$E_{L3} = 3,74\ J$

PARTE 3

13/05/2015

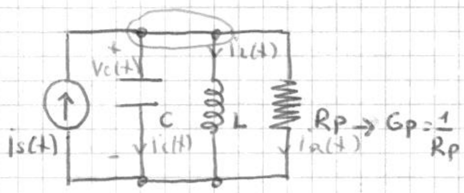
Circuiti del Secondo Ordine

1) Evoluzione continua per $t > \emptyset$

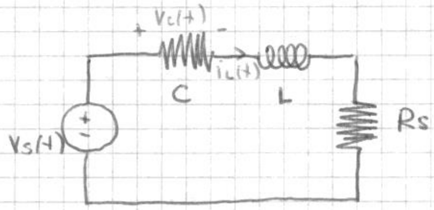
- Libera;
- Forzata ad eccitazione costante e sinusoidale;
- Completa

2) Transitori che conseguono ad un evento critico

RLC //



RLC Σ



Var. di Stato: $i_L(t)$, $V_C(t)$

$i_S(t) = i_C(t) + i_L(t) + i_R(t)$, $t > \emptyset$

$V_C(t) = v(t) = L \frac{d}{dt} i_L(t)$

$L \cdot C \cdot \frac{d^2}{dt^2} i_L + \frac{L}{R_P} \frac{d}{dt} i_L + i_L = i_S(t)$, $t > \emptyset$

$\frac{d^2}{dt^2} i_L + \frac{G_P}{C} \frac{d}{dt} i_L + \frac{1}{LC} \cdot i_L = \frac{1}{LC} \cdot i_S(t)$

Perdite: $V^2(t) \cdot G_P$

$\frac{d^2}{dt^2} V_C + \frac{R_S}{L} \frac{d}{dt} V_C + \frac{1}{LC} V_C = \frac{1}{LC} V_S(t)$, $t > \emptyset$

Perdite: $R_S \cdot i^2(t)$.

$$\frac{d^2}{dt}x + 2\alpha\frac{d}{dt}x + \omega_0^2 x = f(t)\ ,\ t > 0$$

//

$\alpha = \alpha_p = \frac{G_p}{2C}$ rad/sec

Coefficiente di smorzamento

$\omega_0 = \frac{1}{\sqrt{LC}}$ rad/sec

Pulsazione di Risonanza

$f(t) = \omega_0^2\, i_s(t)$

$x(t) = i_L(t)$

Σ

$\alpha = \alpha_s = \frac{R_s}{2L}$

$\omega_0 = \frac{1}{\sqrt{LC}}$

$f(t) = \omega_0^2\, V_s(t)$

$x(t) = V_c(t)$

Risposta Libera

$$\frac{d^2}{dt^2}x + 2\alpha\frac{d}{dt}x + \omega_0^2 x = 0\ ,\ t > 0$$

integrale generale omogenea

$s^2 + 2\alpha s + \omega_0^2 = 0$

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ radici caratteristiche

$\Delta = \alpha^2 - \omega_0^2$ $\Delta \gtreqless 0$; $\alpha \gtreqless \omega_0$

//

$\alpha = \alpha_p = \frac{G_p}{2C} \gtreqless \omega_0 = \frac{1}{\sqrt{LC}}$

$\alpha \gtreqless \omega_0 \longleftrightarrow G_p \gtreqless 2\sqrt{\frac{C}{L}} = G_c$

G_c : conduttanza critica RLC //.

Σ

$\alpha = \alpha_s = \frac{R_s}{2L} \gtreqless \omega_0 = \frac{1}{\sqrt{LC}}$

$\alpha \gtreqless \omega_0 \longleftrightarrow R_s \gtreqless 2\sqrt{\frac{L}{C}} = R_c$

R_c : Resistenza critica RLC Σ.

1) $\alpha > \omega_0$

Sovrasmorzato (Perdite alte)

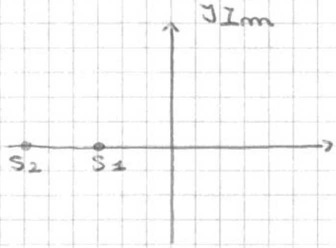
$G_p > G_c = 2\sqrt{\frac{C}{L}}$ (//)

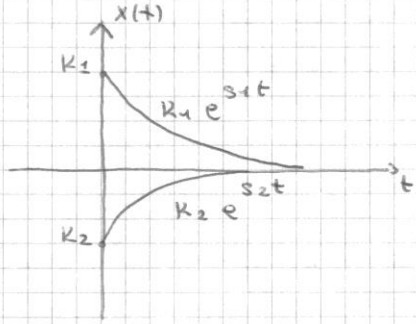
$R_s > R_c = 2\sqrt{\frac{L}{C}}$ (Σ)

$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$ (Reali negative distinte)

$x(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t}, \quad t > \emptyset$

Comportamento Aperiodico





2) $\alpha = \omega_0$

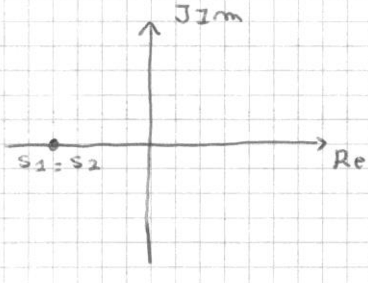
$G_p = G_c = 2\sqrt{\frac{C}{L}}$, $\Delta = \emptyset$ (//)

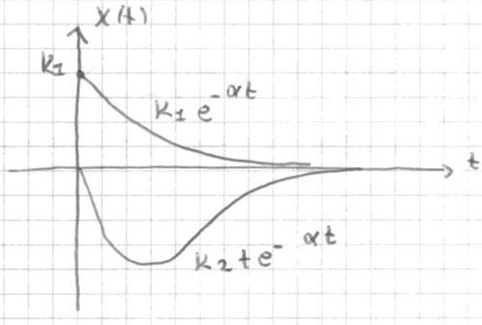
$R_s = R_c = 2\sqrt{\frac{L}{C}}$, $\Delta = \emptyset$ (Σ)

$s_1 = s_2 = -\alpha = -\omega_0$ Smorzamento Critico

$x(t) = k_1 e^{-\alpha t} + k_2 \, t \cdot e^{-\alpha t}$

Comportamento Aperiodico





3) $\alpha < \omega_0$, $\Delta < \emptyset$ Sottosmorzato

$G_p < G_c = 2\sqrt{\frac{C}{L}}$ (//)

$R_s < R_c = 2\sqrt{\frac{L}{C}}$ (Σ)

$S_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2} \rightarrow < \emptyset$

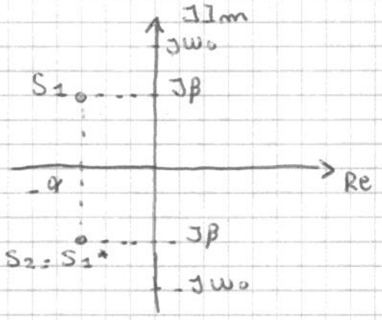
$S_{1,2} = -\alpha \pm J\sqrt{\omega_0^2 - \alpha^2}$

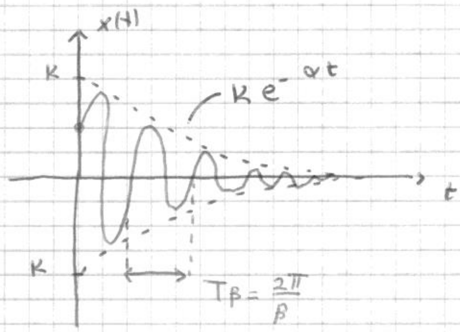
$\beta = \sqrt{\omega_0^2 - \alpha^2}$ rad/sec : Pulsazione naturale di Oscillazione

$\emptyset < \beta < \omega_0$

$S_{1,2} = -\alpha \pm J\beta$

$x(t) = K_1 \cdot e^{-\alpha t} \cos(\beta t) + K_2 e^{-\alpha t} \mathrm{sen}(\beta t)$
$= K \cdot e^{-\alpha t} \cos(\beta t - \theta)$, $t > \emptyset$





Pulsazione dell' Oscillazione effettuata all' interno dell'inviluppo è $\beta < \omega_0$

4) Senza Perdite

$G_p = \emptyset\ \Omega^{-1}$ (//)

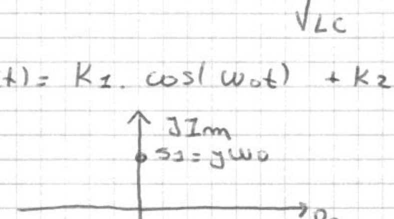
$R_s = \emptyset\ \Omega$ (Σ)

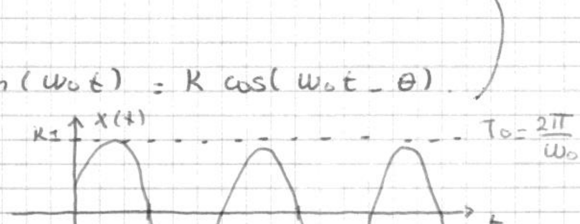
$\alpha = \emptyset$

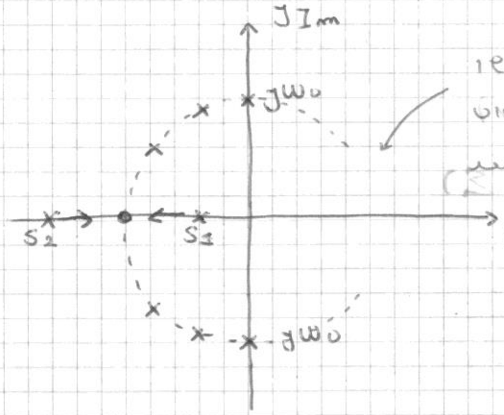
$S_{1,2} = \pm J\omega_0 = \pm J \frac{1}{\sqrt{LC}}$

$x(t) = K_1 \cdot \cos(\omega_0 t) + K_2 \mathrm{sen}(\omega_0 t) = K \cos(\omega_0 t - \theta)$

Oscilla alla frequenza ω_0







il semipiano di destra viene raggiunto solo con un filtro attivo con AO.

↓

uso AO per compensare le perdite.

L, C fissi, vario G_p.

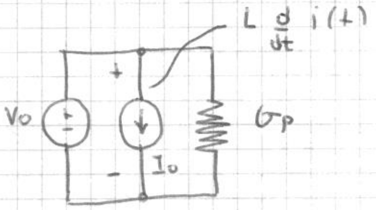
Determinazione di K_1, K_2 (costanti Arbitrarie)

Conosco stato iniziale: $i_L(\emptyset^+) = I_0$, $V_C(\emptyset^+) = V_0$

RLC //, sovrasmorzato

$$\begin{cases} x(t) = i_L(t) = K_1 e^{s_1 t} + K_2 e^{s_2 t} \\ i_L(\emptyset^+) = I_0 \\ \frac{d}{dt} i_L(\emptyset^+) = \frac{V_0}{L} \end{cases}$$

$t = \emptyset^+$



$$i_L(\emptyset) = K_1 + K_2 = I_0$$

$$\frac{d}{dt} i_L(\emptyset^+) = S_1 K_1 + S_2 K_2 = \frac{V_0}{L}$$

$$K_1 = \frac{1}{S_1 - S_2}\left(\frac{V_0}{L} - S_2 I_0\right).$$

$$K_2 = \frac{1}{S_2 - S_1}\left(\frac{V_0}{L} - S_1 I_0\right).$$

fare gli altri casi per casa.

Esercizio:

Circuito RLC // in evoluzione libera

$R_p = 1\,\Omega$, $L = 1\,H$, $C = 1\,F$

$I_0 = 1\,A$, $V_0 = 1\,V$

? $i_L(t)$, $t > \emptyset$

$G_c = 2\sqrt{\frac{C}{L}} = 2\,\Omega^{-1}$, $G_p = \frac{1}{R_p} = 1\,\Omega^{-1} < 2\,\Omega^{-1}$

$\rightarrow$ Caso Sottosmorzato!

$S_{1,2} = -\alpha_p \pm j\beta$

$\alpha_p = \frac{G_p}{2C} = \frac{1}{2}$ rad/sec

$\omega_0 = \frac{1}{\sqrt{LC}} = 1$ rad/se

$\beta = \sqrt{\omega_0^2 - \alpha_p^2} = \sqrt{1 - 1/4} = \frac{\sqrt{3}}{2}$ rad/sec

$S_{1,2} = -\frac{1}{2} \pm j\frac{\sqrt{3}}{2}$.

$$\begin{cases} i_L(t) = k_1\, e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + k_2\, e^{-\frac{1}{2}t} \operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right) \quad , t > \emptyset \\ i_L(\emptyset^+) = 1\,A \\ \frac{d}{dt} i_L(\emptyset^+) = \frac{V_0}{L} = 1\,A/sec \end{cases}$$

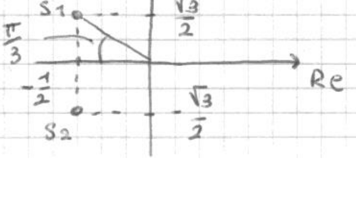
$i_L(\emptyset^+) = k_1 = 1$

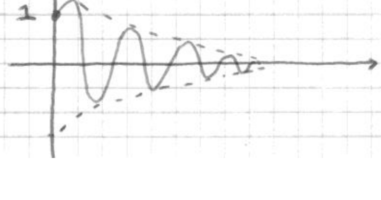
$i_L(t) = e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2}t\right) + k_2\, e^{-\frac{1}{2}t} \operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right)$, $t > \emptyset$.

$\frac{d}{dt} i_L(\emptyset^+) = -\frac{1}{2} + k_2 \frac{\sqrt{3}}{2} = 1\,A/sec$

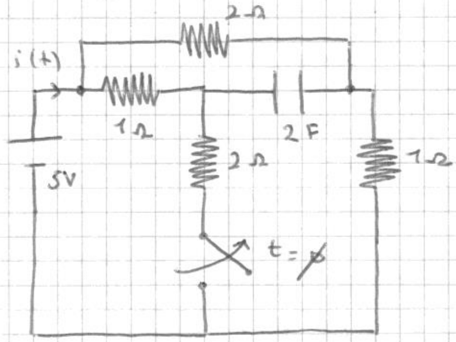
$k_2 = \frac{3}{\sqrt{3}} = \sqrt{3}$

$i_L(t) = e^{-1/2t} \cos\left(\frac{\sqrt{3}}{2}t\right) + \sqrt{3}\, e^{-1/2t} \operatorname{sen}\left(\frac{\sqrt{3}}{2}t\right)$





PARTE 2



l'interruttore, chiuso da lungo tempo, si apre a $t = \emptyset$.

? $i(t),\ t > \emptyset$.

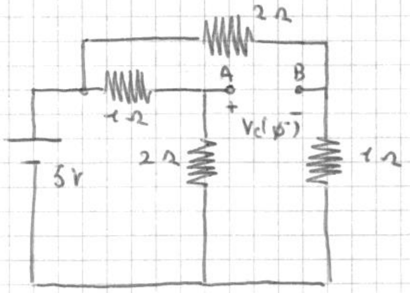
R. staz. 1 — $\emptyset$ — Ris. compl. — R. staz. 2

stato a $t = \emptyset^-$

$$V_c(\emptyset^-) = V_c(\emptyset^+)$$

↑ Circuito non degenere

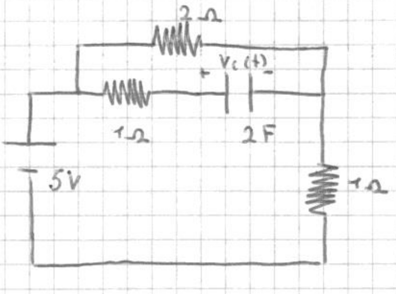
$t = \emptyset^-$ R. staz. 1



$$V_c(\emptyset^-) = V_A - V_B = \frac{10}{3} - \frac{5}{3} = \frac{5}{3}\ V.$$

$$V_c(\emptyset^+) = V_c(\emptyset^-) = \frac{5}{3}\ V.$$

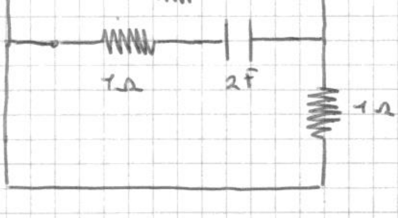
$t > \emptyset$



Circuito I° ordine con eccitazione costante:

$$i(t) = i(\emptyset^+)\, e^{-\frac{t}{\tau_{eq}}} + i(\infty)\left(1 - e^{-\frac{t}{\tau_{eq}}}\right)$$

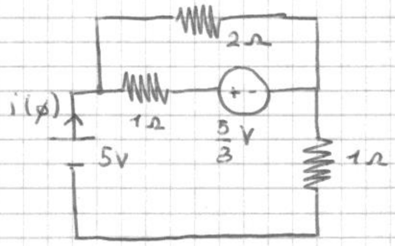
$t > \emptyset$

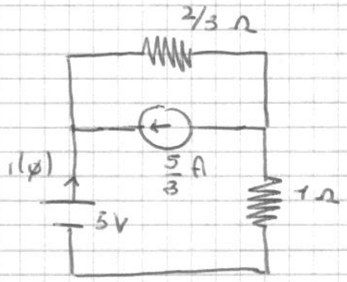


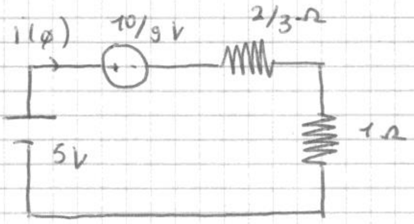
$$\tau_{eq} = C \cdot R_{eq} = \frac{10}{3}\ \text{sec}.$$

$$i(\infty) = \frac{5}{3}\ A.$$

$t = 0^+$ istantanea

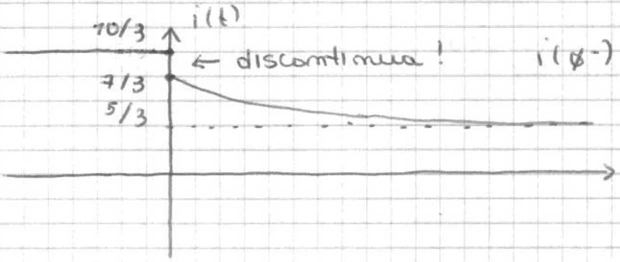






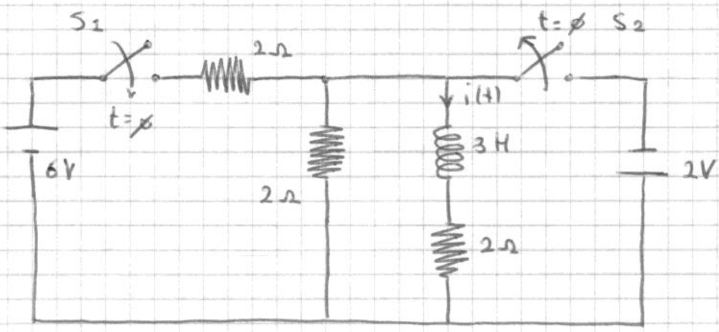
$$i(0) = \frac{5 - 10/9}{\frac{5}{3}} = \frac{7}{3}\ A$$

$$\rightarrow\ i(t) = \frac{7}{3} \cdot e^{-\frac{3}{10}t} + \frac{5}{3}\left(1 - e^{-\frac{3}{10}t}\right), \quad t > 0.$$



$i(0^-) = \frac{10}{3}\ A$.

Esercizio:



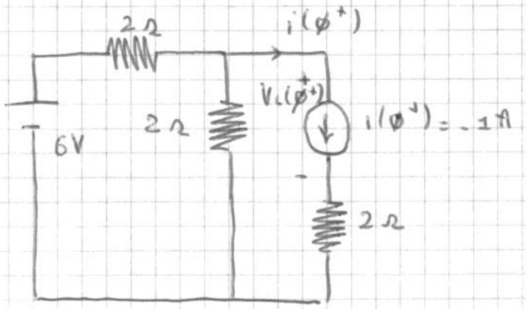
S1 aperto da lungo tempo si chiude a $t = 0$.
S2 chiuso " " " " apre a $t = 0$.

? $i(0^+)$, $\frac{d}{dt}\, i(0^+)$

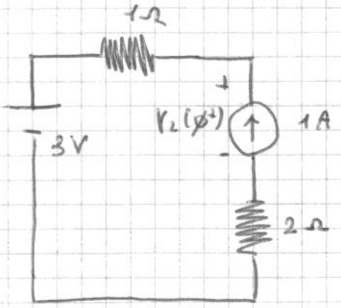
$$i(0^-) = -\frac{2}{2} = -1\ A.$$

Rete non degenere : $i(\phi^+) = i(\phi^-) = -1\,A$.

Istantanea a $t = \phi^+$.



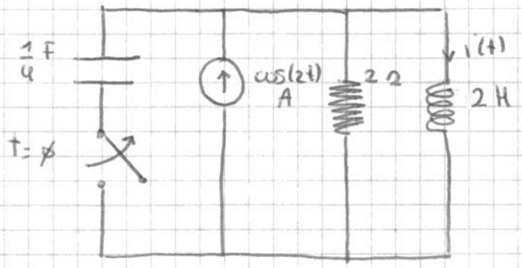
$$L \cdot \frac{d}{dt}\, i(\phi^+) = V_L(\phi^+)$$



$$V_L(\phi^+) = 3+3 = L \cdot \frac{d}{dt}\, i(\phi^+) = 3 \cdot \frac{d}{dt}\, i(\phi^+)$$

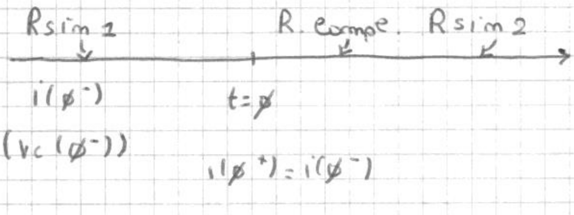
$$\frac{d}{dt}\, i(\phi^+) = 2\ A/sec$$

Esercizio :

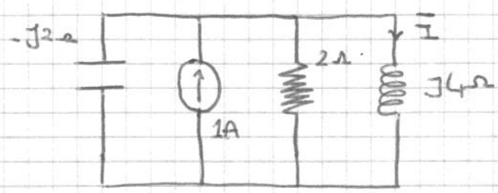


interruttore chiuso da lungo tempo si apre a $t = \phi$.

?. $i(t)$, $t > \phi$, risp. completa.



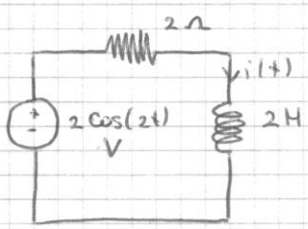
$t = \emptyset$ RSin 1



$$\bar{I} = \frac{\frac{1}{j8}}{\frac{1}{-j2} + \frac{1}{2} + \frac{1}{j4}} = \dots = -\frac{1}{2} - \frac{j}{2} \text{ A}$$

$$i(\emptyset^-) = i(\emptyset^+) = \text{Re}\{\bar{I}\} = -\frac{1}{2} \text{ A}.$$

Risp. Completa $t > \emptyset$



$$i(t) = i_{lib}(t) + i_{for}(t)$$

$$= i(\emptyset^+) \cdot e^{-t} + \text{forzata}$$

Per casa!

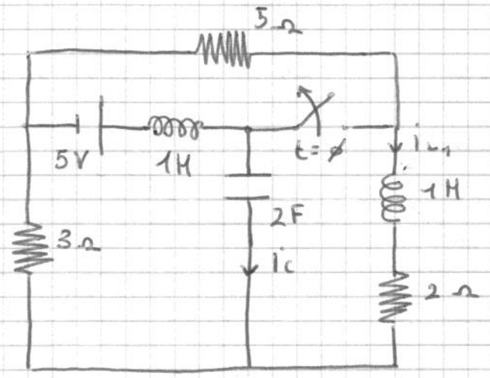
$$\tau_{eq} = \frac{L}{R} = 1 \text{ sec}$$

$$i(t) = \text{TC} + \text{Regime Sin. 2} = K e^{-t} + \frac{2}{\sqrt{4+4}} \cos(2t - 45°)$$

$$i(\emptyset^+) = -\frac{1}{2} = K + \frac{2}{\sqrt{8}} \cos(-45°)$$

$$\rightarrow -\frac{1}{2} = K + \frac{\sqrt{2}}{2} \cdot \frac{\sqrt{2}}{2} \Rightarrow K = -1.$$

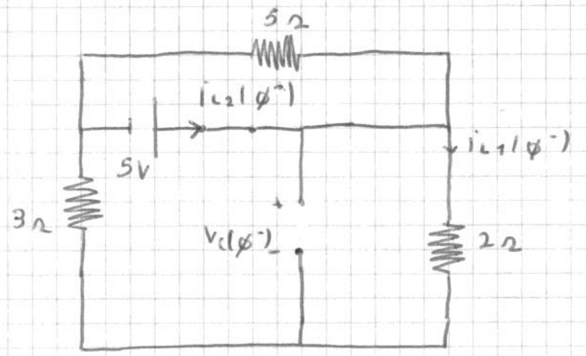
Esercizio:



$t = \emptyset$ si apre interruttore

? $i_c(\emptyset^+)$, $\frac{d}{dt} i_{L_1}(\emptyset^+)$

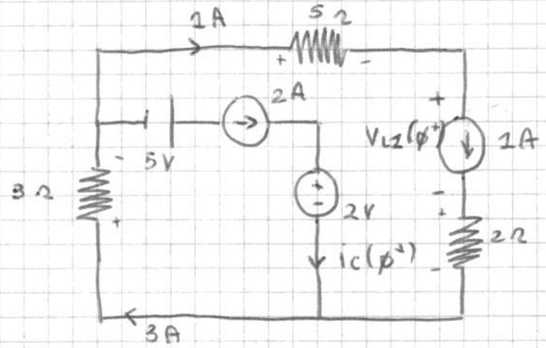
$t = 0^-$ Regime Stazionario



? $V_C(0^-)$
? $i_{L1}(0^-)$
? $i_{L2}(0^-)$

$i_{L1}(0^-) = 1\,A = i_{L1}(0^+)$
$V_C(0^-) = 2V = V_C(0^+)$
$i_{L2}(0^-) = 2\,A = i_{L2}(0^+)$

$t = 0^+$



$i_C(0^+) = 2A$

$3 + 5 + V_{L1}(0^+) + 2 = 0$
$V_{L1}(0^+) = -16\,V.$

$\rightarrow \frac{d}{dt} i_L(t) = -16\ A/sec.$

Piccola Parentesi:

$\sqrt[n]{1} \rightarrow S^n = 1$, $S = |S|\, e^{J\theta}$

$S^n = |S|^n e^{Jn\theta} = 1 \cdot e^{J(0 + 2k\pi)}$

$|S|^n = 1 \longrightarrow |S| = 1 = \sqrt[n]{1}$

$n\theta = 2k\pi$, $k = 0, 1, 2, \dots, n-1$

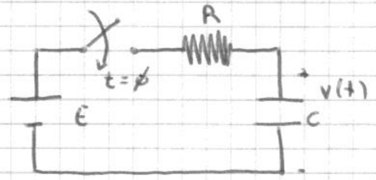
$\theta = \frac{2k\pi}{n}$, $k = 0, 1, \dots, n-1$

$n = 6$, $\theta = \frac{2k\pi}{6}$

$\rightarrow k = 0 \rightarrow \theta = 0$
$k = 1 \rightarrow \theta = \pi/3$
$k = 2 \rightarrow \theta = 2/3\,\pi$
$k = 3 \rightarrow$ ⋮
$k = 4$
$k = 5$

PARTE 3 15/05/2015

Analisi in // nel DT e nel DL di transitori in circuiti del I° ordine.



? $v(t)$, $t > 0$ DT e poi con TL.

$v(0^-) = V_0$

DT:

1) Analisi all'istante critico $(0^-, 0^+)$

$v(0^+) = v(0^-) = V(0)$

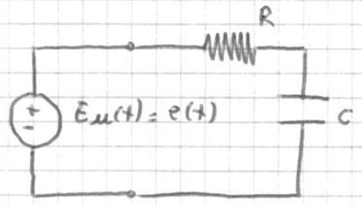
continuità var. di stato circuito non degenere

2) Analisi in evoluzione continua per $t > 0$

$$\begin{cases} RC \cdot \dfrac{d}{dt} v + v = E & , t > 0 \\ v(0^+) = V(0^-) = V_0 \end{cases}$$

$$v(t) = V_{LIB}(t) + V_{FOR}(t) = V_0 \, e^{-\frac{t}{\tau}} + E\left(1 - e^{-\frac{t}{\tau}}\right), \quad t > 0$$

DL: Verifica con TL



$t > 0^-$

$t > -\varepsilon$

$$\begin{cases} RC \cdot \dfrac{d}{dt} v + v = e(t) = E \cdot u(t) \\ v(0^-) = V_0 \end{cases}$$

$$\int_{0^-}^{+\infty} \left(RC \frac{d}{dt} v + v\right) e^{-st} \, dt = \int_{0^-}^{+\infty} e(t) \, e^{-st} \, dt$$

$$= RC\left[s V(s) - v(0^-)\right] + V(s) = E(s) = \frac{E}{s}$$

$$(sRC + 1) V(s) = RC \, V(0^-) + \frac{E}{s} \rightarrow V(s) = \frac{RC}{sRC + 1} V_0 + \frac{E}{s(sRC + 1)}$$

$$= V_{LIB}(s) + V_{FOR}(s)$$

$$V_{LIB}(s) = \frac{V_0}{s + \frac{1}{RC}} \xrightarrow{\mathcal{L}^{-1}} V_{LIB}(t) = V_0 \cdot e^{-\frac{t}{RC}} \cdot u(t)$$

$$V_{FOR}(s) = \frac{E}{\tau} \cdot \frac{1}{s+\frac{1}{\tau}} \cdot \frac{1}{s}$$

$$F(s) = \frac{1}{s+\frac{1}{\tau}} \cdot \frac{1}{s} = \frac{A}{s} + \frac{B}{s+\frac{1}{\tau}}$$

$$A = \lim_{s \to 0} s\,F(s) = \tau$$

$$B = \lim_{s \to -\frac{1}{\tau}} \left(s+\frac{1}{\tau}\right) F(s) = -\tau$$

$$V_{FOR}(s) = \frac{E}{s} - \frac{E}{s+\frac{1}{\tau}} \xrightarrow{\mathcal{L}^{-1}} V_{FOR}(t) = E\,u(t) - E\,e^{-\frac{t}{\tau}}\,u(t)$$

OK! Il risultato è lo stesso.

Se $F(s) = \frac{N(s)}{D(s)}$, s_0 polo I° ordine $\to A = \frac{N(s_0)}{D'(s_0)}$.

Esercizio Teorico (circuito II° Ordine)

? $i(t)$, $t \geq 0$

Risposta Forzata

$i(0^-) = 0$, $V_C(0^-) = 0$

1) Analisi $(0^-, 0^+)$, continuità var. di stato

circuito non degenere $\longrightarrow$ $i_L(0^+) = 0$

$V_C(0^+) = 0$

2) Evoluzione continua per $t > 0$.

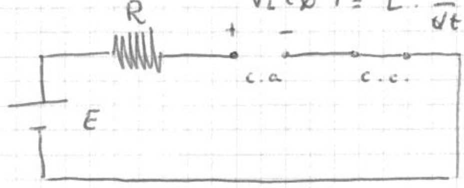
$$L \cdot \frac{d}{dt} i + R i + V_C(0^+) + \frac{1}{C}\int_{0^+}^{t} i(t')\,dt' = E \quad , t > 0$$

Posso derivare:

$$\begin{cases} L \cdot \frac{d^2}{dt^2} i + R \frac{d}{dt} i + \frac{1}{C} i = 0 \quad , t > 0 \\ i(0^+) = 0 \\ \frac{d}{dt} i(0^+) = ? = \frac{E}{L} \end{cases}$$

$t = 0^+$

$V_L(0^+) = L \cdot \frac{d}{dt} i(0^+) = E$



Caso Sottosmorzato:

$R < R_c = 2\sqrt{\frac{L}{C}}$ $\qquad s_{1,2} = -\alpha \pm j\beta$

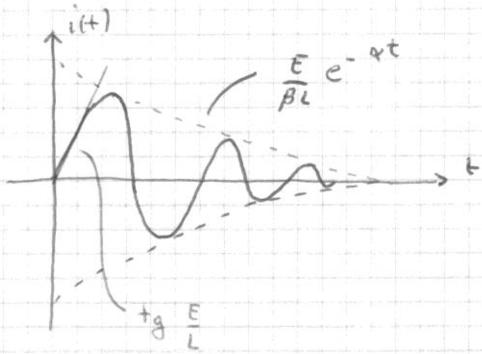
$i(t) = \cancel{K_1 \cdot e^{-\alpha t} \cos(\beta t)} + K_2 \cdot e^{-\alpha t} \, \text{sen}(\beta t) \quad, t > 0$

$i(0^+) = 0 = K_1$

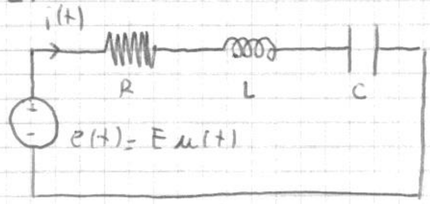
$\rightarrow i(t) = K_2 \cdot e^{-\alpha t} \cdot \text{sen}(\beta t) \quad, t > 0$

$\frac{d}{dt} i(0^+) = \beta \cdot K_2 = \frac{E}{L} \quad \rightarrow K_2 = \frac{E}{\beta \cdot L}$

$i(t) = \frac{E}{\beta \cdot L} e^{-\alpha t} \, \text{sen}(\beta t) \cdot u(t)$



TL:



$L \cdot \frac{d}{dt} i + Ri + V_C(0^-) + \frac{1}{C}\int_{0^-}^{t} i(t')\,dt'\,dt = E u(t) \quad, t > 0^-$

$t > -\varepsilon$

$L[sI(s) - i(0^-)] + RI(s) + \frac{V_C(0^-)}{s} + \frac{1}{sC} \cdot I(s) = \frac{E}{s}$.

$(sL + R + \frac{1}{sC}) I(s) = L i(0^-) - \frac{V_C(0^-)}{s} + \frac{E}{s}$.

$$I(s) = \frac{E}{S} \cdot \frac{1}{SL + R + \frac{1}{SC}} + \frac{L\, i(0^-) - \frac{v_C(0^-)}{S}}{SL + R + \frac{1}{SC}}$$

$$= I_{FOR}(s) + I_{LIB}(s)$$

Devo calcolare $I_{FOR}(s)$

$$I_{FOR}(s) = \frac{E}{L} \cdot \frac{1}{s^2 + \frac{R}{L}s + \frac{1}{LC}} = \frac{E}{L} \cdot \frac{1}{s^2 + 2\alpha s + \omega_0^2}$$

Poli f.t. coincidono con radici caratteristiche

Caso sottosmorzato:

$R < R_C = 2\sqrt{\frac{L}{C}}$

Poli: $s_{1,2} = \alpha \pm j\beta$

$$F(s) = \frac{1}{s^2 + 2\alpha s + \omega_0^2} = \frac{1}{(s - s_1)(s - s_2)} = \frac{1}{[s - (-\alpha + j\beta)]\,[s - (-\alpha - j\beta)]}$$

$$= \frac{A}{s - (-\alpha + j\beta)} + \frac{B}{s - (-\alpha - j\beta)}$$

$\Rightarrow$ Teorema: $B = A^*$.

$$i(t) = \frac{E}{L}\Big[\underbrace{A\, e^{(-\alpha + j\beta)t}}_{f(t)} + \underbrace{A^*\, e^{(-\alpha - j\beta)t}}_{f^*(t)}\Big]u(t)$$

$$i(t) = \frac{E}{L}\, 2\,\mathrm{Re}\left[A \cdot e^{(-\alpha + j\beta)t}\right]u(t)$$

$$= 2 \cdot \frac{E}{L}\, e^{-\alpha t}\, \mathrm{Re}\left[A \cdot e^{j\beta t}\right]u(t)$$

$$A = \lim_{s \to -\alpha + j\beta} (s - (-\alpha + j\beta))\, F(s) = \left.\frac{1}{s - (-\alpha - j\beta)}\right|_{s = -\alpha + j\beta}$$

$$= \frac{1}{-\alpha + j\beta + \alpha + j\beta} = \frac{1}{j2\beta} = -j \cdot \frac{1}{2\beta}.$$

$$i(t) = \frac{2E}{L} \cdot e^{-\alpha t} \cdot \mathrm{Re}\left[-\frac{j}{2\beta} \cdot (\cos\beta t + j\,\mathrm{sen}\,\beta t)\right]u(t)$$

$$= \frac{E}{\beta L} \cdot e^{-\alpha t}\, \mathrm{sen}(\beta t) \cdot u(t)$$

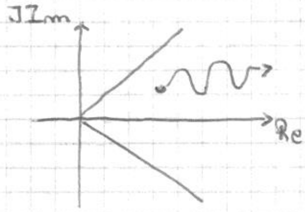
$$I(s) = \frac{E}{L} \cdot \frac{1}{s^2 + s \cdot \frac{R}{L} + \frac{1}{LC}}$$

$$i(0^+) = \lim_{s \to \infty} s\, I(s) = 0$$

$$\frac{d}{dt} i(0^+) = \lim_{s \to \infty} s^2 I(s) = \frac{E}{L}.$$

Se $I(s)$ contiene e^{-s}

$$i(0^+) = \lim_{s \to \infty} s\, I(s) \qquad (Re\, s \to +\infty)$$



$$s_{1,2} = \alpha \pm \sqrt{\alpha^2 - \omega_0^2} \quad rad/sec$$

radici caratteristiche $\Leftrightarrow$ frequenze naturali

Luogo Frequenze Naturali

$$s^2 + 2\alpha s + \omega_0^2 = 0$$

$$s_{1,2} = -\alpha \pm \sqrt{\alpha^2 - \omega_0^2}$$

L, C fissi, Vario G_p //

Varia R_s Σ

Fattore di qualità di un circuito RLC:

$$Q = \frac{\omega_0}{2\alpha}$$

$$//: Q_p = \frac{\omega_0}{2\alpha_p} = \frac{\omega_0}{2\frac{G_p}{2C}} = \frac{\omega_0 C}{G_p} \propto \frac{1}{G_p}$$

$$\Sigma: Q_s = \frac{\omega_0}{2\alpha_s} = \frac{\omega_0}{2\frac{R_s}{2L}} = \frac{\omega_0 L}{R_s} \propto \frac{1}{R_s}$$

Sovrasmorzamento : $\alpha > \omega_0$, $Q < 1/2$

Smorzamento critico : $\alpha = \omega_0$, $Q = 1/2$

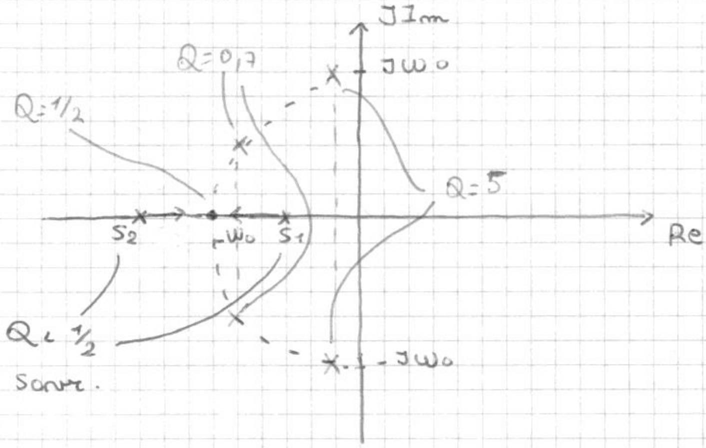
Sottosmorzamento : $\alpha < \omega_0$, $Q > 1/2$

No Perdite : $\alpha = 0$, $Q \to \infty$

$S_1 \cdot S_2 = \omega_0^2 = \text{cost}$

$S_1 + S_2 = 2\alpha$

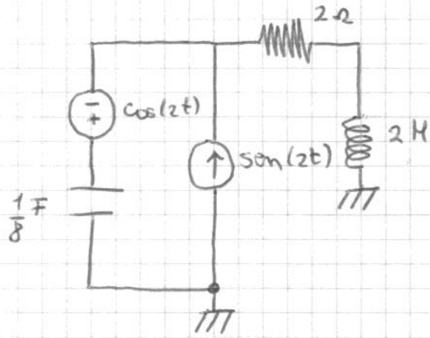
$\downarrow G_p \quad \downarrow \alpha_p \quad \uparrow Q$



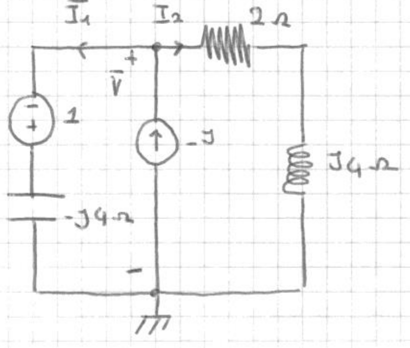
$Q > 5 \quad , \quad S_{1/2} \simeq -\dfrac{\omega_0}{Q} \pm J\omega_0$

PARTE 2

3/03/2010



Verificare T. Boucherot per circuito in figura



$$\overline{V} = \frac{-1 \cdot \frac{J}{4} - J}{\frac{J}{4} + \frac{1}{2 + J4}} = \ldots = -5 - J10 \; V$$

$\overline{V} = -1 + \overline{I}_1(-J4)$

$\overline{I}_1 = \dfrac{\overline{V} + 1}{-J4} = \ldots = \dfrac{5}{2} - J \; A$

$-J = \overline{I}_1 + \overline{I}_2 \rightarrow \overline{I}_2 = -J - \overline{I}_1 = -\dfrac{5}{2} A$

$$\overline{P}_{\substack{gen.\ cor.\\ erogata}} = \frac{1}{2}\,\overline{V}\,j = \frac{1}{2}\left(-5-j10\right)j = 5 - j\frac{5}{2} \ \text{VA}.$$

$$\overline{P}_{\substack{gen.\ ten.\\ erogata}} = \frac{1}{2}\cdot 1\cdot \overline{I}_1^{*} = \frac{1}{2}\cdot\left(\frac{5}{2}+j\right) = \frac{5}{4} + j\frac{1}{2} \ \text{VA}.$$

$$\overline{P}_{\substack{gen.\ ri\\ erogata}} = \frac{25}{4} - j2$$

$$P_{\substack{R\\ ass.}} = \frac{1}{2}\cdot 2\cdot I_{2M}^2 = \frac{25}{4} \ \text{Watt}$$

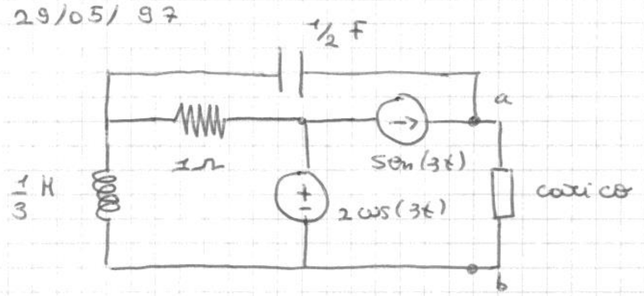
$$Q_{\substack{L\\ ass.}} = \frac{1}{2}X_L I_{2M}^2 = \frac{1}{2}\cdot\cancel{4}\cdot\frac{25}{\cancel{4}} = \frac{25}{2} \ \text{VAR}$$

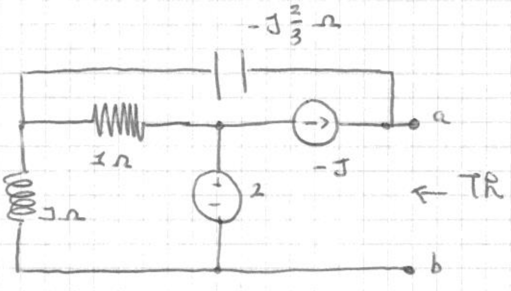
$$Q_{\substack{C\\ ass.}} = -\frac{1}{2}X_C\, I_{1M}^2 = -\frac{1}{2}\cdot\cancel{4}\cdot\frac{29}{\cancel{4}} = -\frac{29}{2} \ \text{VAR}$$

$$Q_L + Q_C = -2 \ \text{VAR}.$$

Ok! Verificato.

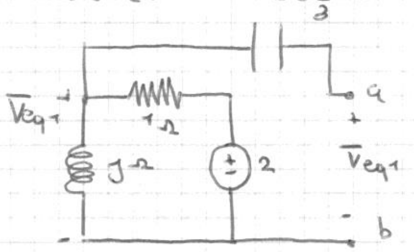
29/05/97



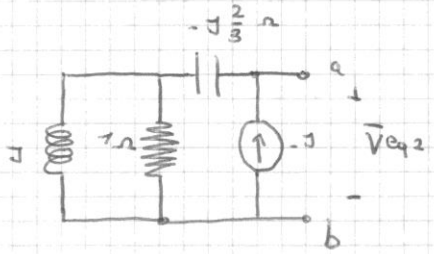


$$\overline{Z}_{eq} = -j\frac{2}{3} + 1\,||\,j = -j\frac{2}{3} + \frac{j}{1+j} = \ldots = \frac{1}{2} - j\cdot\frac{1}{6}.$$

P.s.e.



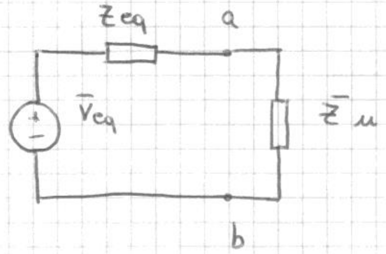
$$\overline{V}_{eq_1} = \frac{2j}{1+j} = \cancel{2}\cdot\frac{j(1-j)}{\cancel{2}} = 1 + j \ \text{V}$$



$$\bar{V}_{eq2} = -j \cdot \bar{Z}_{eq} = -j\left(\frac{1}{2} - j\frac{1}{6}\right)$$

$$= -\frac{1}{6} - j \cdot \frac{1}{2} \text{ V}$$

$$\bar{V}_{eq} = \bar{V}_{eq1} + \bar{V}_{eq2} = \frac{5}{6} + j\,\frac{1}{2} \text{ V}.$$



$$\bar{Z}_u = \bar{Z}_{eq}^{*} = \frac{1}{2} + j\frac{1}{6}\ \Omega.$$

$$P_d = P_{u,MAX} = \frac{1}{8} \cdot \frac{V_{eqM}^{2}}{Re_{eq}}$$

$$= \frac{1}{8} \cdot \frac{\frac{25}{36} + \frac{1}{4}}{\frac{1}{2}} = 0,236 \text{ W}.$$

20/05/2015

Continuo Analisi II° Ordine

$$I_f(s) = \frac{E}{L} \frac{1}{s^2+\frac{R}{L}s+\frac{1}{LC}} \xrightarrow{\mathcal{L}^{-1}} i(t) = \frac{E}{L}\cdot\frac{1}{s_1-s_2}\left[e^{s_1 t}-e^{s_2 t}\right]u(t)$$

Sovrasm.: $R > R_c = 2\sqrt{\frac{L}{C}}$ $s_{1,2}$ reali neg. distinte

Smorz. Critico: $R = R_c$ $s_1 = s_2 = -\alpha = -\omega_0$

$$I_f(s) = \frac{E}{L}\cdot\frac{1}{(s+\alpha)^2} \xrightarrow{\mathcal{L}^{-1}} i_f(t) = \frac{E}{L}\, t\, e^{-\alpha t}\cdot u(t)$$

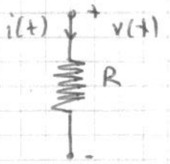
Metodo simbolico di Laplace per circuiti LTI.

1) Relazioni costitutive di R, L, C nel dominio di s;

2) Leggi di Kirchhoff, nel dominio di s.

-> circuito simbolico nel dominio di s.

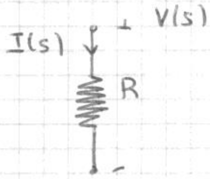
R ideale



$v(t), i(t)$ trasformate con Laplace.

$v(t) = R\cdot i(t)$

DT.

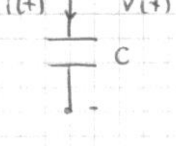


$V(s) = R\cdot I(s)$

$[V(s)]$ = Volt . sec

$[I(s)]$ = Ampere . sec

C ideale



$$i(t) = C\cdot\frac{d}{dt}v(t)$$

$$I(s) = C\left[sV(s) - V(\emptyset^-)\right]$$

DT.

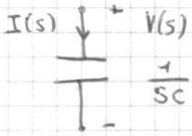
1) C scarico a $t = \emptyset^-$, $V(\emptyset^-) = \emptyset$

$I(s) = \frac{V(s)}{\frac{1}{sC}}$ $Z_c(s) = \frac{1}{sC}$: impedenza di C in Dominio di s.

$\left[\frac{1}{sC}\right] = \Omega \rightarrow I(s) = \frac{V(s)}{Z_c(s)}$: Legge di Ohm simbolica di C.

$y_C(s) = sC = \frac{1}{Z_C(s)}$: ammettenza di C.

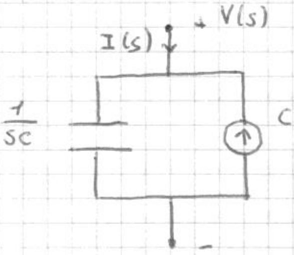
C scarico a $\emptyset^-$ è un impedenza di valore $\frac{1}{sC}$.



$\frac{1}{sC} = Z_C(s)$

2) C carico a $t = \emptyset^-$

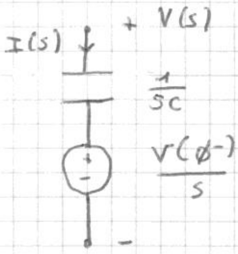
$I(s) = sC\,V(s) - C\,v(\emptyset^-)$



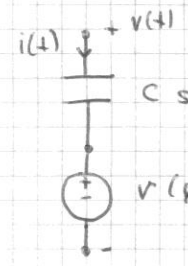
$C\,v(\emptyset^-) = q(\emptyset^-)$

A sec = coulomb.

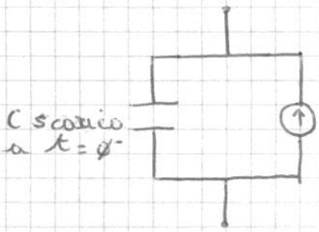
$V(s) = \frac{1}{sC} \cdot I(s) + \frac{v(\emptyset^-)}{s}$



C carico a $t = \emptyset^-$

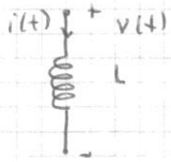


C carico a $t = \emptyset^-$



$C\,v(\emptyset^-)\,\delta(t) = q(\emptyset^-)\,\delta(t)$

L ideale



$v(t) = L \cdot \frac{d}{dt} i(t)$

$V(s) = L\,[\,s \cdot I(s) - i(\emptyset^-)\,]$

1) L scarico a $t = \emptyset^-$, $i(\emptyset^-) = \emptyset$.

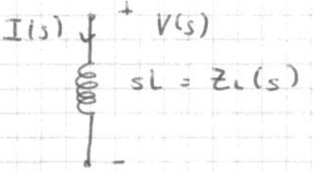
$V(s) = sL \cdot I(s)$, $Z_L(s) = sL$: impedenza di L in dominio di s.

$[Z_L(s)] = \Omega$

$I(s) = \frac{V(s)}{Z_L(s)} = \frac{V(s)}{sL}$

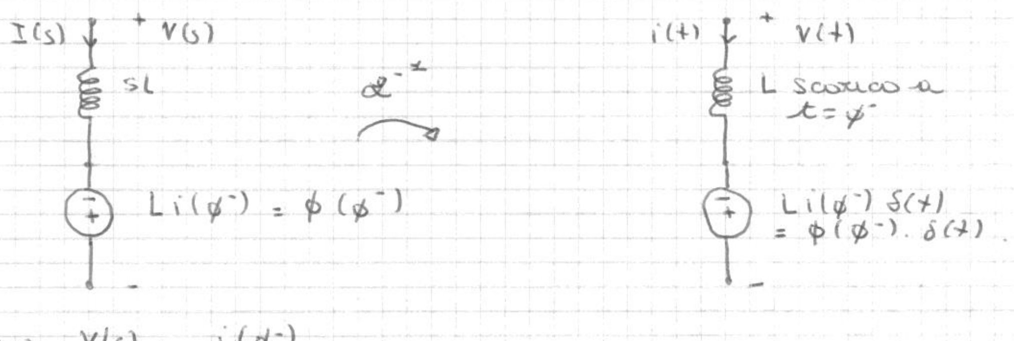
$Y_L(s) = \frac{1}{sL}$: ammettenza di L

L scarico a $t = \emptyset^-$ è un impedenza di valore sL.

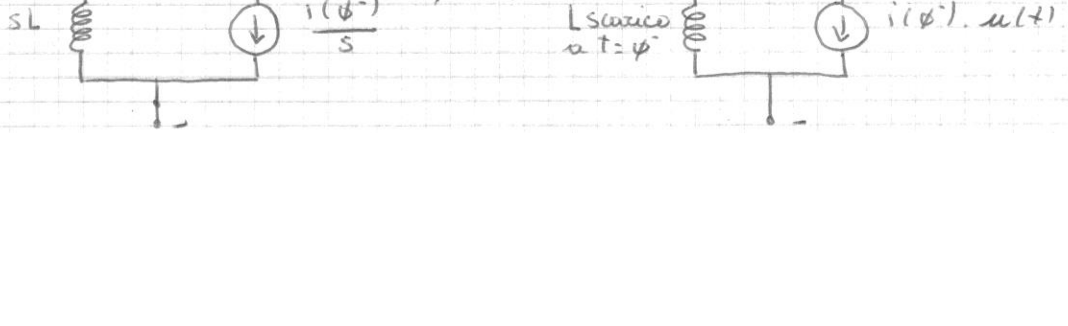


2) L carico a $t = \emptyset^-$

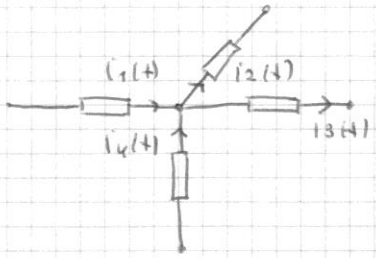
$V(s) = sL\,I(s) - L\,i(\emptyset^-)$



$I(s) = \frac{V(s)}{sL} + \frac{i(\emptyset^-)}{s}$



Leggi di Kirchhoff nel dominio di S.



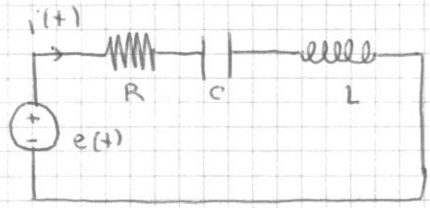
$$i_1(t) - i_2(t) - i_3(t) + i_4(t) = 0$$

$$I_1(s) - I_2(s) - I_3(s) + I_4(s) = 0.$$

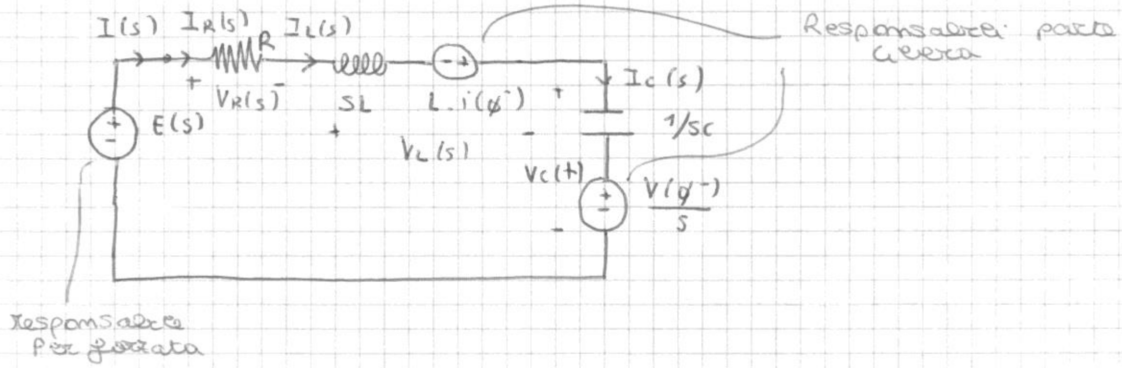
T.L. di correnti soddisfano KI.

T.L. di tensioni soddisfano KV.

Esercizio Teorico



? $I(s)$



$$KV: E(s) = V_R(s) + V_L(s) + V_C(s)$$

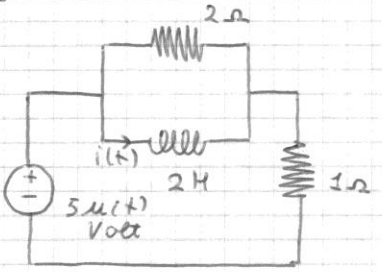
$$= R \cdot I_R(s) + sL \cdot I_L(s) - L\,i(0^-) + \frac{1}{sC} \cdot I_C(s) + \frac{V(0^-)}{s}$$

$$KI: I(s) = I_R(s) = I_L(s) = I_C(s)$$

$$\to \left(R + sL + \frac{1}{sC}\right) I(s) = E(s) + L\,i(0^-) - \frac{V(0^-)}{s}$$

Da cui ricavo $I(s)$

Esercizio :



$i(\phi^-) = 2$ Amp.

? $i(t)$, $t > \phi$

1) DT

2) TL

$i(\phi^+) = i(\phi^-) = 2$ Amp.

$t > \phi$

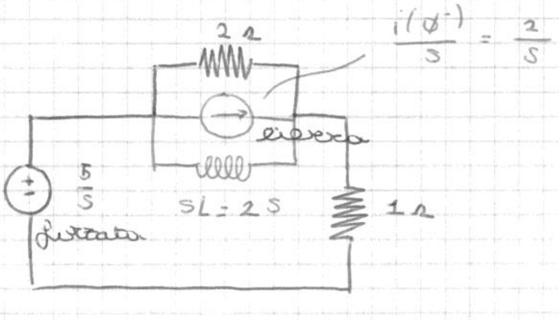
$$i(t) = \underset{\underset{2}{\shortparallel}}{i(\phi^-)}\, e^{-t/\tau_{eq}} + \underset{\underset{5}{\shortparallel}}{i(\infty)}\left(1 - e^{-t/\tau_{eq}}\right)$$

$$\frac{L}{R_{eq}} = 3 \text{ sec}$$

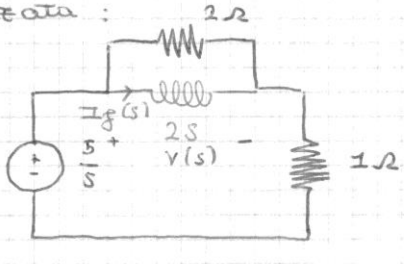
$$i(t) = \underbrace{2e^{-\frac{t}{3}}}_{i_\ell(t)} + \underbrace{5\left(1 - e^{-t/3}\right)}_{i_f(t)} \quad , t > \phi$$

N.B. nel caso non si analizzi var di stato le due parti non sono rispettivamente parte forzata e parte libera

Circuito Simbolico



Forzata :



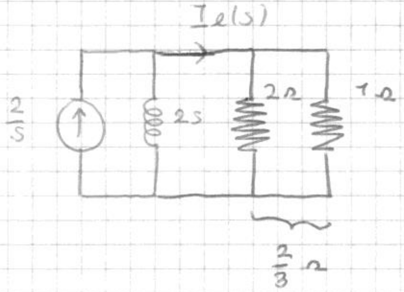
$$Z_{\parallel}(s) = \frac{4s}{2+2s} = \frac{2s}{s+1}\ \Omega$$

$$V(s) = \frac{5}{s} \cdot \frac{Z_{\parallel}(s)}{Z_{\parallel}(s)+1} = \frac{5}{s} \cdot \frac{\frac{2s}{s+1}}{\frac{2s}{s+1}+1}$$

$$= \frac{5}{s} \cdot \frac{2s}{3s+1} = \frac{10}{3} \cdot \frac{1}{s+\frac{1}{3}}$$

$$I_f(s) = \frac{V(s)}{2s} = \frac{5}{3} \cdot \frac{1}{s} \cdot \frac{1}{s+\frac{1}{3}} = \frac{5}{\cancel{3}}\left(\frac{3}{s} - \frac{\cancel{3}}{s+\frac{1}{3}}\right) = \frac{5}{s} - \frac{5}{s+\frac{1}{3}}$$

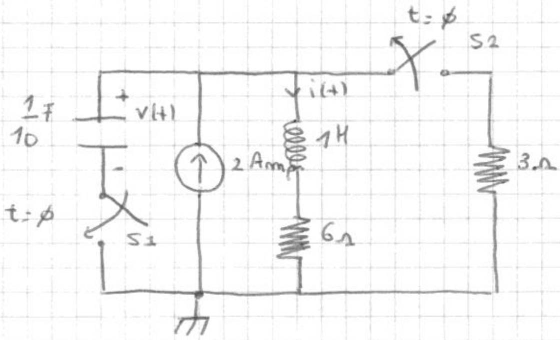
$i_g(t) = 5u(t) - 5\, e^{-\frac{t}{3}}\, u(t)$



$$I_e(s) = \frac{2}{s} \cdot \frac{\frac{3}{2}}{\frac{3}{2} + \frac{1}{2s}} = 2 \cdot \frac{1}{s + \frac{1}{3}}$$

$$i_e(t) = 2\, e^{-t/3}\, u(t).$$

04/07/2011



S1 aperto da lungo tempo si chiude a $t = \phi$.
S2 chiuso da lungo tempo si apre a $t = \phi$.

$v(\phi^-) = 2$ V

$v(\phi^-) = 2$ V

? $i(t)$, $t > \phi$

1) nel D.T.

2) con T.L.

Reg. Staz. 1 — Risp. Completa — Reg. Staz 2 → t

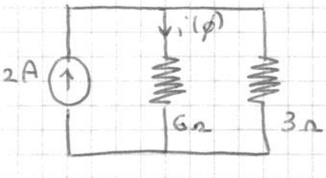
stato a $t = \phi^-$: $v(\phi^-)$, $i(\phi^-)$

$t = \phi$

$v(\phi^-) = v(\phi^+)$

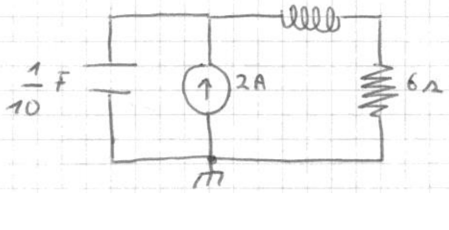
$i(\phi^-) = i(\phi^+)$

$v(\phi^-) = 2V = v(\phi^+)$



$i(\phi^-) = \frac{2}{3}\ A = i(\phi^+)$

$t > \phi$



? che tipo di circuito è per $t > \emptyset$

In evoluzione libera RLC serie II° ordine
↑
Spenti i gen. ri

Completa:

C = TC + Regime Stat. 2

$i(t) = \ldots + i(\infty)$, $t > \emptyset$

? In che caso siamo

$R_c = 2\sqrt{\frac{L}{C}} = 2\sqrt{10}$

$R = 6\,\Omega < R_c = 2\sqrt{10}$

-> Caso Sottosmorzato

$S_{1,2}$ complesse , parte reale neg. , coniugate

$S_{1,2} = -\alpha_s \pm j\beta$

$\alpha_s = \frac{R_s}{2L} = 3$ rad/sec , $\beta = \sqrt{\omega_0^2 - \alpha_s^2} = 1$ rad/sec

$\omega_0 = \frac{1}{\sqrt{LC}} = \sqrt{10}$ rad/sec

$S_{1,2} = -3 \pm j$: radici caratteristiche

$i(t) = K_1 \cdot e^{-3t} \cos(t) + K_2 \cdot e^{-3t} \operatorname{sen}(t) + i(\infty)$, $t > \emptyset$

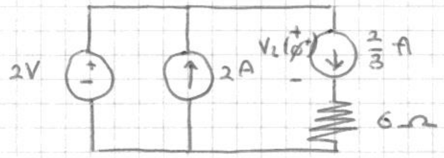
$i(\infty) = 2A$ -> cc. al posto di L
ca al posto di C

$$\begin{cases} i(t) = K_1 \cdot e^{-3t} \cos(t) + K_2 e^{-3t} \operatorname{sen}(t) + 2, & t > \emptyset . \\ i(\emptyset^+) = \frac{2}{3}\,A \\ \frac{d}{dt} i(\emptyset^+) = ? = -2\,A/\text{sec} \end{cases}$$

$\rightarrow K_1 + 2 = \frac{2}{3} \rightarrow K_1 = -\frac{4}{3}$

$-3K_1 + K_2 = -2 \rightarrow K_2 = -6$

$t = 0^+$



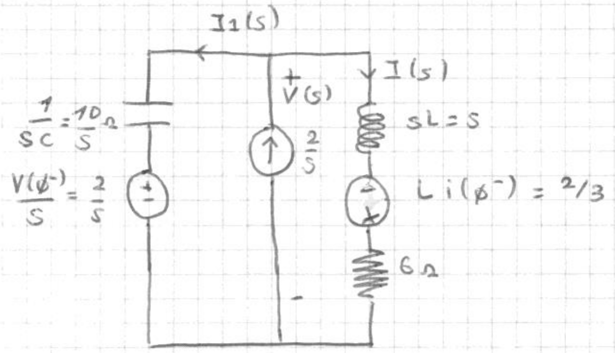
$V_L(0^+) = L \cdot \frac{d}{dt} i(0^+)$, $L = 1$

$= \frac{d}{dt} i(0^+)$

$V_L(0^+) + 6 \cdot \frac{2}{3} - 2 = 0 \quad \rightarrow \quad V_L = 0^+ = -2V = \frac{d}{dt} i(0^+)$

Verifica con Trasformata di Laplace

$t > 0$



$$V(s) = \frac{\frac{2}{s} \cdot \frac{3}{10} + \frac{2}{s} - \frac{2}{3} \frac{1}{s+6}}{\frac{s}{10} + \frac{1}{s+6}}$$

$= \ldots$

$\frac{2}{s} = I_1(s) + I(s)$

$s\,I(s) - \frac{2}{3} + 6 \cdot I(s) - \frac{2}{s} - \frac{10}{s} I_1(s) = 0$.

$s\,I(s) - \frac{2}{3} + 6 \cdot I(s) - \frac{2}{s} - \frac{10}{s}\left(\frac{2}{s} - I(s)\right) = 0$

$I(s)\left(s + 6 + \frac{10}{s}\right) - \frac{2}{3} - \frac{2}{s} - \frac{20}{s^2} = 0$

$$I(s) = \frac{\frac{2}{3} + \frac{2}{s} + \frac{20}{s^2}}{\frac{s^2 + 6s + 10}{s}} = \frac{\frac{2s^2 + 6s + 60}{3s^2}}{\frac{s^2 + 6s + 10}{s}} = \frac{2s^2 + 6s + 60}{3s(s^2 + 6s + 10)}.$$

$$I(s) = \frac{2}{3} \cdot \underbrace{\frac{s^2 + 3s + 30}{s(s^2 + 6s + 10)}}_{F(s)}$$

$s_{1,2} = -3 \pm \sqrt{-1} = -3 \pm j$

OK sono le radici caratteristiche

$$F(s) = \frac{A}{s} + \frac{B}{s - (-3 + j)} + \frac{B^*}{s - (-3 - j)}$$

$A = \lim_{s \to 0} s F(s) = 3$

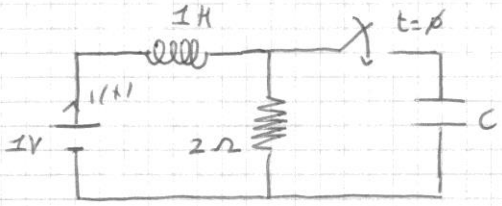
$B = \lim_{s \to -3+j} (s - (-3 + j)) F(s) = \ldots = -1 + j \frac{9}{2}$

$i(t) = 2 \cdot u(t) + \frac{1}{3} \cdot 2 \operatorname{Re}\left[B e^{(-3+j)t}\right] u(t)$

$$= 2u(t) + \frac{4}{3} Re\left[\left(-1 + j\frac{9}{2}\right) e^{jt}\right] e^{-3t} \cdot u(t)$$

$$= 2u(t) + \frac{4}{3} e^{-3t} \left(-\cos(t) - \frac{9}{2} \operatorname{sen}(t)\right) u(t)$$

Esercizio



$t>0$, $\omega_0 = 2\alpha$

C inizialmente scarico

? $i(t)$, $t>0$ nel DT.

$t>0$: RLC Parallelo.

$$\omega_0 = \frac{1}{\sqrt{LC}} = 2\alpha_{\parallel}, \quad \text{dove} \quad \alpha_{\parallel} = \frac{G_p}{2C} = \frac{1/2}{2C} = \frac{1}{4C}$$

$$\cancel{2} \cdot \frac{1}{{}_2\cancel{4}C} = \frac{1}{\sqrt{C}} \to C = \frac{1}{4} F$$

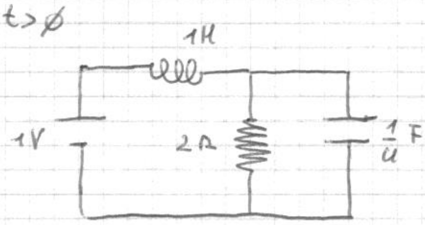
$t = 0^-$

$V(0^-) = 0 \quad = V(0^+)$

$i(0^-) = \frac{1}{2} A \quad = i(0^+)$

$i(t) = TC + R.\ staz.$, $R.\ staz = \frac{1}{2} A$.

$t>0$



$G_p < G_c = 2\sqrt{\frac{C}{L}}$

$\frac{1}{2} < 2\sqrt{\frac{1/4}{1}} \to 1/2 < 1.$

Caso sottosmorzato

$S_{1,2} = -\alpha_{\parallel} \pm j\beta$

$\alpha_{\parallel} = \frac{G_p}{2C} = \frac{1/2}{1/2} = 1 \text{ rad/sec}$

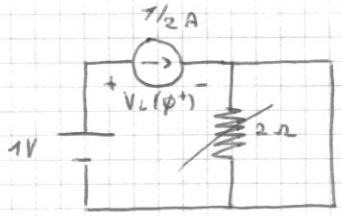
$\omega_0 = \frac{1}{\sqrt{LC}} = 2 \text{ rad/sec}$

$\to S_{1,2} = -1 \pm \sqrt{3}\, j$

$\beta = \sqrt{\omega_0^2 - \alpha_{\parallel}^2} = \sqrt{3} \text{ rad/sec}$

$$\begin{cases} i(t) = K_1 e^{-t} \cos(\sqrt{3}t) + K_2 e^{-t} \operatorname{sen}(\sqrt{3}t) + 1/2 & , t > \emptyset \\ i(\emptyset^+) = 1/2 \\ \frac{d}{dt} i(\emptyset^+) = ? = 1 \end{cases}$$

$t = \emptyset^+$



$$V_L(\emptyset^+) = L \cdot \frac{d}{dt} i(\emptyset^+)$$

$$= \frac{d}{dt} i(\emptyset^+) = 1$$

$$\frac{1}{2} = K_1 + \frac{1}{2} \quad \rightarrow K_1 = \emptyset$$

$$-K_1 + \sqrt{3} \cdot K_2 = 1 \quad \rightarrow K_2 = \frac{1}{\sqrt{3}}.$$

$$i(t) = \frac{1}{\sqrt{3}} e^{-t} \operatorname{sen}(\sqrt{3}t) + \frac{1}{2}, \quad t > \emptyset.$$

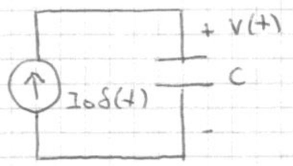
Con Laplace : Ricontrolla!

$$\underline{I}(s) = \frac{1}{2} \cdot \frac{1}{s} \cdot \frac{s^2 + 4s + 4}{s^2 + 2s + 4}$$

$$i(\infty) = \lim_{s \to 0} s\underline{I}(s) = \frac{4}{8} = \frac{1}{2}.$$

22/05/2015

C con eccitazione impulsiva



$v(\varnothing^-) = \varnothing$.

$[I_0] = A \cdot sec = $ Coulomb

$i(t) = C \cdot \frac{d}{dt} v(t) = I_0\, \delta(t)$

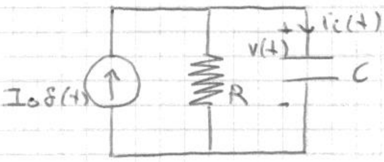
$C \int_{\varnothing^-}^{\varnothing^+} \frac{d}{dt} v(t) = I_0 \int_{\varnothing^-}^{\varnothing^+} \delta(t)\, dt$

$C\,[\,v(\varnothing^+) - \underbrace{\cancel{v(\varnothing^-)}}_{\varnothing}\,] = I_0 \cdot 1 \quad \rightarrow \quad v(\varnothing^+) = \frac{I_0}{C}$ Volt

c'è quindi una discontinuità.

Impulso di corrente in C $\Rightarrow$ discontinuità var. di stato

RC con eccitazione impulsiva



$v(\varnothing^-) = \varnothing$? $v(t)$, $t > \varnothing^-$

? $i_c(t)$

il condensatore è scarico (c.c.) quindi la corrente tenderà a passare di lì

$I_0\delta(t)$, $t > \varnothing^-$

$I_0\delta(t) = C \cdot \frac{d}{dt} v(t) + \frac{v(t)}{R}$, $t > \varnothing^-$

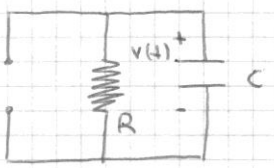
$C\,[\,v(\varnothing^+) - \underbrace{\cancel{v(\varnothing^-)}}_{\varnothing}\,] + \frac{1}{R} \underbrace{\cancel{\int_{\varnothing^-}^{\varnothing^+} v(t)\, dt}}_{\varnothing} = I_0 \cdot 1$

funzione limitata integrata fra $(-\varepsilon, \varepsilon)$.

$v(\varnothing^+) = \frac{I_0}{C}$ Volt

$(0^-, 0^+) \rightarrow v(0^+) = \frac{I_0}{C}$

$t > 0$



$v(t) = v(0^+)\, e^{-t/RC}$, $t > 0$

C si scarica su R.

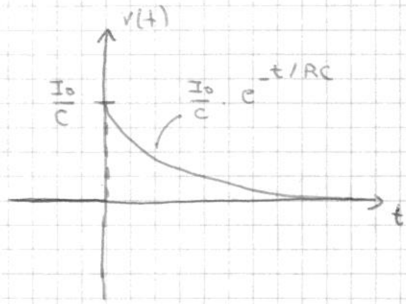
$$v(t) = \frac{I_0}{C} \cdot e^{-t/RC} \cdot u(t)$$

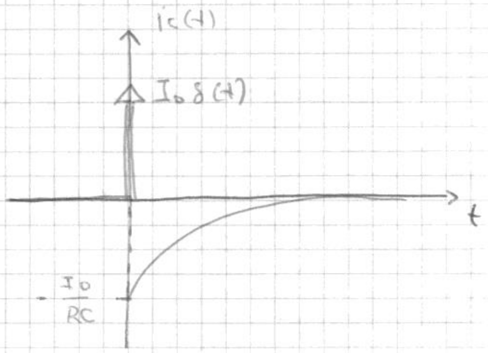
$$i_C(t) = C \cdot \frac{d}{dt} v(t) = \cancel{C} \cdot \frac{d}{dt}\left(\frac{I_0}{\cancel{C}} \cdot e^{-t/RC} \cdot u(t) \right)$$

derivata nel senso delle funzioni impulsive

$$= -\frac{I_0}{RC} \cdot e^{-t/RC} \cdot u(t) + I_0\, \delta(t)$$

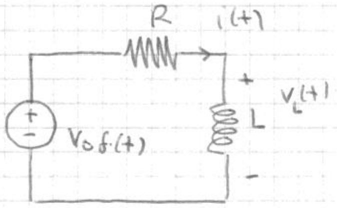
in quanto v(t) discontinua in 0.





Inizialmente tutta la corrente passa su C, poi C si scarica su R.

RL con eccitazione impulsiva



$i(0^-) = 0$ $\quad$? $i(t)$, $V_L(t)$

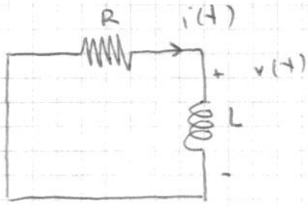
$(0^-, 0^+) \longrightarrow V_L(t) \simeq V_0\,\delta(t)$ $\quad$ in quanto L scarico è un c.a.

$$L \cdot \frac{d}{dt} i(t) \simeq V_0\,\delta(t)$$

$$L\,[i(0^+) - i(0^-)] \simeq V_0 \cdot 1$$

$$i(0^+) \simeq \frac{V_0}{L}$$

$t > 0$



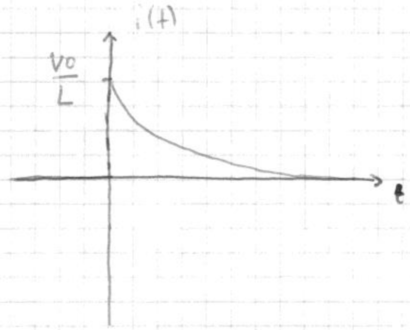
Scarico di L su R

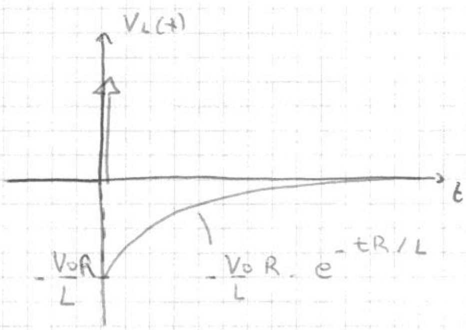
$$i(t) = i(0^+) \cdot e^{-t \cdot R/L}, \quad t > 0$$

$$= \frac{V_0}{L} \cdot e^{-tR/L}, \quad t > 0$$

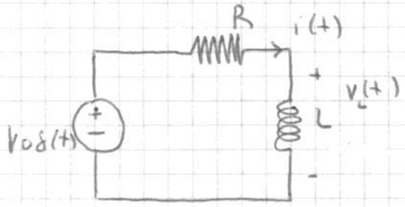
$$i(t) = \frac{V_0}{L} \cdot e^{-tR/L} \cdot u(t)$$

$$V_L(t) = L \cdot \frac{d}{dt} i(t) = V_0\,\delta(t) - \frac{V_0}{L} \cdot R \cdot e^{-tR/L} \cdot u(t)$$





Verifica con T. L.



$i(\phi^-) = \phi$

$L \cdot \frac{d}{dt} i + Ri = V_0 \, \delta(t) \quad , \quad t > \phi$

$$\int_{\phi^-}^{+\infty} \left(L \cdot \frac{d}{dt} i + Ri \right) e^{-st} \, dt = \int_{\phi^-}^{+\infty} V_0 \delta(t) \, e^{-st} \, dt$$

$$sL\, I(s) - L \underbrace{\cancel{i(\phi^-)}}_{\phi} + R I(s) = V_0$$

$$(sL + R)\, I(s) = V_0$$

$$I(s) = \frac{V_0}{sL + R} = \frac{V_0}{L} \cdot \frac{1}{s + \frac{R}{L}}$$

$$i(t) = \frac{V_0}{L} \, e^{-tR/L} \cdot u(t)$$, verificato!

$$i(\phi^+) = \lim_{s \to \infty} s I(s) = \frac{V_0}{L}.$$

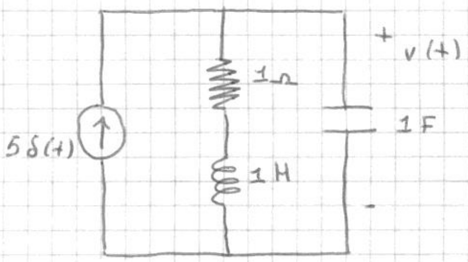
$$v(t) = L \cdot \frac{d}{dt} i(t)$$

$$V(s) = s \cdot L\, I(s) - L \underbrace{\cancel{i(\phi^-)}}_{\phi}$$

$$V(s) = V_0 \cdot \frac{s}{s + \frac{R}{L}} = V_0 \left(1 - \frac{\frac{R}{L}}{s + \frac{R}{L}} \right)$$

$$v(t) = V_0\, \delta(t) - V_0 \frac{R}{L} \, e^{-tR/L} \cdot u(t)$$, verificato!

16 / 12 / 1996 Per Casa.



? $v(t)$, $t > \phi$ risp. forzata

$v(\phi^-) = \phi$

$i_L(\phi^-) = \phi$

1) D. T.

2) T. L.

$$v(t) = \frac{10}{\sqrt{3}} \, e^{-\frac{1}{2}t} \cos\left(\frac{\sqrt{3}}{2} t - \frac{\pi}{6} \right).$$

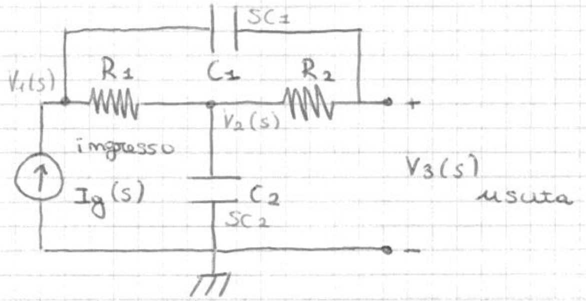
Funzioni di Rete di un circuito LTI

1 ingresso, tensione impressa da gen. re indip. di tensione o corrente impressa da gen. re indip. di corrente.

1 uscita: tensione o corrente che interessa

$$H(s) = \frac{\text{T.L. uscita (risp. forzata)}}{\text{T.L. ingresso}}$$

Li, Ci, scarichi a $t = \emptyset^-$.



? $H(s) = \dfrac{V_3(s)}{I_g(s)}$

$$\vec{\vec{Y}}(s) \cdot \vec{V}(s) = \vec{I}(s)$$

↑

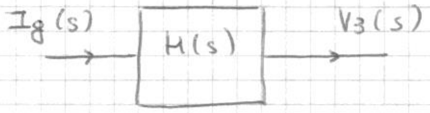
matrici ammettenze

$$\begin{pmatrix} G_1+sC_1 & -G_1 & -sC_1 \\ -G_1 & G_1+G_2+sC_2 & -G_2 \\ -sC_1 & -G_2 & G_1+sC_1 \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_2(s) \\ V_3(s) \end{pmatrix} = \begin{pmatrix} I_g(s) \\ \emptyset \\ \emptyset \end{pmatrix}$$

$$V_3(s) = \frac{\det\begin{pmatrix} G_1+sC_1 & -G_1 & I_g(s) \\ -G_1 & G_1+G_2+sC_2 & \emptyset \\ -sC_1 & -G_2 & \emptyset \end{pmatrix}}{\det \vec{\vec{Y}}(s)}$$

$$= I_g(s) \cdot \underbrace{\frac{\det\begin{pmatrix} -G_1 & G_1+G_2+sC_2 \\ -sC_1 & -G_2 \end{pmatrix}}{\det \vec{\vec{Y}}(s)}}_{H(s)}$$

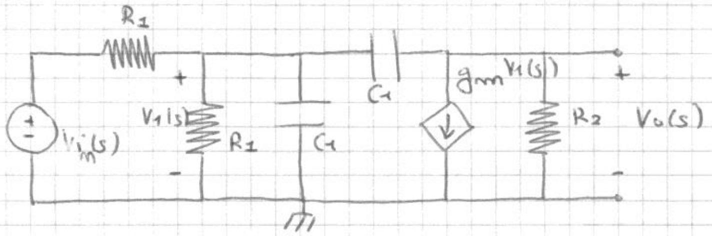
$$H(s) = \frac{\det(\ldots)}{\det \vec{\vec{Y}}(s)} = \frac{s^2 + \left(\frac{G_1}{C_2} + \frac{G_2}{C_2}\right)s + \frac{G_1G_2}{C_1C_2}}{(G_1+G_2)s^2 + \frac{G_1G_2}{C_1}s}$$



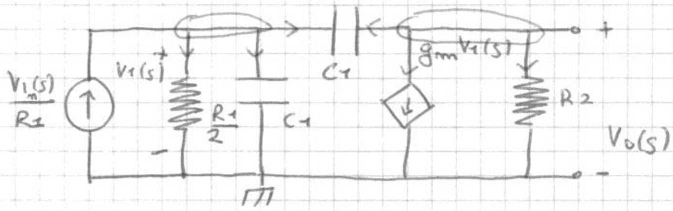
$H(s) = Z_T(s)$ impedenza di Trasferimento

Funzione di Rete: razionali, a coefficienti reali e non dipendono da grandezza imposta dall'eccitazione.

Es. Transistor ad alta frequenza.



$$H(s) = \frac{V_o(s)}{V_i(s)} = T_V(s)$$



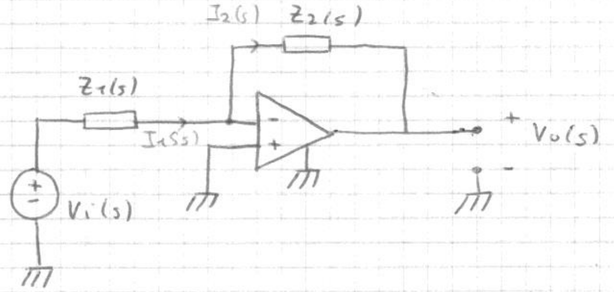
$$\begin{cases} V_1(s)\dfrac{R_1}{2} + V_1(s)sC_1 + (V_1(s) - V_0(s))sC_1 = \dfrac{V_{in}(s)}{R_1} \\ (V_0(s) - V_1(s))sC_1 + g_m V_1(s) + \dfrac{V_0(s)}{R_2} = \emptyset \end{cases}$$

$$\begin{pmatrix} \dfrac{2}{R_1} + 2sC_1 & -sC_1 \\ -sC_1 + g_m & sC_1 + \dfrac{1}{R_2} \end{pmatrix} \begin{pmatrix} V_1(s) \\ V_0(s) \end{pmatrix} = \begin{pmatrix} \dfrac{V_{IN}(s)}{R_1} \\ \emptyset \end{pmatrix}$$

$$H(s) = \frac{V_0(s)}{V_{in}(s)} = \frac{\dfrac{C_1}{R_1}s - \dfrac{g_m}{R_1}}{3C_1^2 s^2 + \left(2\dfrac{C_1}{R_1} + C_1 g_m\right)s + \dfrac{2}{R_1 R_2}}$$

Per ogni valore dei parametri il filtro è asintoticamente stabile.

Funzioni di Rete circuiti con A. O.



$$? \; T_V(s) = \frac{V_o(s)}{V_i(s)} = - \frac{Z_2(s)}{Z_1(s)}$$

$$I_1(s) = I_2(s)$$

$$\frac{V_i(s) - \emptyset}{Z_1(s)} = \frac{\emptyset - V_o(s)}{Z_2(s)}$$

$$Z_1(s) = R, \quad Z_2(s) = C$$

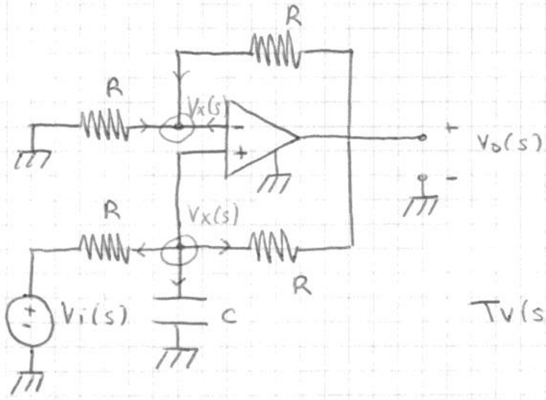
$$T_V(s) = - \frac{Z_2(s)}{Z_1(s)} = - \frac{1}{SCR} = \frac{V_o(s)}{V_i(s)}$$

$$V_o(s) = - \frac{1}{SRC} \cdot V_i(s)$$

$$V_o(t) = - \frac{1}{RC} \int V_i(t)\, dt$$: Integratore invertente.

Se scambio R con C ottengo un derivatore.

Integratore non Invertente (di DEBOO).



$$T_V(s) = \frac{V_o(s)}{V_i(s)} = \frac{2}{RCS}.$$

2 incognite: $V_0(s)$, $V_x(s)$

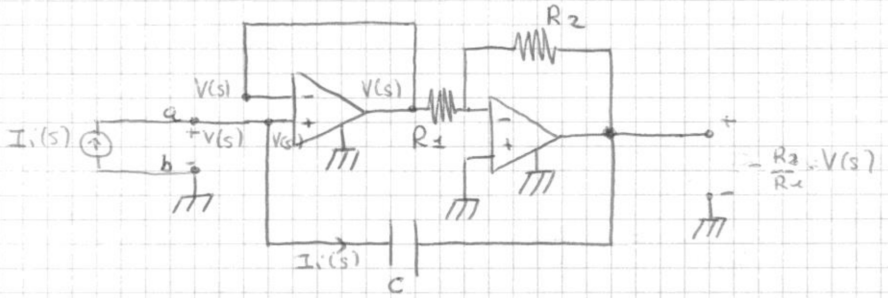
2 KI.

$$V_x(s) = \frac{V_0(s)}{2}$$

$$\frac{\frac{V_0(s)}{2} - V_i(s)}{R} + \frac{\frac{V_0(s)}{2} - V_0}{R} + \frac{V_0}{2} \cdot SC = \emptyset$$

$$V_0(s) = \frac{2}{RCS} \cdot V_i(s)$$

moltiplicatore di capacità



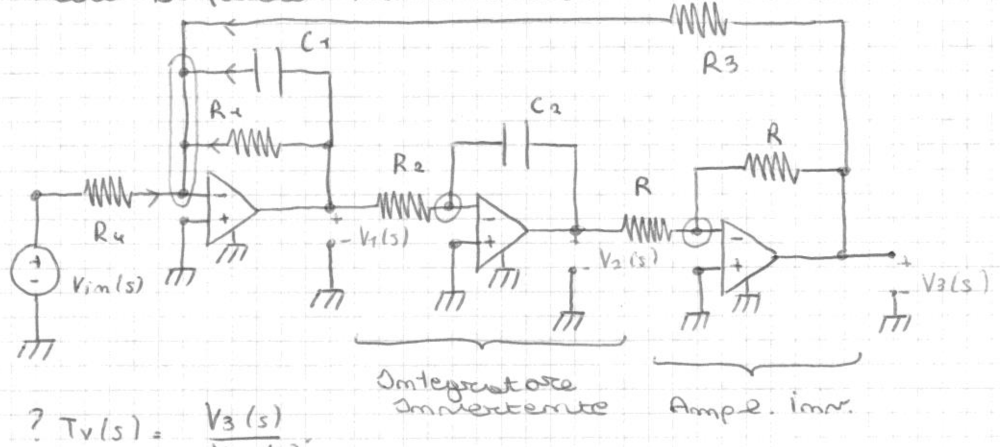
$? \; Z(s) = \frac{V(s)}{I(s)}$

$$I_i(s) = \left[V(s) - \left(-\frac{R_2}{R_1} V(s)\right)\right] SC = V(s)\left[1 + \frac{R_2}{R_1}\right] SC$$

$$Z(s) = \frac{V(s)}{I(s)} = \frac{1}{\left(1 + \frac{R_2}{R_1}\right) SC} = \frac{1}{S \cdot C_{eq}}, \quad C_{eq} = \left(1 + \frac{R_2}{R_1}\right) \cdot C.$$

mantengo la corrente su C costante ma aumento la d.d.p. ai capi di C.

Filtro Biquad Passa Basso.



? $T_v(s) = \frac{V_3(s)}{V_{in}(s)}$

3 incognite: $V_1(s), V_2(s), V_3(s)$

3 KI

$$\begin{cases} V_2(s) = -\frac{1}{sR_2C_2} V_1(s) \quad \rightarrow \quad V_1 = -sR_2C_2V_2 = sR_2C_2V_3 \\ V_3(s) = -V_2(s) \\ \frac{V_{in}(s)}{R_4} + \frac{V_1(s)}{R_1} + V_1(s) \cdot sC_1 + \frac{V_3(s)}{R_3} = \emptyset \end{cases}$$

$$H(s) = T_v(s) = -\frac{1}{R_2R_4C_1C_2} \cdot \frac{1}{s^2 + s \cdot \frac{1}{R_1C_1} + \frac{1}{R_2R_3C_1C_2}}$$

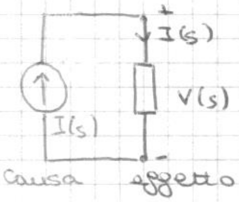
Filtro stabile!

$T_v(s) = -\frac{K}{s^2+as+b}$ filtro passa basso del secondo ordine

$\frac{V_2(s)}{V_{in}(s)}$: Filtro Passa Banda!

Cenni su Bipoli 27/05/2015

L_i, C_i scarichi a $t = 0^-$.



$Z(s) = \frac{V(s)}{I(s)}$, $V(s) = Z(s) \cdot I(s)$

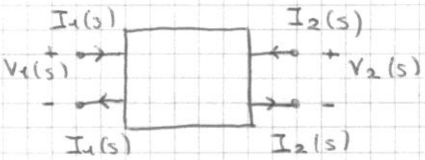
contr. in corrente

relazione costitutiva

Se contr. in tensione : $Y(s) = \frac{I(s)}{V(s)}$

$I(s) = Y(s) \cdot V(s)$.

Quadripolo (Rete 2 Porte)



Grandezze di Porta : $V_1(s)$, $V_2(s)$, $I_1(s)$, $I_2(s)$

Relazioni costitutive reti due porte:

2 legami indip. ti che la rete stabilisce fra le quattro grandezze di porta V_1, V_2, I_1, I_2.

Grandezze ind. ti : I_1, I_2
(Cause)

Grandezze dip. ti : V_1, V_2.
(Effetti)

$$\begin{cases} V_1(s) = Z_{11}(s) I_1(s) + Z_{12}(s) \cdot I_2(s) \\ V_2(s) = Z_{21}(s) I_1(s) + Z_{22}(s) \cdot I_2(s) \end{cases}$$

Rappresentazione a parametri Z.

sovrapposizione degli effetti

Cause : V_1, V_2
Effetti : I_1, I_2

$$\begin{cases} I_1(s) = Y_{11}(s) \cdot V_1(s) + Y_{12}(s) \cdot V_2(s) \\ I_2(s) = Y_{21}(s) \cdot V_1(s) + Y_{22}(s) \cdot V_2(s) \end{cases}$$

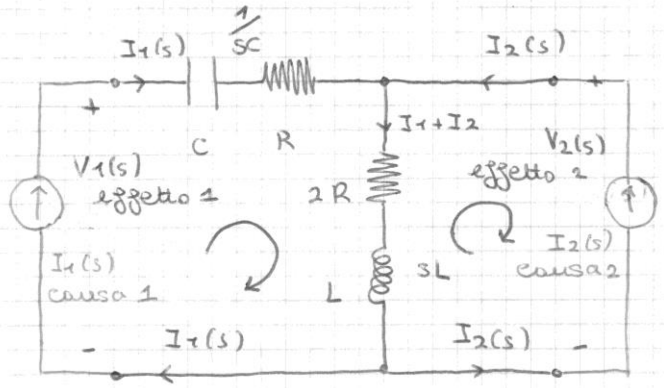
Rappresentazione a parametri Y.

Cause : I_1, V_2
Effetti: V_1, I_2

$$\begin{cases} V_1 = R_{11} \cdot I_1 + R_{12} \cdot V_2 \\ I_2 = R_{21} \cdot I_1 + R_{22} \cdot V_2 \end{cases}$$

Rappresentazione a parametri R.

Esercizio :



È una rete due porte per definizione.

? Rappr. Param. Z.

$$\begin{cases} V_1(s) = \left(\frac{1}{sC} + R\right) I_1 + (2R + sL)(I_1 + I_2) \\ V_2(s) = (2R + sL)(I_1 + I_2) \end{cases}$$

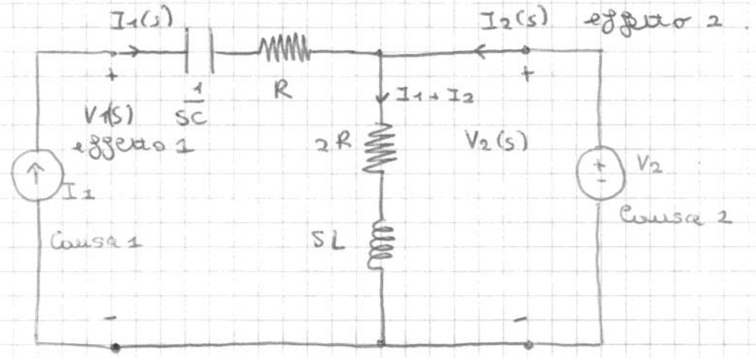
$$V_1(s) = \underbrace{\left(3R + sL + \frac{1}{sC}\right)}_{Z_{11}(s)} I_1 + \underbrace{(2R + sL)}_{Z_{12}(s)} I_2$$

$$V_2(s) = \underbrace{(2R + sL)}_{Z_{21}(s)} I_1 + \underbrace{(2R + sL)}_{Z_{22}(s)} \cdot I_2$$

mat. Z simmetrica

? Rappr. Param. h

Cause: I_1, V_2



$$\begin{cases} V_1(s) = \underbrace{\left(\frac{1}{sC} + R\right)}_{h_{11}} I_1 + \underbrace{1}_{h_{12}} \cdot V_2 \\ I_2(s) = \underbrace{-1}_{h_{21}} \cdot I_1 + \underbrace{\frac{1}{sL+2R}}_{h_{22}} \cdot V_2 \end{cases}$$

mat. h antisimmetrica

Parametri Z (di circuito aperto)

Cause: I_1, I_2

Effetti: V_1, V_2

$$\begin{cases} V_1 = Z_{11}(s) \cdot I_1 + Z_{12}(s) \cdot I_2 \\ V_2 = Z_{21}(s) \cdot I_1 + Z_{22}(s) \cdot I_2 \end{cases}$$

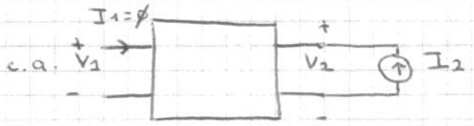
$$\begin{pmatrix} V_1 \\ V_2 \end{pmatrix} = \begin{pmatrix} Z_{11}(s) & Z_{12}(s) \\ Z_{21}(s) & Z_{22}(s) \end{pmatrix} \begin{pmatrix} I_1 \\ I_2 \end{pmatrix}$$

$$\vec{V} = \vec{\vec{Z}}(s) \cdot \vec{I}$$

Definizione Operativa:

$Z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=\emptyset}$ impedenza alla porta 1 con porta 2 a c.a.

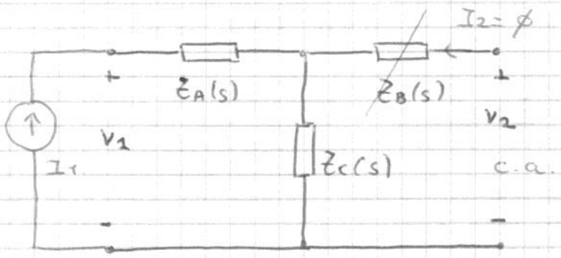
$Z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=\emptyset}$ impedenza di trasferimento da porta 1 a porta 2 con porta 2 a c.a.



$$Z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=\emptyset}$$

$$Z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=\emptyset}$$

Impedenze a T



$$Z_{11}(s) = \left.\frac{V_1}{I_1}\right|_{I_2=\emptyset}$$

$$Z_{21}(s) = \left.\frac{V_2}{I_1}\right|_{I_2=\emptyset}$$

$$Z_{11}(s) = Z_A(s) + Z_C(s)$$

$$Z_{21}(s) = Z_C(s)$$

$$Z_{22}(s) = \left.\frac{V_2}{I_2}\right|_{I_1=\emptyset} = Z_B(s) + Z_C(s)$$

$$Z_{12}(s) = \left.\frac{V_1}{I_2}\right|_{I_1=\emptyset} = Z_C(s)$$

$$\underline{\underline{Z}}_{\text{"T"}} = \begin{pmatrix} Z_A + Z_C & Z_C \\ Z_C & Z_B + Z_C \end{pmatrix}$$

$\underline{\underline{Z}}^t = \underline{\underline{Z}}$: simmetrica

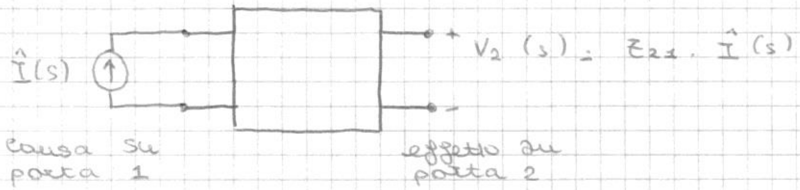
La rete che ha $\underline{\underline{Z}}$ simmetrica è una rete di tipo reciproco (rispetto ai parametri z).

Proprietà:

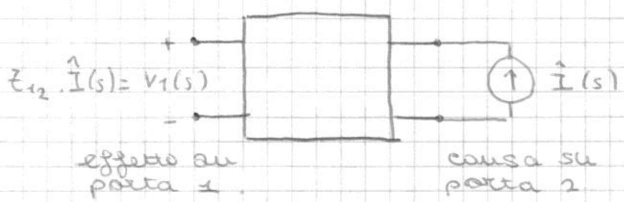
Una rete due porte che è costituita da soli bipoli è necessariamente RECIPROCA.

Rete Reciproca:

$$Z_{21} = \frac{V_2}{I_1}\Bigg|_{I_2=\emptyset} = Z_{12} = \frac{V_1}{I_2}\Bigg|_{I_1=\emptyset}$$



$Z_{12} = Z_{21}$

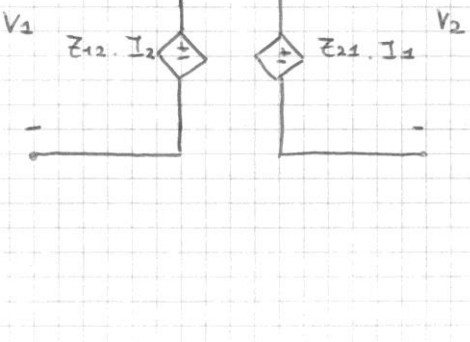


"In una rete reciproca l'effetto non cambia, scambiando fra di loro, posizione di causa ed effetto".

Circuito Equivalente a Parametri z.

$$\begin{cases} V_1 = Z_{11} \cdot I_1 + Z_{12} \cdot I_2 \\ V_2 = Z_{21} \cdot I_1 + Z_{22} \cdot I_2 \end{cases}$$

Relazioni costitutive.

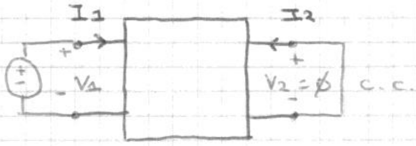


Parametri y (di corto circuito)

Cause: V_1, V_2

Effetti: I_1, I_2

$$\begin{cases} I_1 = y_{11} V_1 + y_{12} V_2 \\ I_2 = y_{21} V_1 + y_{22} V_2 \end{cases}$$



$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=\emptyset}$

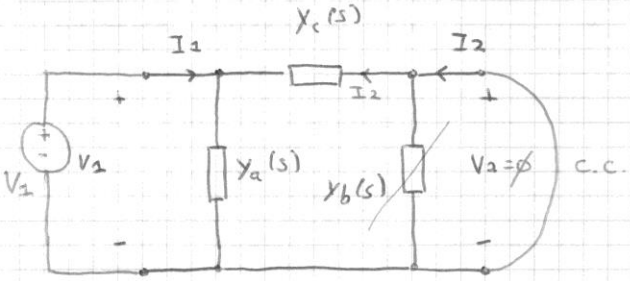
$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=\emptyset}$

ammettenza di trasferimento da porta 1 a porta 2

$y_{22} = \left.\frac{I_2}{V_2}\right|_{V_1=\emptyset}$

$y_{12} = \left.\frac{I_1}{V_2}\right|_{V_2=\emptyset}$

Impedenze a "π" (Triangolo)



$y_{11} = \left.\frac{I_1}{V_1}\right|_{V_2=\emptyset} = y_a + y_c$

$y_{21} = \left.\frac{I_2}{V_1}\right|_{V_2=\emptyset} = -y_c$

con $I_2 = -y_c \cdot V_1$

Poi, invertendo:

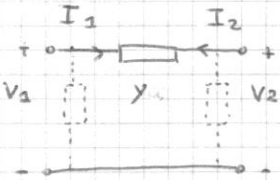
$y_{22} = y_b + y_c$

$y_{12} = -y_c.$

$$\underline{\underline{Y}}_{''\pi''} = \begin{pmatrix} y_a + y_c & -y_c \\ -y_c & y_b + y_c \end{pmatrix}$$

Reciproca $\Leftrightarrow \underline{\underline{Y}} = \underline{\underline{Y}}^t$.

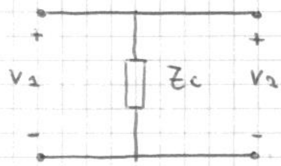
Inoltre: $\underline{\underline{Z}} = \underline{\underline{Y}}^{-1}$.



$I_1 = -I_2$, $\nexists$ Parametri Z

I_1 e I_2 dip. ti

$$\exists \; \underline{\underline{Y}} = \begin{pmatrix} y_c & -y_c \\ -y_c & y_c \end{pmatrix}$$



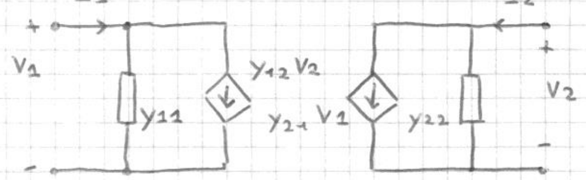
$\exists \; \underline{\underline{Z}}$, $\nexists \; \underline{\underline{Y}}$. in quanto V_1 e V_2 dip. ti

Trasformatore Ideale

$$\begin{cases} V_1 = n V_2 \\ I_1 = -\dfrac{1}{n} I_2 \end{cases}$$

$\nexists \; \underline{\underline{Z}}$, $\nexists \; \underline{\underline{Y}}$. in quanto V_1 e V_2 e I_1 e I_2 dip. ti.

Circuito Equivalente a Parametri y.



$$\begin{cases} I_1 = y_{11} V_1 + y_{12} V_2 \\ I_2 = y_{21} V_1 + y_{22} V_2 \end{cases}$$

Stabilità Rispetto alla funzione di rete H(s)

$$H(s) = \frac{N(s)}{D(s)}$$

Asintoticamente Stabile
$\Leftrightarrow$ tutti i poli di $H(s)$ hanno $Re < \emptyset$.

Stabilità Marginale
$\Leftrightarrow$ tutti i poli di $H(s)$ hanno $Re \leq \emptyset$ e quelli con $Re = \emptyset$ sono semplici.

Instabile in tutti gli altri casi.

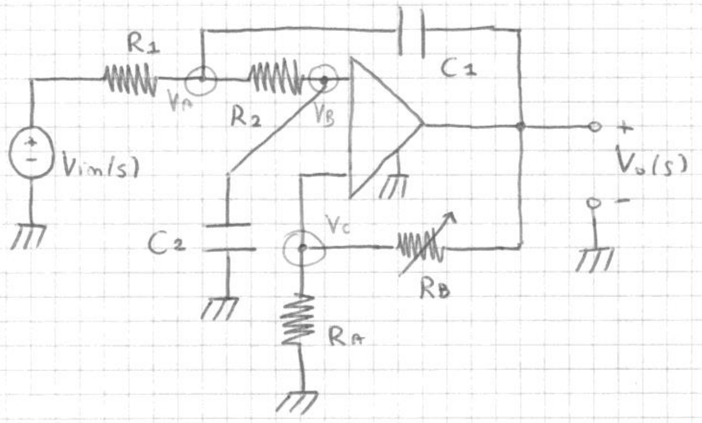
La stabilità è una proprietà intrinseca di tutti i circuiti passivi RLC.

Proprietà: Sia $H(s)$ una funzione di rete di un circuito passivo RLC.
I poli di $H(s)$ possono essere dei seguenti tipi:
1) Poli a $Re < \emptyset$ (anche multipli);
2) Poli a $Re = \emptyset$ (necessariamente semplici)
3) Polo per $s \to \infty$ (necessariamente semplice)
esempio: $\frac{s^2}{s+1} \sim s$

Perché non ci può essere un polo doppio in ∞?
Perché se metto $z = \frac{1}{s}$, riporto tutto sullo $\emptyset$ (cambio L con C) ma non ci può essere un polo doppio in $\emptyset$.

Per un circuito attivo la stabilità non è garantita a priori.

Filtro del II° Ordine con A.O.



$$? \; T_V(s) = \frac{V_o(s)}{V_{in}(s)}$$

$$K = 1 + \frac{R_B}{R_A}.$$

$$H(s) = \frac{K \cdot \frac{1}{R_1R_2C_1C_2}}{s^2 + s\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - K \cdot \frac{1}{R_2C_2}\right) + \frac{1}{R_1R_2C_1C_2}}$$

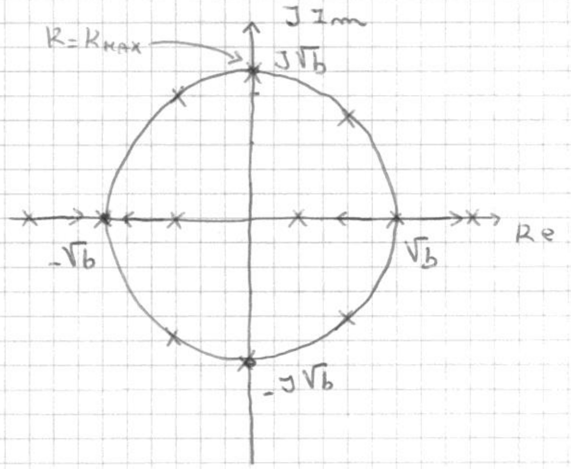
$$K_{MAX} = \frac{\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2}}{\frac{1}{R_2C_2}}$$

$K < K_{MAX}$: 2 Poli a $Re < \varnothing$ Asint. Stab.

$K = K_{MAX}$: 2 Poli a $Re = \varnothing$ semplici Marg. Stab.

$K > K_{MAX}$: 2 Poli a $Re > \varnothing$ Instab.

$$b = -\frac{1}{R_1R_2C_1C_2}$$



$$V_B = V_0 \cdot \frac{R_A}{R_A + R_B} = \frac{V_0}{K}$$

$$V_B = V_A \cdot \frac{\frac{1}{SC_2}}{\frac{1}{SC_2} + R_2} = \frac{V_A}{1 + SC_2R_2}$$

$$V_A = V_B(1 + SR_2C_2) = (1 + SR_2C_2)\frac{V_0}{K}$$

$$\frac{V_A - V_{in}}{R_1} + \frac{V_A - V_B}{R_2} + (V_A - V_0)SC_1 = \emptyset$$

$$V_A\left(\frac{1}{R_1} + \frac{1}{R_2} + SC_1\right) - \frac{V_B}{R_2} - V_0SC_1 = \frac{V_{in}}{R_1}$$

$$(1 + SR_2C_2)\left(\frac{1}{R_1} + \frac{1}{R_2} + SC_1\right)\frac{V_0}{K} - \frac{V_0}{KR_2} - V_0SC_1 = \frac{V_{in}}{R_1}$$

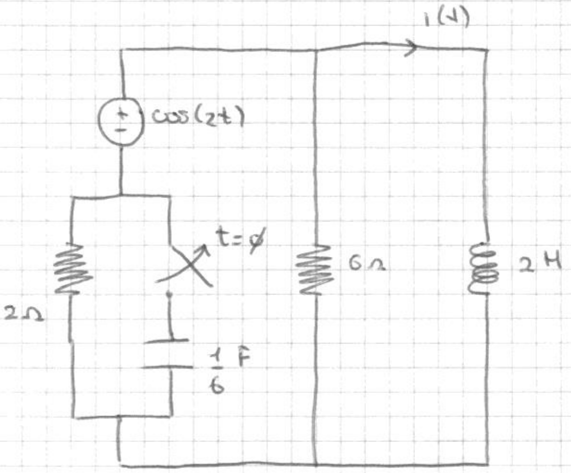
$$V_0\left[S^2R_2C_1C_2 + SR_2C_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + \frac{1}{R_1} + \frac{1}{R_2} + SC_1 - \frac{1}{R_2} - KSC_1\right] = K\frac{V_{in}}{R_1}$$

$$\frac{V_0}{V_{in}} = \frac{\frac{K}{R_1}}{S^2R_2C_1C_2 + S\left(R_2C_2\left(\frac{1}{R_1} + \frac{1}{R_2}\right) + C_1 - KC_1\right) + \frac{1}{R_1}}$$

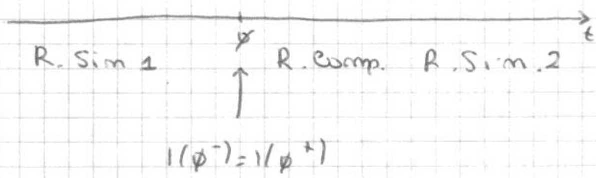
$$\frac{V_0}{V_{in}} = \frac{K}{R_1R_2C_1C_2} \cdot \frac{1}{S^2 + S\left(\frac{1}{R_1C_1} + \frac{1}{R_2C_1} + \frac{1}{R_2C_2} - \frac{K}{R_2C_2}\right) + \frac{1}{R_1R_2C_1C_2}}$$

$$\frac{V_0}{V_{in}} = \frac{K}{S^2 + aS + b}$$

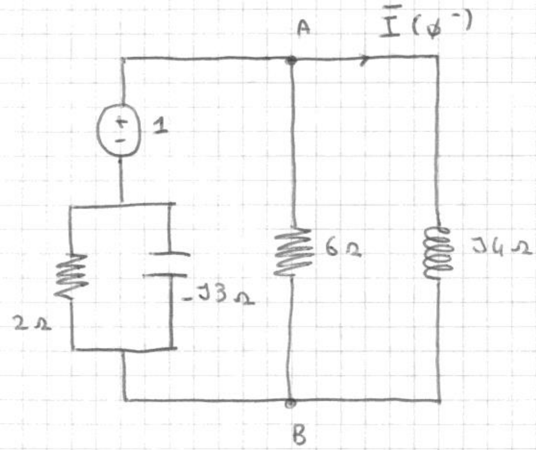
Es. 15/01/2009



Determinare i(t) , t ≥ ∅.



$t = \emptyset^-$



MILLMANN:

$$V_{AB} = \frac{1 \cdot \frac{1}{2 \,||\, -j3}}{\frac{1}{2 \,||\, -j3} + \frac{1}{6} + \frac{1}{j4}}$$

$$2 \,||\, -j3 = \frac{-2 \cdot j3}{2 - j3}$$

$$= \frac{-j6}{2-j3} \cdot \frac{2+j3}{2+j3} = \frac{18 - j12}{13}$$

$$V_{AB} = \frac{\frac{13}{18 - j12}}{\frac{13}{18-j12} + \frac{1}{6} + \frac{1}{j4}}$$

...

29/05/2015

Parametri Ibridi R

Cause : I_1 , V_2

Effetti : V_1 , I_2

$$\begin{cases} V_1 = R_{11} I_1 + R_{12} V_2 \\ I_2 = R_{21} I_1 + R_{22} V_2 \end{cases}$$

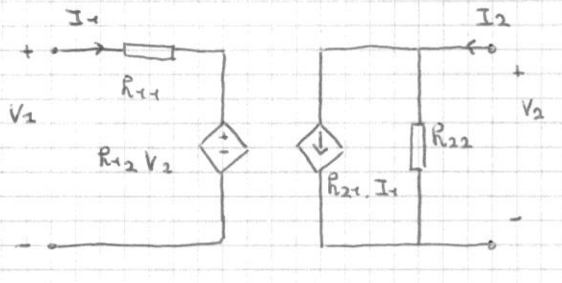
$$R_{11} = \left.\frac{V_1}{I_1}\right|_{V_2=\emptyset} = \left.\frac{1}{\frac{I_1}{V_1}}\right|_{V_2=\emptyset} = \frac{1}{y_{11}}$$

$$R_{21} = \left.\frac{I_2}{I_1}\right|_{V_2=\emptyset} = \left.\frac{I_2}{V_1}\cdot\frac{V_1}{I_1}\right|_{V_2=\emptyset} = \frac{y_{21}}{y_{11}}$$: funzione di trasferimento in corrente

$$R_{22} = \left.\frac{I_2}{V_2}\right|_{I_1=\emptyset} = \left.\frac{1}{\frac{V_2}{I_2}}\right|_{I_1=\emptyset} = \frac{1}{z_{22}}$$

$$R_{12} = \left.\frac{V_1}{V_2}\right|_{I_1=\emptyset} = \left.\frac{V_1}{I_2}\cdot\frac{I_2}{V_2}\right|_{I_1=\emptyset} = \frac{z_{12}}{z_{22}}$$ funzione di trasferimento in tensione.

Circuito Equivalente a Parametri R.



Reciprocità $\Leftrightarrow$ $R_{12} = -R_{21}$

Determinare $\vec{\vec{R}}$ in funzione di $\vec{\vec{Z}}$ e verificare che, in termini di R, la reciprocità equivale a $R_{12} = -R_{21}$.

$$\vec{\vec{Z}} = \begin{cases} V_1 = z_{11} I_1 + z_{12} I_2 \\ V_2 = z_{21} I_1 + z_{22} I_2 \end{cases} \rightarrow I_2 = \underbrace{-\frac{z_{21}}{z_{22}}}_{R_{21}} I_1 + \underbrace{\frac{1}{z_{22}}}_{R_{22}} V_2$$

$$\vec{\vec{R}} = \begin{cases} V_1 = R_{11} I_1 + R_{12} V_2 \\ I_2 = R_{21} I_1 + R_{22} V_2 \end{cases}$$

$$V_1 = Z_{11} I_1 - \frac{Z_{12} Z_{21}}{Z_{22}} I_1 + \frac{Z_{12}}{Z_{22}} V_2$$

$$\overrightarrow{\overrightarrow{h}} = \frac{1}{Z_{22}} \cdot \begin{pmatrix} \det \overrightarrow{\overrightarrow{Z}} & Z_{12} \\ -Z_{21} & 1 \end{pmatrix}$$

Rete Reciproca : $Z_{12} = Z_{21} \Rightarrow h_{12} = -h_{21}$.

Parametri di Trasmissione (Diretta)

Cause : V_2, $-I_2$

Effetti : V_1, I_1

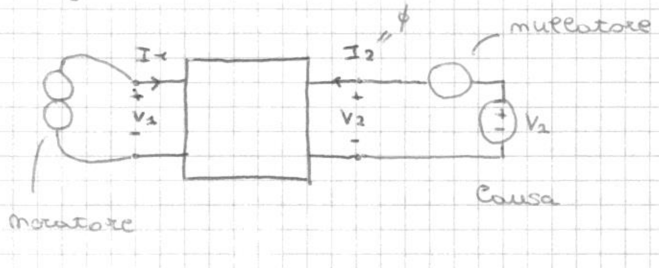
$$\begin{cases} V_1 = A V_2 + B(-I_2) \\ I_1 = C V_2 + D(-I_2) \end{cases}$$

Matrice di Trasmissione Diretta : $\overrightarrow{\overrightarrow{T}} = \begin{pmatrix} A & B \\ C & D \end{pmatrix}$

$A = \left.\frac{V_1}{V_2}\right|_{I_2 = \phi}$: funzione di trasferimento in tensione da P2 a P1.

$C = \left.\frac{I_1}{V_2}\right|_{I_2 = \phi}$: ammettenza // // da P2 a P1

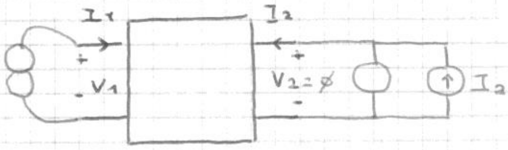
Def. Operativa :



$$B = \left.\frac{V_1}{-I_2}\right|_{V_2=\phi}$$: impedenza di trasferimento

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2=\phi}$$: funz. di trasferimento in corrente

Def. Operativa



Reciprocità : $\det \overline{\overline{T}} = AD - BC = 1.$

? $\overline{\overline{Z}}$ in funzione di $\overline{\overline{T}}$

$$\begin{cases} V_1 = AV_2 + B(-I_2) \\ I_1 = CV_2 + D(-I_2) \end{cases} \rightarrow V_2 = \underbrace{\frac{1}{C}}_{Z_{21}} I_1 + \underbrace{\frac{D}{C}}_{Z_{22}} I_2$$

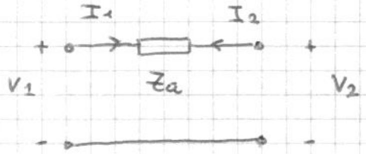
$$\begin{cases} V_1 = Z_{11} I_1 + Z_{12} I_2 \\ V_2 = Z_{21} I_1 + Z_{22} I_2 \end{cases}$$

$$V_1 = \frac{A}{C} \cdot I_1 + \frac{AD}{C} \cdot I_2 - BI_2$$

$$\overline{\overline{Z}} = \frac{1}{C} \begin{pmatrix} \cancel{\frac{1}{C}} A & \det \overline{\overline{T}} \\ 1 & D \end{pmatrix}$$

Reciprocità : $Z_{12} = Z_{21} \Rightarrow \det \overline{\overline{T}} = 1.$

Esercizio Teorico

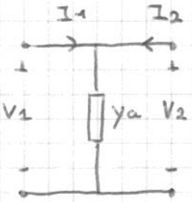


$$\overrightarrow{\overrightarrow{T}} = \begin{pmatrix} 1 & Z_a \\ \emptyset & 1 \end{pmatrix}$$

$V_1 = Z_a \cdot I_1 + V_2$.

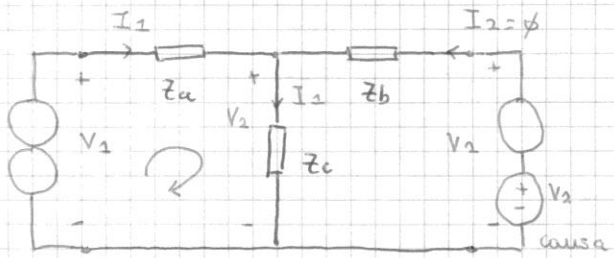
$$\begin{cases} V_1 = 1 \cdot V_2 + Z_a(-I_2) \\ I_1 = \emptyset \cdot V_2 + 1(-I_2) \end{cases}$$ ok !

Per casa :



verificare che $\overrightarrow{\overrightarrow{T}} = \begin{pmatrix} 1 & \emptyset \\ y_a & 1 \end{pmatrix}$

Esercizio Teorico :



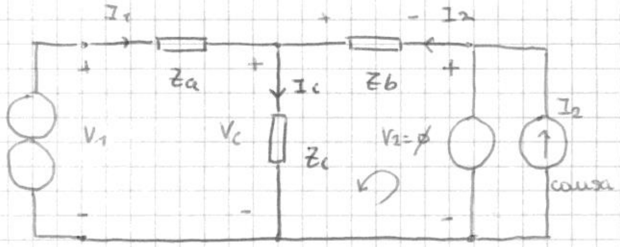
? ABCD con la def.

$$A = \left.\frac{V_1}{V_2}\right|_{I_2 = \emptyset} = 1 + \frac{Z_a}{Z_c}$$

$$C = \left.\frac{I_1}{V_2}\right|_{I_2 = \emptyset} = \frac{1}{Z_c}$$

$$I_1 = \frac{V_2}{Z_c}$$

$$V_1 = Z_a \cdot I_1 + V_2 = Z_a \cdot \frac{V_2}{Z_c} + V_2$$



$$B = \left.\frac{V_1}{-I_2}\right|_{V_2 = \emptyset} = Z_a + Z_b + \frac{Z_a Z_b}{Z_c}$$

$$D = \left.\frac{I_1}{-I_2}\right|_{V_2 = \emptyset} = 1 + \frac{Z_b}{Z_c}$$

$$V_c = -Z_b \cdot I_2$$

$$I_c = \frac{V_c}{Z_c} = -\frac{Z_b}{Z_c} \cdot I_2$$

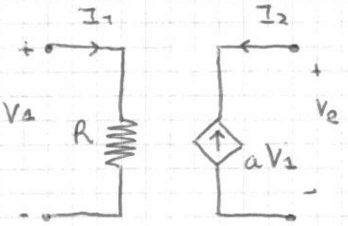
$$I_1 = I_c - I_2 = \left(1 + \frac{Z_b}{Z_c}\right)(-I_2)$$

$$V_1 = Z_a \cdot I_1 - Z_b \cdot I_2 = Z_a\left(1+\frac{Z_b}{Z_c}\right)(-I_2) - Z_b I_2$$

$$\overrightarrow{\overrightarrow{T}} = \begin{pmatrix} 1+\frac{Z_a}{Z_c} & Z_a + Z_b + \frac{Z_a Z_b}{Z_c} \\ \frac{1}{Z_c} & 1+\frac{Z_b}{Z_c} \end{pmatrix}$$

$\det \overrightarrow{\overrightarrow{T}} = 1$, OK!

14/09/1998



$a \neq \emptyset$

? z, y, h, g, ABCD

N.B. gen.re controllato è una rete due porte.

$$\begin{cases} V_1 = R \cdot i_1 \\ i_2 = -aV_1 = -aRi_1 \end{cases}$$

i_1, i_2 sono dip.ti $\Rightarrow \nexists\ \overrightarrow{\overrightarrow{z}}$

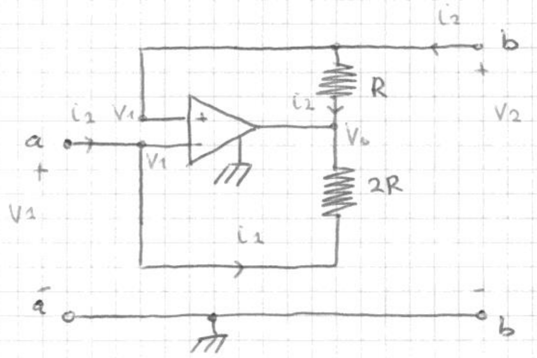
V_1, i_2 sono dip.ti $\Rightarrow \nexists\ g$

$\exists\ \overrightarrow{\overrightarrow{y}}$ $\begin{cases} i_1 = \frac{1}{R} V_1 + \emptyset \cdot V_2 \\ i_2 = -aV_1 - \emptyset \cdot V_2 \end{cases}$ $\overrightarrow{\overrightarrow{y}} = \begin{pmatrix} 1/R & \emptyset \\ -a & \emptyset \end{pmatrix}$

$\exists\ \overrightarrow{\overrightarrow{h}}$ $\begin{cases} V_1 = R i_1 + \emptyset V_2 \\ i_2 = -aRi_1 + \emptyset V_2 \end{cases}$ $\overrightarrow{\overrightarrow{h}} = \begin{pmatrix} R & \emptyset \\ -aR & \emptyset \end{pmatrix}$

$\exists$ ABCD $\begin{cases} V_1 = \emptyset \cdot V_2 + \frac{1}{a}(-i_2) \\ i_1 = \emptyset \cdot V_2 + \frac{1}{aR}(-i_2) \end{cases}$ $\overrightarrow{\overrightarrow{T}} = \begin{pmatrix} \emptyset & 1/a \\ \emptyset & 1/aR \end{pmatrix}$

31/10/2001



? z, y, ABCD.

$V_1 = V_2$

V_1, V_2 dip.ti $\Rightarrow \nexists \, \vec{\vec{y}}$

$V_1 = 2Ri_1 + V_0$

$V_2 = Ri_2 + V_0$

$V_1 = V_2 \quad \rightarrow \quad 2\cancel{R}i_1 + \cancel{V_0} = \cancel{R}i_2 + \cancel{V_0}$

$i_2 = 2i_1$

i_1, i_2 dip.ti $\Rightarrow \nexists \, \vec{\vec{z}}$

$$\begin{cases} V_1 = V_2 \\ i_1 = \frac{1}{2} i_2 \end{cases} \qquad \vec{\vec{T}} = \begin{pmatrix} 1 & \emptyset \\ \emptyset & -\frac{1}{2} \end{pmatrix}$$

$\det \vec{\vec{T}} \neq 1$: Rete non reciproca.

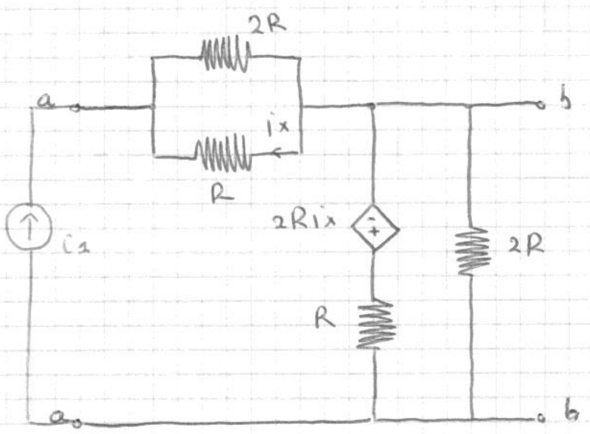
INIC

$$\begin{cases} V_1 = K_1 V_2 \\ i_1 = K_2 i_2 \end{cases} \qquad K_1, K > \emptyset \qquad \begin{cases} V_1 = V_2 \\ i_1 = i_2 \end{cases}$$

Invertitore di corrente

Rende una resistenza $R < \emptyset$.

ES. 29/03/2006



? z_{11}, z_{21}, h_{11}, h_{21} con le def.

$i_x = -\frac{2}{3} i_1$

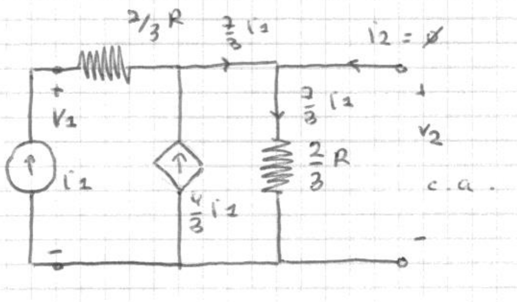
$$z_{11} = \left.\frac{V_1}{i_1}\right|_{i_2=\emptyset}$$

$$z_{21} = \left.\frac{V_2}{i_1}\right|_{i_2=\emptyset}$$

$$h_{11} = \left.\frac{V_1}{i_1}\right|_{V_2=\emptyset}$$

$$h_{21} = \left.\frac{i_2}{i_1}\right|_{V_2=\emptyset}$$

Causa sempre i_1.

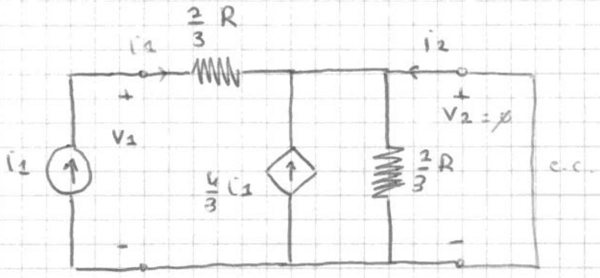


Param z.

$$z_{11} = \left.\frac{V_1}{i_1}\right|_{i_2=\emptyset} = \frac{20}{9} R$$

$$z_{21} = \left.\frac{V_2}{i_1}\right|_{i_2=\emptyset} = \frac{14}{9} R$$

$$V_1 = \frac{2}{3} R \cdot i_1 + \frac{2}{3} R \cdot \frac{7}{3} i_1 = \frac{20}{9} R \cdot i_1$$



Parametri h

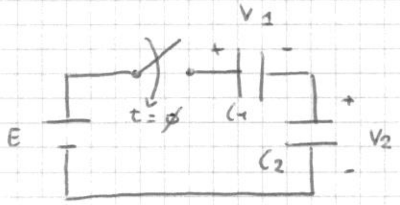
$$h_{11} = \frac{V_1}{i_1}\Big|_{V_2=\emptyset} = \frac{2}{3}R$$

$$h_{21} = \frac{i_2}{i_1}\Big|_{V_2=\emptyset} = -\frac{7}{3}.$$

$$i_1 + \frac{4}{3}i_1 + i_2 = \emptyset$$

PARTE 2

Rete Degenere



$$V_1(\emptyset^-) = V_2(\emptyset^-) = \emptyset$$

$$?\ V_1(\emptyset^+),\ V_2(\emptyset^+)$$

Rete Degenere con maglia di C
$\Rightarrow$ discontinuità var. di stato

$$V_1(\emptyset^+) + V_2(\emptyset^+) = E$$

$$i_1(t) = i_2(t) \qquad t \in [\emptyset^-, \emptyset^+]$$

$$i_1 = C_1 \cdot \frac{d}{dt} V_1(t) = i_2 = C_2 \cdot \frac{d}{dt} V_2(t)$$

$$C_1 \int_{\emptyset^-}^{\emptyset^+} \frac{d}{dt} V_1(t)\, dt = C_2 \int_{\emptyset^-}^{\emptyset^+} V_2(t)\, dt$$

$$C_1\left(V_1(\emptyset^+) - V_1(\emptyset^-)\right) = C_2\left(V_2(\emptyset^+) - V_2(\emptyset^-)\right)$$

$V_1(\emptyset^-) = V_2(\emptyset^-) = \emptyset$ ipotesi iniziale

$$C_1 V_1(\emptyset^+) = C_2 V_2(\emptyset^+)$$

$$\begin{cases} V_1(\emptyset^+) + V_2(\emptyset^+) = E \\ C_1 V_1(\emptyset^+) = C_2 V_2(\emptyset^+) \end{cases}$$

$$\frac{C_2}{C_1} V_2(\emptyset^+) + V_2(\emptyset^+) = E$$

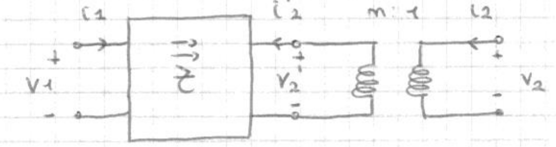
$$V_2(\emptyset^+) = \frac{E}{1 + \frac{C_2}{C_1}} = \frac{E C_1}{C_1 + C_2}$$

$$V_1(\emptyset^+) = \frac{E C_2}{C_1 + C_2}$$

N.B: In questo circuito Tellegen non torna. Potenza erogata da gen. re è doppia di quella accumulata dai C. Causa fenomeni di scintillazione

03/06/2015

Interconnessione di reti due Porte



$$\begin{cases} V'_2 = m V_2 \\ i'_2 = \frac{1}{m} i_2 \end{cases}$$

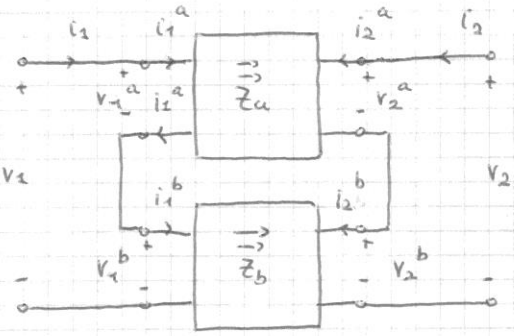
$$\vec{\vec{Z}} = \begin{pmatrix} z_{11} & \frac{z_{12}}{m} \\ \frac{z_{21}}{m} & \frac{z_{22}}{m^2} \end{pmatrix}$$

$$z_{11} = \left.\frac{V_1}{i_1}\right|_{i_2=\emptyset} = \left.\frac{V_1}{i_1}\right|_{i'_2=\emptyset}$$ z_{11} non cambia

$$z_{21} = \left.\frac{V_2}{i_1}\right|_{i_2=\emptyset} = \frac{1}{m}\frac{V'_2}{i_1} = \frac{z_{21}}{m}$$

Da verificare le altre due relazioni.

Connessione in serie fra reti due Porte



$$\begin{cases} i_1 = i_1^a = i_1^b \\ V_1 = V_1^a + V_1^b \end{cases}$$

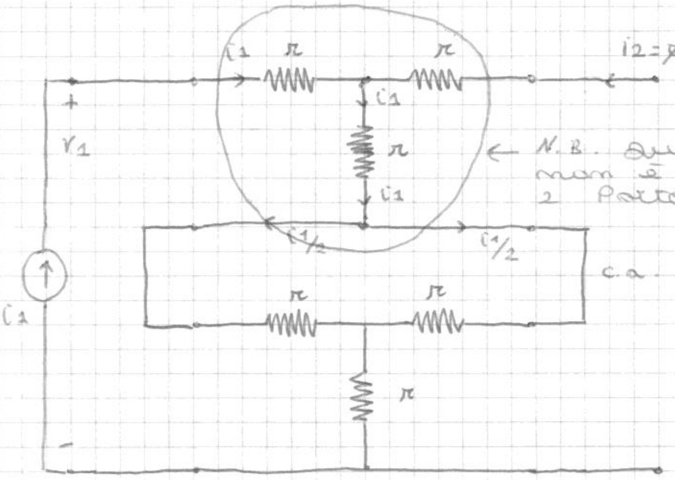
$$\begin{cases} i_2 = i_2^a = i_2^b \\ V_2 = V_2^a + V_2^b \end{cases}$$

$$\vec{\vec{Z}} = \vec{\vec{Z}}_a + \vec{\vec{Z}}_b \quad \text{— DIM.}$$

$$V_1 = V_1^a + V_1^b = z_{11}^a \cdot i_1^a + z_{12}^a \cdot i_2^a + z_{11}^b i_1^b + z_{12}^b \cdot i_2^b$$
$$= \underbrace{(z_{11}^a + z_{11}^b)}_{z_{11}} i_1 + \underbrace{(z_{12}^a + z_{12}^b)}_{z_{12}} i_2$$

Stessa cosa per V_2.

ESERCIZIO TEORICO

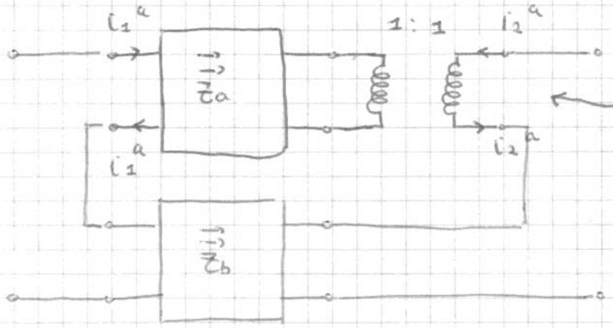


← N.B. Questa non è una rete 2 Porte!

RETE 2 PORTE NON INTRINSECA

$$\vec{\vec{Z}}_a = \begin{pmatrix} 2r & r \\ r & 2r \end{pmatrix}$$

$z_{11} = \frac{7}{2} r$ ed invece dovrebbe venire $2r$.

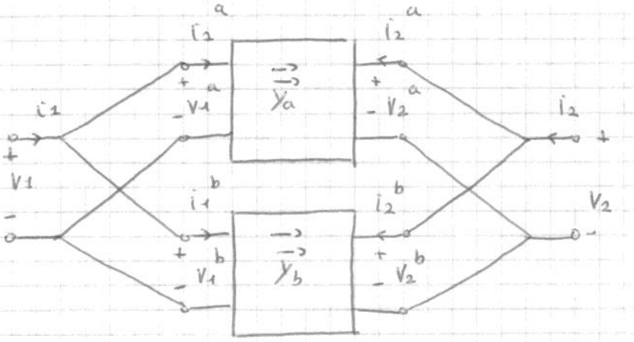


Rete 2 Porte Intrinseca.

↑

Collegamento Serie Valido: $\vec{\vec{Z}} = \vec{\vec{Z}}_a + \vec{\vec{Z}}_b$

Collegamento in Parallelo fra reti 2 Porte



$$\begin{cases} V_1 = V_1^a = V_1^b \\ i_1 = i_1^a + i_1^b \end{cases}$$

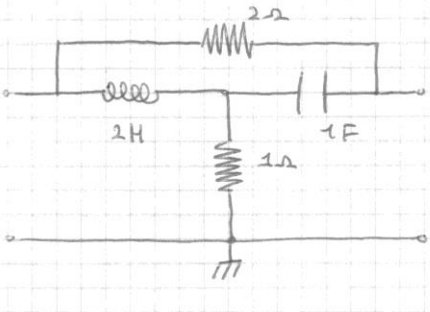
$$\begin{cases} V_2 = V_2^a = V_2^b \\ i_2 = i_2^a + i_2^b \end{cases}$$

$$i_2 = i_2^a + i_2^b = y_{21}^a \cdot V_1^a + y_{22}^a \cdot V_2^a + y_{21}^b \cdot V_1^b + y_{22}^b \cdot V_2^b$$

$$= \underbrace{(y_{21}^a + y_{21}^b)}_{y_{21}} V_1 + \underbrace{(y_{22}^a + y_{22}^b)}_{y_{22}} V_2$$

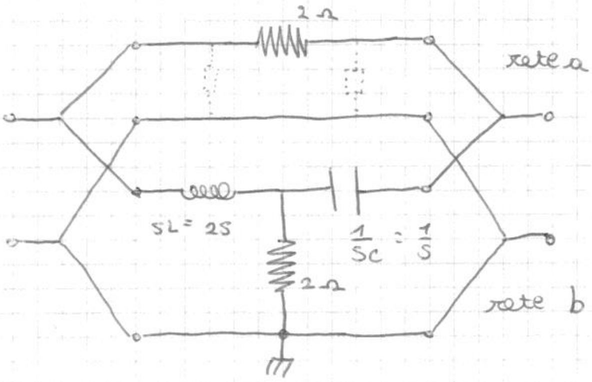
$$\vec{\vec{y}} = \vec{\vec{y}}_a + \vec{\vec{y}}_b$$

ESERCIZIO TEORICO



? $\vec{\vec{y}}$ Rete due porte

Risolta con Laplace scomponendo in due reti in parallelo.

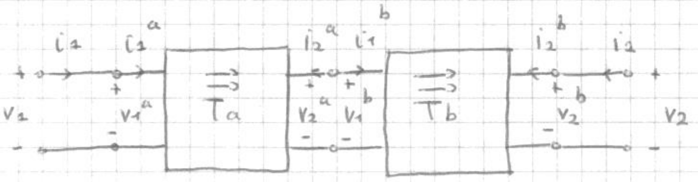


$$\vec{\vec{y}} = \vec{\vec{y}}_a + \vec{\vec{y}}_b$$

$$\vec{\vec{Y}}_a = \begin{pmatrix} 1/2 & -1/2 \\ -1/2 & 1/2 \end{pmatrix}$$

$$\vec{\vec{Z}}_b = \begin{pmatrix} 2s+1 & 1 \\ 1 & \frac{1}{s}+1 \end{pmatrix} \qquad \vec{\vec{Y}}_b = \vec{\vec{Z}}_b^{\,-1} = \frac{\begin{pmatrix} \frac{1}{s}+1 & -1 \\ -1 & 2s+1 \end{pmatrix}}{\det \vec{\vec{Z}}_b}$$

Collegamento in cascata fra reti 2 Porte



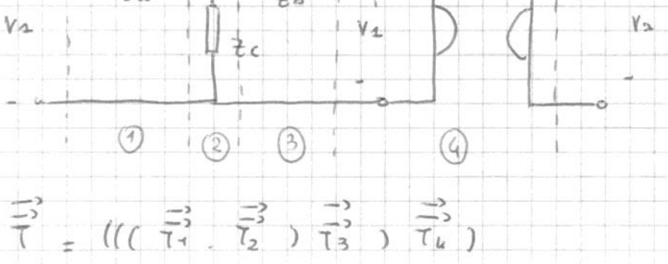
$$\vec{\vec{T}} = \vec{\vec{T}}_a \cdot \vec{\vec{T}}_b$$

$$\begin{cases} v_1 = v_1^a \\ i_1 = i_1^a \end{cases} \qquad \begin{cases} v_2^a = v_1^b \\ i_2^a = -i_1^b \end{cases} \qquad \begin{cases} v_2 = v_2^b \\ i_2 = i_2^b \end{cases}$$

$$\begin{pmatrix} v_1 \\ i_1 \end{pmatrix} = \begin{pmatrix} v_1^a \\ i_1^a \end{pmatrix} = \vec{\vec{T}}_a \begin{pmatrix} v_2^a \\ -i_2^a \end{pmatrix} = \vec{\vec{T}}_a \begin{pmatrix} v_1^b \\ i_1^b \end{pmatrix} = \vec{\vec{T}}_a \cdot \vec{\vec{T}}_b \begin{pmatrix} v_2^b \\ -i_2^b \end{pmatrix}$$

$$= \underbrace{\vec{\vec{T}}_a \cdot \vec{\vec{T}}_b}_{\vec{\vec{T}}} \cdot \begin{pmatrix} v_2 \\ -i_2 \end{pmatrix}$$

ESERCIZIO TEORICO



$? \, \vec{\vec{T}}$

$$\vec{\vec{T}} = (((\vec{\vec{T}}_1 \cdot \vec{\vec{T}}_2)\, \vec{\vec{T}}_3)\, \vec{\vec{T}}_4)$$

$$\vec{\vec{T}}_1 = \begin{pmatrix} 1 & Z_a \\ \emptyset & 1 \end{pmatrix}$$

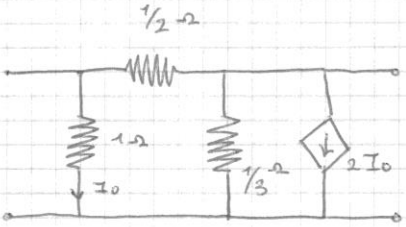
$$\vec{\vec{T}}_3 = \begin{pmatrix} 1 & Z_b \\ \emptyset & 1 \end{pmatrix}$$

$$\vec{\vec{T}}_2 = \begin{pmatrix} 1 & \emptyset \\ \frac{1}{Z_c} & 1 \end{pmatrix}$$

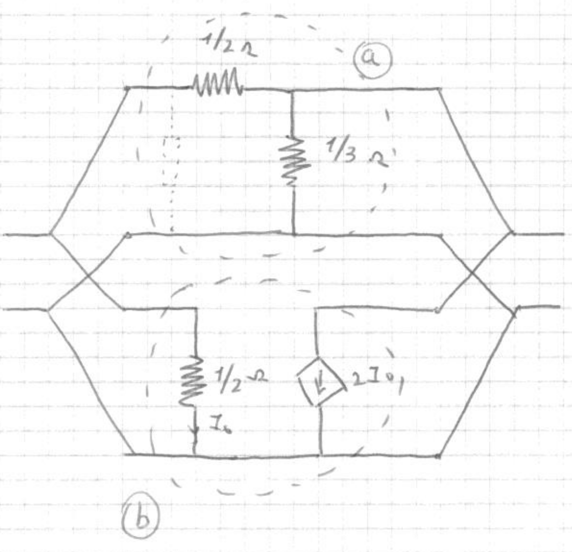
$$\begin{cases} V_1' = -R_0 i_2 \\ V_2 = R_0 i_1' \end{cases} \qquad \begin{cases} V_1' = R_0(-i_2) \\ i_1' = \frac{1}{R_0} V_2 \end{cases}$$

$$\vec{\vec{T}}_4 = \begin{pmatrix} \emptyset & R_0 \\ \frac{1}{R_0} & \emptyset \end{pmatrix} \leftarrow \text{Rete antireciproca}$$

ES.



? $\vec{\vec{y}}$ della rete due porte pensando la rete come parallelo fra due sottoreti.



$$\vec{\vec{y}} = \vec{\vec{y}}_a + \vec{\vec{y}}_b$$

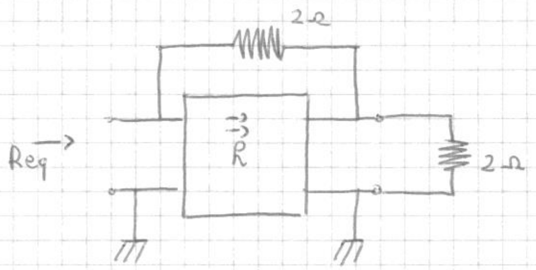
$$\vec{\vec{y}}_a = \begin{pmatrix} 2 & -2 \\ -2 & 5 \end{pmatrix}$$

$$\vec{\vec{y}}_b = \begin{pmatrix} 1 & \emptyset \\ 2 & \emptyset \end{pmatrix}$$

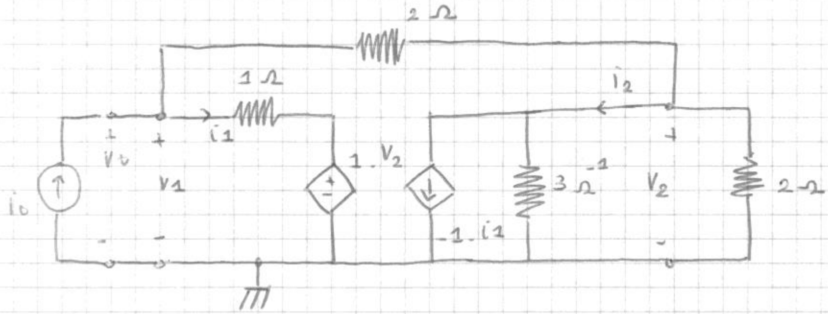
$$\vec{\vec{y}} = \begin{pmatrix} 3 & -2 \\ \emptyset & 5 \end{pmatrix}$$

ES.

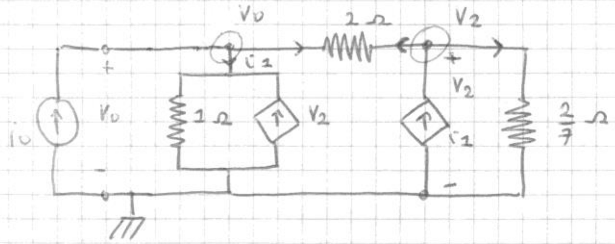
? R_{eq}



$$\vec{\vec{R}} = \begin{pmatrix} 1 & 1 \\ -1 & 3 \end{pmatrix}$$



$$R_{eq} = \frac{V_0}{I_0}$$



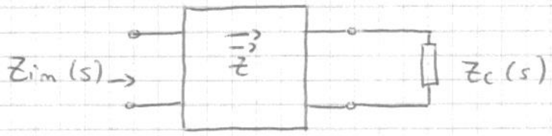
$$\begin{cases} i_0 = i_1 + \dfrac{V_0 - V_2}{2} \\ i_1 = V_2 \cdot \dfrac{7}{2} + \dfrac{V_2 - V_0}{2} \end{cases}$$

$$i_1 = V_0 - 1 \cdot V_2$$

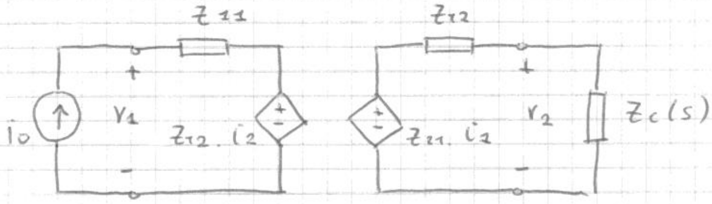
$$\ldots \quad \frac{V_0}{I_0} = \frac{10}{21} \ \Omega .$$

Es.

? $Z_{in}(s)$



$$\overline{\overline{Z}} = \begin{pmatrix} Z_{11} & Z_{12} \\ Z_{21} & Z_{22} \end{pmatrix} \qquad \begin{cases} V_1 = Z_{11} \cdot i_1 + Z_{12} \cdot i_2 \\ V_2 = Z_{21} \cdot i_1 + Z_{22} \cdot i_2 \end{cases}$$



$$Z_{in}(s) = \frac{V_1}{i_0}$$

$$i_1 = i_0$$

$$i_2 = -\frac{Z_{21} \cdot i_0}{Z_{22} + Z_c}$$

$$V_1 = Z_{12} \cdot i_2 + Z_{11} \, i_1 = -\frac{Z_{12} \, Z_{21} \, i_0}{Z_{22} + Z_c} + Z_{11} \cdot i_0$$

$$Z_{in} = \frac{V_1}{i_0} = Z_{11} - \frac{Z_{12} \cdot Z_{21}}{Z_{22} + Z_c}$$